Mechanism Design with Correlated Distributions
Mechanism Design with Correlated Distributions

\[ \nu(\theta) \]

\[ \pi(\theta|\theta) \]
Mechanism Design with Correlated Distributions

\[ v(\theta) \]

\[ \pi(\theta | \theta) \]
Mechanism Design with Correlated Distributions
Mechanism Design with Correlated Distributions
Mechanism Design with Correlated Distributions

\[ \nu(\theta) \]

\[ \pi(\theta | \theta) \]
Mechanism Design with Correlated Distributions
Mechanism Design with Correlated Distributions
Necessary and Sufficient Condition

\[ v(\theta) \]

\[ \pi(\theta | \theta) \]
Necessary and Sufficient Condition
Necessary and Sufficient Condition
Necessary and Sufficient Condition
Necessary and Sufficient Condition
Theorem: Full Surplus Extraction with a Bayesian Mechanism (AAAI 2016)

For a given \((\pi, \Theta, \Omega)\), full surplus extraction is possible for a Bayesian mechanism if and only if there exists a concave function \(G : \mathbb{R}^{|\Omega|} \to \mathbb{R}\) such that \(G(\pi(\bullet | \theta)) = \nu(\theta)\).
What if we have access to samples and there is “sufficient” correlation?
What if we have access to samples and there is “sufficient” correlation?

Theorem: Learning is Impossible (AAMAS 17)

For any finite number of samples, there exists a distribution for which the optimal learned mechanism is no better than the ex-post mechanism.
Consistent Distributions

\[ v(\theta) \]

\[ \pi(\theta|\theta) \]
Consistent Distributions

\[ \pi(\theta | \theta) \]
Consistent Distributions
Consistent Distributions

\[ v(\theta) \]

\[ \pi(\theta | \theta) \]
Consistent Distributions
Consistent Distributions

Definition: Set of Consistent Distributions

A set of distributions, \( \mathcal{P}(\hat{\pi}) \), is a consistent set of distributions for the estimated distribution \( \hat{\pi} \) if the true distribution, \( \pi \), is guaranteed to be in \( \mathcal{P}(\hat{\pi}) \) and \( \hat{\pi} \in \mathcal{P}(\hat{\pi}) \).
Robust Mechanism Design

The diagram shows a scatter plot with two axes: $v(\theta)$ on the vertical axis and $\pi(\text{ debunk } | \theta)$ on the horizontal axis. There are two points marked on the plot: one at approximately $(0.2, 1)$ and another at $(0.8, 5)$. The axes range from $-2$ to $6$ for both $v(\theta)$ and $\pi(\text{ debunk } | \theta)$. The plot suggests that as $\theta$ increases, $v(\theta)$ decreases, and $\pi(\text{ debunk } | \theta)$ increases.
Robust Mechanism Design

The diagram illustrates the relationship between $\nu(\theta)$ and $\pi(\theta | \theta)$. The x-axis represents $\pi(\theta | \theta)$ with values ranging from 0 to 1, and the y-axis represents $\nu(\theta)$ with values ranging from -2 to 6.
Robust Mechanism Design

The diagram shows a graph with the x-axis labeled \( \pi(\theta|\theta) \) and the y-axis labeled \( v(\theta) \). The graph includes a line and two points marked with circles. The line passes through the origin and extends to the right, indicating a linear relationship between the variables.
Robust Mechanism Design
Robust Mechanism Design

![Graph showing robust mechanism design with axes labeled as $v(\theta)$ and $\pi(\theta | \theta)$]
Robust Mechanism Design
Linear Program for Robust Mechanisms

\[
\max_{x(\theta, \omega), p(\theta, \omega)} \sum_{\theta, \omega} \hat{\pi}(\theta, \omega)x(\theta, \omega)
\]

subject to **Robust Individual Rationality (IR):**
\[
\sum_{\omega \in \Omega} \pi(\omega | \theta) U(\theta, \theta, \omega) \geq 0 \quad \forall \theta \in \Theta, \omega \in \Omega, \pi \in \mathcal{P}(\hat{\pi})
\]

and subject to **Robust Incentive Compatibility (IC):**
\[
\sum_{\omega \in \Omega} \pi(\omega | \theta) U(\theta, \theta, \omega) \geq \sum_{\omega \in \Omega} \pi(\omega | \theta) U(\theta, \theta', \omega)
\quad \forall \theta, \theta' \in \Theta, \omega \in \Omega, \pi \in \mathcal{P}(\hat{\pi})
\]

