New Directions in Automated Mechanism Design

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Hanrui Zhang (Duke)
Yu Cheng (Duke → IAS → UIC)
Mechanism design

Make decisions based on the preferences (or other information) of one or more agents (as in social choice)

Focus on strategic (game-theoretic) agents with privately held information; have to be incentivized to reveal it truthfully

Popular approach in design of auctions, matching mechanisms, …
Sealed-bid auctions
(on a single item)

Bidder $i$ determines how much the item is worth to her ($v_i$)
Writes a bid ($v'_i$) on a piece of paper
How would you bid? How much would I make?
**First price:** Highest bid wins, pays bid
**Second price:** Highest bid wins, pays next-highest bid
**First price with reserve:** Highest bid wins iff it exceeds $r$, pays bid
**Second price with reserve:** Highest bid wins iff it exceeds $r$, pays next highest bid or $r$ (whichever is higher)
Revelation Principle

Anything you can achieve, you can also achieve with a truthful (AKA incentive compatible) mechanism.
Revelation Principle

Anything you can achieve, you can also achieve with a truthful (AKA incentive compatible) mechanism.

[Diagram showing a software agent taking action 4, leading to the original mechanism, which then leads to a new mechanism. The new mechanism reports type B, which is accepted.]
Automated mechanism design input

**Instance** is given by

- Set of possible *outcomes*
- Set of *agents*
  - For each agent
  - set of possible *types*
  - *probability distribution* over these types

**Objective function**

Gives a value for each outcome for each combination of agents’ types

E.g., social welfare, revenue

**Restrictions** on the mechanism

- Are payments allowed?
- Is randomization over outcomes allowed?

What versions of *incentive compatibility (IC)* & *individual rationality (IR)* are used?
How hard is designing an optimal deterministic mechanism (without reporting costs)?

[C. & Sandholm UAI’02, ICEC’03, EC’04]

<table>
<thead>
<tr>
<th>NP-complete (even with 1 reporting agent):</th>
<th>Solvable in polynomial time (for any constant number of agents):</th>
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<tbody>
<tr>
<td>1. Maximizing social welfare (no payments)</td>
<td>1. Maximizing social welfare (not regarding the payments) (VCG)</td>
</tr>
<tr>
<td>2. Designer’s own utility over outcomes (no payments)</td>
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<tr>
<td>3. General (linear) objective that doesn’t regard payments</td>
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<tr>
<td>4. Expected revenue</td>
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1 and 3 hold even with no IR constraints
Positive results (randomized mechanisms)

[C. & Sandholm UAI’02, ICEC’03, EC’04]

- Use linear programming
- Variables:
  \[ p(o | \theta_1, \ldots, \theta_n) = \text{probability that outcome } o \text{ is chosen given types } \theta_1, \ldots, \theta_n \]
  (maybe) \[ \pi_i(\theta_1, \ldots, \theta_n) = i\text{'s payment given types } \theta_1, \ldots, \theta_n \]
- Strategy-proofness constraints: for all \( i, \theta_1, \ldots, \theta_n, \theta_i' \):
  \[ \sum_o p(o | \theta_1, \ldots, \theta_n)u_i(\theta_i, o) + \pi_i(\theta_1, \ldots, \theta_n) \geq \sum_o p(o | \theta_1, \ldots, \theta_i', \ldots, \theta_n)u_i(\theta_i, o) + \pi_i(\theta_1, \ldots, \theta_i', \ldots, \theta_n) \]
- Individual-rationality constraints: for all \( i, \theta_1, \ldots, \theta_n \):
  \[ \sum_o p(o | \theta_1, \ldots, \theta_n)u_i(\theta_i, o) + \pi_i(\theta_1, \ldots, \theta_n) \geq 0 \]
- Objective (e.g., sum of utilities)
  \[ \sum_{\theta_1, \ldots, \theta_n} p(\theta_1, \ldots, \theta_n) \sum_i (\sum_o p(o | \theta_1, \ldots, \theta_n)u_i(\theta_i, o) + \pi_i(\theta_1, \ldots, \theta_n)) \]
- Also works for BNE incentive compatibility, ex-interim individual rationality notions, other objectives, etc.
- For deterministic mechanisms, can still use mixed integer programming: require probabilities in \( \{0, 1\} \)
  – Remember typically designing the optimal deterministic mechanism is NP-hard
A simple example

One item for sale (free disposal)
2 agents, IID valuations: uniform over \{1, 2\}
Maximize expected revenue under ex-interim IR, Bayes-Nash equilibrium
How much can we get?
(What is optimal expected welfare?)

