Due Date: November 18, 2005

Problem 1: Let P be a set of n points in $\mathbb{R}^2$. Show that P can be preprocessed in $O(n^2 \log n)$ time into a data structure of size $O(n^2)$ so that the number of points lying below a query line can be counted in $O(\log n)$ time. (**Hint:** *Use duality.*)

Problem 2: Let P be a set of n points in $\mathbb{R}^2$. Show that the number of triples of P that span an isosceles triangle is $O(n^{7/3})$. Show that the number of triples that form a right-angle triangle is also $O(n^{7/3})$. (**Hint:** *Invoke the bound on point-line incidences n times, once for each point.*)

Problem 3: (i) Let $e = pq$ be a segment in $\mathbb{R}^2$ and ℓ a line in $\mathbb{R}^2$. Let e^* be the double wedge defined by the lines p^* and q^* that does not contain the vertical line, where p^*, q^* are the lines dual to p and q. Show that e intersects ℓ if ℓ^* lies in e^*.

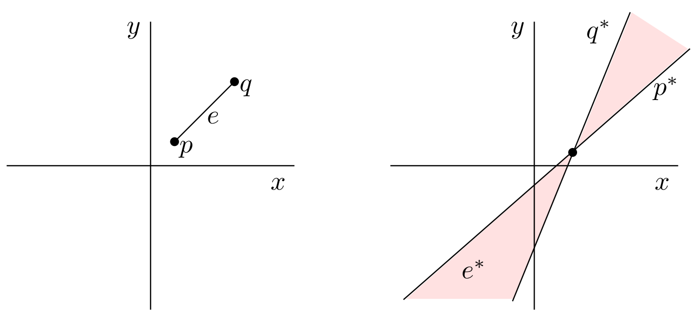

Figure 1: The dual e^* of the edge e

(ii) Let S be a set of n segments in $\mathbb{R}^2$ and L a set of n lines in $\mathbb{R}^2$. Let $\chi(S, L)$ be the number of line-segment pairs in $L \times S$ that intersect. Show that $\chi(L, S)$ can be computed in $O(n^{3/2})$ time. (**Hint:** *Partition S into $\sqrt{n}$ subsets $S_1, \ldots, S_k$ and count $\chi(S_i, L)$ for each i using (i).*)

Extra Credit. Improve the running time to $O(n^{4/3} \log n)$ using $1/r$-cuttings.

Problem 4: (i) Let S be a set of n points and L a set of m lines. Show that there exist two points $p, q \in S$ so that the segment pq crosses $O(m/\sqrt{n})$ lines of L.
(**Hint:** *Use the result on $1/r$-cuttings.*)

(ii) Let S and L be as above. Describe an algorithm for computing a spanning tree of S so that, on average, each line of L intersects $O(\sqrt{n})$ edges of the spanning tree. (**Hint:** *If you define the weight of an edge pq to be the number of lines of L intersecting it, estimate an upper bound on the weight of a minimum spannig tree of S, using (i) repeatedly.*)