and subject to an allocation constraint:
\[
0 \leq p(\theta, \omega) \leq 1 \quad \forall \theta \in \Theta, \omega \in \Omega
\]
Linear Program for Robust Mechanisms

\[
\max_{x(\theta, \omega), \rho(\theta, \omega)} \sum_{\theta, \omega} \hat{\pi}(\theta, \omega)x(\theta, \omega)
\]

subject to Robust Individual Rationality (IR):

\[
\sum_{\omega \in \Omega} \pi(\omega | \theta) U(\theta, \theta, \omega) \geq 0 \quad \forall \theta \in \Theta, \omega \in \Omega, \pi \in \mathcal{P}(\hat{\pi})
\]

and subject to Robust Incentive Compatibility (IC):

\[
\sum_{\omega \in \Omega} \pi(\omega | \theta) U(\theta, \theta, \omega) \geq \sum_{\omega \in \Omega} \pi(\omega | \theta) U(\theta, \theta', \omega)
\]

\[
\forall \theta, \theta' \in \Theta, \omega \in \Omega, \pi \in \mathcal{P}(\hat{\pi})
\]

and subject to an allocation constraint:

\[
0 \leq \rho(\theta, \omega) \leq 1 \quad \forall \theta \in \Theta, \omega \in \Omega
\]
Linear Program for Robust Mechanisms

\[
\max_{x(\theta, \omega), p(\theta, \omega)} \sum_{\theta, \omega} \hat{\pi}(\theta, \omega)x(\theta, \omega)
\]

subject to **Robust Individual Rationality (IR):**

\[
\sum_{\omega \in \Omega} \pi(\omega|\theta)U(\theta, \theta, \omega) \geq 0 \quad \forall \theta \in \Theta, \omega \in \Omega, \pi \in \mathcal{P}(\hat{\pi})
\]

and subject to **Robust Incentive Compatibility (IC):**

\[
\sum_{\omega \in \Omega} \pi(\omega|\theta)U(\theta, \theta, \omega) \geq \sum_{\omega \in \Omega} \pi(\omega|\theta)U(\theta, \theta', \omega) \quad \forall \theta, \theta' \in \Theta, \omega \in \Omega, \pi \in \mathcal{P}(\hat{\pi})
\]

and subject to an allocation constraint:

\[
0 \leq p(\theta, \omega) \leq 1 \quad \forall \theta \in \Theta, \omega \in \Omega
\]
Robust Mechanism Design

$v(\theta)$

$\pi(\theta|\theta)$
Robust Mechanism Design

\[ \nu(\theta) \]

\[ \pi(\theta | \theta) \]
Robust Mechanism Design
Robust Mechanism Design

\[ v(\theta) \]

\[ \pi(\text{ain foot}|\theta) \]
$\epsilon$-Robust Mechanism Design

Robust is not sufficient

- All results and intuition for robust mechanism design carries over to restricted $\epsilon$-robust mechanism design
Robust is not sufficient

Definition: Set of $\varepsilon$-Consistent Distributions

A set of distributions, $\mathcal{P}_\varepsilon(\hat{\pi})$, is an $\varepsilon$-consistent set of distributions for the estimated distribution $\hat{\pi}$ if the true distribution, $\pi$, is in $\mathcal{P}_\varepsilon(\hat{\pi})$ with probability $1 - \varepsilon$ and $\hat{\pi} \in \mathcal{P}_\varepsilon(\hat{\pi})$.
Robust is not sufficient

Definition: Set of $\epsilon$-Consistent Distributions

A set of distributions, $\mathcal{P}_\epsilon(\hat{\pi})$, is an $\epsilon$-consistent set of distributions for the estimated distribution $\hat{\pi}$ if the true distribution, $\pi$, is in $\mathcal{P}_\epsilon(\hat{\pi})$ with probability $1 - \epsilon$ and $\hat{\pi} \in \mathcal{P}_\epsilon(\hat{\pi})$.

