One Equilibrium Is Not Enough: Computing Game-Theoretic Solutions to Act Strategically

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<tbody>
<tr>
<td>0, 0</td>
<td>-1, 2</td>
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<td>-1, 1</td>
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<tr>
<td>2, 2</td>
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</tr>
<tr>
<td>-7, -8</td>
<td>0, 0</td>
<td>2, 1</td>
</tr>
</tbody>
</table>

Vincent Conitzer
Duke University

My wonderful co-authors (alphabetically):

Multiple entities with different interests

How can AI help? Prediction markets

Kidney exchanges

Auctions

Rating/voting systems

Donation matching

overview: C.
CACM March 2010

Security

THIS TALK
Closer to home…

Multiagent systems

Game playing

Goal: Blocked(Room0)

Goal: Clean(Room0)
Some microeconomic theory tools for AI

GAME THEORY

A > B > C
B > A > C
C > B > A

\[ \begin{array}{cc}
2, 2 & -1, 0 \\
-7, -8 & 0, 0 \\
\end{array} \]

B wins

SOCIAL CHOICE

THIS TALK

\[ v_1 = 42 \]
\[ v_2 = 30 \]
\[ v_3 = 20 \]

1 wins, pays 30

MECHANISM DESIGN

Josh Letchford

Dima Korzhyk

Lirong Xia

Mingyu Guo

Liad Wagman
Penalty kick example

Is this a "rational" outcome? If not, what is?
Penalty kick
(also known as: matching pennies)
Security example

BCN terminal 2A

BCN terminal 2B

action

action
Security game

<table>
<thead>
<tr>
<th></th>
<th>2A</th>
<th>2B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33</td>
<td>0, 0</td>
<td>-1, 2</td>
</tr>
<tr>
<td>0.67</td>
<td>-1, 1</td>
<td>0, 0</td>
</tr>
</tbody>
</table>
Recent deployments in security

- Tambe’s TEAMCORE group at USC
- Airport security
  - Where should checkpoints, canine units, etc. be deployed?
  - Deployed at LAX and another US airport, being evaluated for deployment at all US airports
- Federal Air Marshals
- Coast Guard
- …
“Should I buy an SUV?”
(also known as the Prisoner’s Dilemma)

<table>
<thead>
<tr>
<th></th>
<th>Purchasing + Gas Cost</th>
<th>Accident Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cost:</strong></td>
<td><strong>Cost:</strong></td>
<td><strong>Cost:</strong></td>
</tr>
<tr>
<td>SUV</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Compact Car</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Computational aspects of dominance: Gilboa, Kalai, Zemel Math of OR ’93; C. & Sandholm EC ’05, AAAI’05; Brandt, Brill, Fischer, Harrenstein TOCS ‘11

<table>
<thead>
<tr>
<th></th>
<th>-10, -10</th>
<th>-7, -11</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cost:</strong></td>
<td><strong>Cost:</strong></td>
<td><strong>Cost:</strong></td>
</tr>
<tr>
<td>SUV</td>
<td>-11, -7</td>
<td>-8, -8</td>
</tr>
<tr>
<td>Compact Car</td>
<td></td>
<td></td>
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</tbody>
</table>
“Chicken”

- Two players drive cars towards each other
- If one player goes straight, that player wins
- If both go straight, they both die

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>D</strong></td>
<td>0, 0</td>
<td>-1, 1</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>1, -1</td>
<td>-5, -5</td>
</tr>
</tbody>
</table>
Nash equilibrium [Nash ‘50]

• A profile (= strategy for each player) so that no player wants to deviate

\[
\begin{array}{cc}
D & S \\
D & 0, 0 & -1, 1 \\
S & 1, -1 & -5, -5 \\
\end{array}
\]

• This game has another Nash equilibrium in mixed strategies – both play D with 80%
The presentation game

<table>
<thead>
<tr>
<th>Pay attention (A)</th>
<th>Do not pay attention (NA)</th>
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</thead>
<tbody>
<tr>
<td>2, 2</td>
<td>-1, 0</td>
</tr>
<tr>
<td>-7, -8</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

- Pure-strategy Nash equilibria: (E, A), (NE, NA)
- Mixed-strategy Nash equilibrium:
  ((4/5 E, 1/5 NE), (1/10 A, 9/10 NA))
  – Utility -7/10 for presenter, 0 for audience
Modeling and representing games

**THIS TALK**
(Unless specified otherwise)

<table>
<thead>
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<th>L</th>
<th>R</th>
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<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>4</td>
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Normal-form games

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<tbody>
<tr>
<td>U</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>4</td>
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Bayesian games

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<tr>
<td>D</td>
<td>4</td>
<td>2</td>
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Extensive-form games

Stochastic games

Action-graph games

Graphical games

[Lei, 2013] [Bhat & Leyton-Brown, UAI’04] [Jiang, Leyton-Brown, Bhat GEB’11]

MAIDs

[Koller & Milch. IJCAI’01/GE’03]
Computing a single Nash equilibrium

“Together with factoring, the complexity of finding a Nash equilibrium is in my opinion the most important concrete open question on the boundary of P today.”

Christos Papadimitriou, *STOC’01*

• PPAD-complete to compute one Nash equilibrium, even in a two-player game [Daskalakis, Goldberg, Papadimitriou *STOC’06*; Chen & Deng *FOCS’06*]
  • still holds for FPTAS / smoothed poly [Chen, Deng, Teng *FOCS’06*]
• Is one Nash equilibrium all we need to know?
A useful reduction (SAT → game)

[C. & Sandholm IJCAI’03, Games and Economic Behavior ‘08]

(Earlier reduction with weaker implications: Gilboa & Zemel GEB ‘89)

Formula: \((x_1 \text{ or } -x_2) \text{ and } (-x_1 \text{ or } x_2)\)

Solutions:
- \(x_1=\text{true}, x_2=\text{true}\)
- \(x_1=\text{false}, x_2=\text{false}\)

Game:

<table>
<thead>
<tr>
<th></th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>+(x_1)</th>
<th>-(x_1)</th>
<th>+(x_2)</th>
<th>-(x_2)</th>
<th>((x_1) or -(x_2))</th>
<th>(-(x_1) or (x_2))</th>
<th>default</th>
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<tr>
<td>(x_1)</td>
<td>-2,-2</td>
<td>-2,-2</td>
<td>0,-2</td>
<td>0,-2</td>
<td>2,-2</td>
<td>2,-2</td>
<td>-2,-2</td>
<td>-2,-2</td>
<td>0,1</td>
</tr>
<tr>
<td>(x_2)</td>
<td>-2,-2</td>
<td>-2,-2</td>
<td>2,-2</td>
<td>2,-2</td>
<td>0,-2</td>
<td>0,-2</td>
<td>-2,-2</td>
<td>-2,-2</td>
<td>0,1</td>
</tr>
<tr>
<td>+(x_1)</td>
<td>-2,0</td>
<td>-2,2</td>
<td>1,1</td>
<td>-2,-2</td>
<td>1,1</td>
<td>1,1</td>
<td>-2,0</td>
<td>-2,2</td>
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</tr>
<tr>
<td>-(x_1)</td>
<td>-2,0</td>
<td>-2,2</td>
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<td>1,1</td>
<td>1,1</td>
<td>1,1</td>
<td>-2,2</td>
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</tr>
<tr>
<td>+(x_2)</td>
<td>-2,2</td>
<td>-2,0</td>
<td>1,1</td>
<td>1,1</td>
<td>1,1</td>
<td>-2,-2</td>
<td>-2,2</td>
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<td>-2,2</td>
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(\(x_1\) or -\(x_2\)) (-\(x_1\) or \(x_2\))

- Every satisfying assignment (if there are any) corresponds to an equilibrium with utilities 1, 1
- Exactly one additional equilibrium with utilities \(\varepsilon, \varepsilon\) that always exists
Some algorithm families for computing Nash equilibria of 2-player normal-form games

- for both $i$, for any $s_i \in S_i - X_i$, $p_i(s_i) = 0$
- for both $i$, for any $s_i \in X_i$, $\sum p_i(s_i)u_i(s_i, s_{-i}) = u_i$
- for both $i$, for any $s_i \in S_i - X_i$, $\sum p_i(s_i)u_i(s_i, s_{-i}) \leq u_i$