- All results and intuition for robust mechanism design carries over to restricted $\epsilon$-robust mechanism design
Parameterized Bayesian IC and IR with $\varepsilon$

\[ \varepsilon \approx .05 \]
Parameterized Bayesian IC and IR with $\epsilon$

$\epsilon \approx .05$
Parameterized Bayesian IC and IR with $\epsilon$

\[ \epsilon \approx 0.05 \]
Parameterized Bayesian IC and IR with $\epsilon$

$\epsilon \approx 0.05$
Parameterized Bayesian IC and IR with $\epsilon$

$\epsilon \approx 1$

$\pi(\theta | \theta)$

$v(\theta)$
Parameterized Bayesian IC and IR with $\varepsilon$

$$\varepsilon \approx 1$$
Parameterized Bayesian IC and IR with $\epsilon$

$\epsilon \approx 0$

$v(\theta)$

$\pi(\theta | \theta)$
Parameterized Bayesian IC and IR with $\epsilon$

\[ \epsilon \approx 0 \]
Revenue Guarantee for $\varepsilon$-Robust Mechanism Design

**How do $\varepsilon$-robust mechanisms perform?**

- By our impossibility result, may perform arbitrarily badly
- We require that beliefs by $\gamma$-separated
Revenue Guarantee for $\epsilon$-Robust Mechanism Design

How do $\epsilon$-robust mechanisms perform?

- By our impossibility result, may perform arbitrarily badly
- We require that beliefs by $\gamma$-separated
Revenue Guarantee for $\epsilon$-Robust Mechanism Design

*How do $\epsilon$-robust mechanisms perform?*

- By our impossibility result, may perform arbitrarily badly
- We require that beliefs by $\gamma$-separated
How do $\epsilon$-robust mechanisms perform?

- By our impossibility result, may perform arbitrarily badly
- We require that beliefs by $\gamma$-separated

\[
\pi(\omega_H) = 1
\]

\[
\pi(\omega_L) = 1
\]

\[
\pi(\omega_M) = 1
\]
Revenue Guarantee for $\varepsilon$-Robust Mechanism Design

How do $\varepsilon$-robust mechanisms perform?

- By our impossibility result, may perform arbitrarily badly
- We require that beliefs by $\gamma$-separated

\[
\pi(\omega_H) = 1
\]

\[
\pi(\omega_L) = 1 \quad \pi(\omega_M) = 1
\]
How do $\epsilon$-robust mechanisms perform?

- By our impossibility result, may perform arbitrarily badly
- We require that beliefs by $\gamma$-separated

$$\pi(\omega_H) = 1$$

$$\pi(\omega_L) = 1$$

$$\pi(\omega_M) = 1$$
How do $\varepsilon$-robust mechanisms perform?

- By our impossibility result, may perform arbitrarily badly
- We require that beliefs by $\gamma$-separated

\[ \pi(\omega_H) = 1 \]

\[ \pi(\omega_L) = 1 \]

\[ \pi(\omega_M) = 1 \]
Revenue Guarantee for $\epsilon$-Robust Mechanism Design

How do $\epsilon$-robust mechanisms perform?

- By our impossibility result, may perform arbitrarily badly
- We require that beliefs by $\gamma$-separated

\[ \pi(\omega_H) = 1 \]

\[ \pi(\omega_L) = 1 \]

\[ \pi(\omega_M) = 1 \]
Revenue Guarantee for $\varepsilon$-Robust Mechanism Design

How do $\varepsilon$-robust mechanisms perform?

- By our impossibility result, may perform arbitrarily badly
- We require that beliefs by $\gamma$-separated

\[ \pi(\omega_H) = 1 \]

\[ \pi(\omega_L) = 1 \]

\[ \pi(\omega_M) = 1 \]
Revenue Guarantee for $\varepsilon$-Robust Mechanism Design

**How do $\varepsilon$-robust mechanisms perform?**

- By our impossibility result, may perform arbitrarily badly
- We require that beliefs by $\gamma$-separated

---

**Sample Complexity of $\varepsilon$-Robust Mechanism Design**

The sample complexity for constructing an $\varepsilon$-robust mechanism that is an additive $k$-approximation to the optimal mechanism is $O(\text{poly}(\frac{1}{k}, \frac{1}{\gamma}, |\Theta|, |\Omega|, v(|\Theta|)))$. (Proof)

---

\[ \pi(\omega_L) = 1 \quad \pi(\omega_M) = 1 \]
Simulations

- True distribution is discretized bivariate normal distribution
- Sample from the true distribution $N$ times
- Use Bayesian methods to estimate the distribution
- Calculate empirical confidence intervals for elements of the distribution
- Parameters unless otherwise specified:
  - Correlation = .5
  - $\epsilon = .05$
  - $\Theta = \{1, 2, \ldots, 10\}$
  - $|\Omega| = 10$
  - $v(\theta) = \theta$