**Lemke-Howson** [J. SIAM ‘64]

Exponential time due to Savani & von Stengel [FOCS’04 / Econometrica’06]

Search over supports / MIP

[Dickhaut & Kaplan, Mathematica J. ‘91]
[Porter, Nudelman, Shoham AAAI’04 / GEB’08]
[Sandholm, Gilpin, C. AAAI’05]

Special cases / subroutines

[C. & Sandholm AAAI’05, AAMAS’06; Benisch, Davis, Sandholm AAAI’06 / JAIR’10; Kontogiannis & Spirakis APPROX’11; Adsul, Garg, Mehta, Sohoni STOC’11; …]

Approximate equilibria

[Brown ’51 / C. ’09 / Goldberg, Savani, Sørensen, Vente ’11; Althöfer ’94, Lipton, Markakis, Mehta ’03, Daskalakis, Mehta, Papadimitriou ’06, ’07, Feder, Nazerzadeh, Saberi ’07, Tsaknakis & Spirakis ‘07, Spirakis ‘08, Bosse, Byrka, Markakis ‘07, …]
Sidestepping the problems

(one solution concept is not enough...?)
Nash is not optimal if one player can commit

Unique Nash equilibrium

Suppose the game is played as follows:
- Player 1 commits to playing one of the rows,
- Player 2 observes the commitment and then chooses a column

Optimal strategy for player 1: commit to Down
Commitment to mixed strategies

- Sometimes also called a Stackelberg (mixed) strategy

\[
\begin{array}{cc}
0 & 1 \\
.49 & 1, 1 & 3, 0 \\
.51 & 0, 0 & 2, 1 \\
\end{array}
\]
Observing the defender’s distribution in security

This argument is not uncontroversial… [Pita, Jain, Tambe, Ordóñez, Kraus AIJ’10; Korzhyk, Yin, Kiekintveld, C., Tambe JAIR’11; Korzhyk, C., Parr AAMAS’11]
Computing the optimal mixed strategy to commit to

[C. & Sandholm EC’06, von Stengel & Zamir GEB’10]

• Separate LP for every column \( c^* \):

\[
\begin{align*}
\text{maximize} & \quad \sum_r p_r u_R(r, c^*) \\
\text{subject to} & \\
\text{for all } c, & \quad \sum_r p_r u_C(r, c) \leq \sum_r p_r u_C(r, c^*) \\
\sum_r p_r & = 1
\end{align*}
\]

\text{leader utility}

\text{follower optimality}

\text{distributional constraint}
Other nice properties of commitment to mixed strategies

- Agrees w. Nash in zero-sum games

- Leader’s payoff at least as good as any Nash eq. or even correlated eq. (von Stengel & Zamir [GEB ‘10]; see also C. & Korzhyyk [AAAI ‘11], Letchford & C. [draft])

- No equilibrium selection problem
Some other work on commitment in unrestricted games

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normal-form games

learning to commit [Letchford, C., Munagala SAGT’09]
uncertain observability [Korzhyk, C., Parr AAMAS’11]
correlated strategies [C. & Korzhyk, AAAI’11]

extensive-form games

commitment in Bayesian games

[C. & Sandholm EC’06; Paruchuri, Pearce, Marecki, Tambe, Ordóñez, Kraus AAMAS’08; Letchford, C., Munagala SAGT’09; Pita, Jain, Tambe, Ordóñez, Kraus AIJ’10; Jain, Kiekintveld, Tambe AAMAS’11]

stochastic games

ongoing work with Korzhyk, Letchford, Parr
Security resource allocation games

[Kiekintveld, Jain, Tsai, Pita, Ordóñez, Tambe AAMAS’09]

• Set of targets $T$
• Set of security resources $\Omega$ available to the defender (leader)
• Set of schedules $S \subseteq 2^T$
• Resource $\omega$ can be assigned to one of the schedules in $A(\omega) \subseteq S$
• Attacker (follower) chooses one target to attack
• Utilities: $U_d^c(t), U_a^c(t)$ if the attacked target is defended,
  $U_d^u(t), U_a^u(t)$ otherwise
• $U_d^c(t) \geq U_d^u(t); U_a^c(t) \leq U_a^u(t)$

\[ \begin{align*}
\omega_1 & \quad t_1 \\
\omega_2 & \quad t_2, t_3 \\
& \quad t_4, t_5
\end{align*} \]
Game-theoretic properties of security resource allocation games [Korzhyk, Yin, Kiekintveld, C., Tambe JAIR’11]

• For the defender:
  Stackelberg strategies are also Nash strategies
  – minor assumption needed
  – not true with multiple attacks

• Interchangeability property for Nash equilibria (“solvable”)
  • no equilibrium selection problem
  • still true with multiple attacks

[Korzhyk, C., Parr IJCAI’11 – poster W. 3:30pm, talk F. 10:30am]
Scalability in security games

basic model

[Kiekintveld, Jain, Tsai, Pita, Ordóñez, Tambe AAMAS’09; Korzhyk, C., Parr, AAAI’10; Jain, Kardeš, Kiekintveld, Ordóñez, Tambe AAAI’10; Korzhyk, C., Parr, IJCAI’11]

games on graphs (usually zero-sum)

[Halvorson, C., Parr IJCAI’09; Tsai, Yin, Kwak, Kempe, Kiekintveld, Tambe AAMAS’11; Jain, Korzhyk, Vaněk, C., Pěchouček, Tambe AAMAS’11; ongoing work with Letchford, Vorobeychik]

Techniques:

compact linear/integer programs

Maximize $U_d^c(t^*) \sum_{a} \sum_{s_{t^*} \in s_{t^*}} c_{a,s} + U_d^u(t^*) \left( 1 - \sum_{a} \sum_{s_{t^*} \in s_{t^*}} c_{a,s} \right)$

Subject to

$\forall a: \sum_{s} c_{a,s} \leq 1$

$\forall t: \sum_{a} \sum_{s \in s_{t^*}} c_{a,s} \leq 1$

$\forall t: U_d^c(t) \sum_{a} \sum_{s \in s_{t^*}} c_{a,s} + U_d^u(t) \left( 1 - \sum_{a} \sum_{s \in s_{t^*}} c_{a,s} \right) \leq U_d^c(t^*) \sum_{a} \sum_{s_{t^*} \in s_{t^*}} c_{a,s} + U_d^u(t^*) \left( 1 - \sum_{a} \sum_{s_{t^*} \in s_{t^*}} c_{a,s} \right)$

Defender utility

Marginal probability of $t^*$ being defended (?)

Distributional constraints

Attacker optimality

strategy generation

$\sigma_h(\theta_h) + u \geq \sigma_h(\theta_h) \cdot u(\theta_h, \theta_h)$

$\sigma_h(\theta_h) \cdot u(\theta_h, \theta_h) + u \geq \sigma_h(\theta_h) \cdot u(\theta_h, \theta_h) + \sigma_h(\theta_h) \cdot u(\theta_h, \theta_h)$

$\vdots$

$u \geq \sigma_h(\theta_h) \cdot u(\theta_h, \theta_h) + \cdots + \sigma_h(\theta_h) \cdot u(\theta_h, \theta_h)$

$= 1$
In summary: AI pushing at some of the boundaries of game theory

AI work in game theory

Learning in games

Game theory

Computation

Representation

Behavioral (humans playing games)

Conceptual (e.g., equilibrium selection)
Funding

Any opinions, conclusions or recommendations are mine and do not necessarily reflect the views of the funding agencies.
Academic family

Tuomas Sandholm (CMU)

Mike Benisch
Andrew Gilpin

Vic Lesser (UMass)
Barbara Grosz
Avi Pfeffer

Kate Larson (Waterloo)

Dima Korzhyk (Ph.D. student)
Josh Letchford (Ph.D. student)
Lirong Xia (Ph.D. student, starting postdoc at Harvard)
Mingyu Guo (Ph.D. 2010, now at U. Liverpool)
Liad Wagman (Ph.D. 2009, now at Illinois Inst. of Tech.)