Status:     OPTIMAL
Objective:  obj = 1.5  (MAXimum)
[nonzero variables:]
p_{t\_1\_1\_o3}    1    (probability of disposal for (1, 1))
p_{t\_2\_1\_o1}    1    (probability 1 gets the item for (2, 1))
p_{t\_1\_2\_o2}    1    (probability 2 gets the item for (1, 2))
p_{t\_2\_2\_o2}    1    (probability 2 gets the item for (2, 2))
pi_{2\_2\_1}    2    (1’s payment for (2, 2))
pi_{2\_2\_2}    4    (2’s payment for (2, 2))


<table>
<thead>
<tr>
<th>Agent 1’s valuation</th>
<th>Agent 2’s valuation</th>
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<tbody>
<tr>
<td>1</td>
<td>0.25 0.25</td>
</tr>
<tr>
<td>2</td>
<td>0.25 0.25</td>
</tr>
</tbody>
</table>

probabilities
A slightly different distribution

One item for sale (free disposal)
2 agents, valuations drawn as on right
Maximize expected revenue under ex-interim IR, Bayes-Nash equilibrium
How much can we get? 
(What is optimal expected welfare?)

| Status:     | OPTIMAL                  |
| Objective:  | obj = 1.749 (MAXimum)   |
| [some of the nonzero payment variables:] | |
| \( \pi_{1 \_1 \_2} \)     | 62501                    |
| \( \pi_{2 \_1 \_2} \)     | -62750                   |
| \( \pi_{2 \_1 \_1} \)     | 2                        |
| \( \pi_{1 \_2 \_2} \)     | 3.992                    |

Agent 1's valuation

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
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<tbody>
<tr>
<td>0.251</td>
<td>0.250</td>
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Agent 2's valuation

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
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<tbody>
<tr>
<td>0.250</td>
<td>0.249</td>
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</table>

You’d better be really sure about your distribution!
A nearby distribution without correlation

One item for sale (free disposal)
2 agents, valuations IID: 1 w/ .501, 2 w/ .499
Maximize expected revenue under ex-interim IR, Bayes-Nash equilibrium
How much can we get?
(What is optimal expected welfare?)

Status: OPTIMAL
Objective: obj = 1.499 (MAXimum)
Cremer-McLean [1985]

For every agent, consider the following matrix $\Gamma$ of conditional probabilities, where $\Theta$ is the set of types for the agent and $\Omega$ is the set of signals (joint types for other agents, or something else observable to the auctioneer)

$$
\Gamma = \begin{bmatrix}
\pi(1|1) & \cdots & \pi(|\Omega||1)
\\
\vdots & \ddots & \vdots
\\
\pi(1||\Theta|) & \cdots & \pi(|\Omega|||\Theta|)
\end{bmatrix}
$$

If $\Gamma$ has rank $|\Theta|$ for every agent then the auctioneer can allocate efficiently and extract the full surplus as revenue (!!)
Standard setup in mechanism design

1. Designer has beliefs about agent’s type (e.g., preferences)
   - 40%: \( v = 10 \)
   - 60%: \( v = 20 \)

2. Designer announces mechanism (typically mapping from reported types to outcomes)
   - \( v = 20 \) →

3. Agent strategically acts in mechanism (typically type report), however she likes at no cost
   - \( v = 20 \)

4. Mechanism functions as specified
The mechanism may have more information about the specific agent!

**application**
- online marketplaces
- selling insurance
- university admissions
- webpage ranking

**information**
- actions taken online
- driving record
- courses taken
- links to page
(0) Agent acts in the world (naively?)

(1) Designer obtains beliefs about agent’s type (e.g., preferences)

(2) Designer announces mechanism (typically mapping from reported types to outcomes)

(3) Agent strategically acts in mechanism (typically type report), however she likes at no cost

(4) Mechanism functions as specified
Attempt 2: Sophisticated agent

(1) Designer has prior beliefs about agent’s type (e.g., preferences)

(2) Designer announces mechanism (typically mapping from reported types to outcomes)

(3) Agent strategically takes possibly costly actions

(4) Mechanism functions as specified

40%: $v = 10$
60%: $v = 20$

Show me pictures of cats
$v = 20$
See also later work by Hardt, Megiddo, Papadimitriou, Wootters [2015/2016]
From Ancient Times…

Jacob and Esau

Trojan Horse
… to Modern Times
Illustration: Barbara Buying Fish From Fred

<table>
<thead>
<tr>
<th>Types ($t \in T$)</th>
<th>Actions ($a \in A$)</th>
<th>Choice Function ($F : T \rightarrow A$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>fresh</td>
<td>accept</td>
<td>fresh $\rightarrow$ accept</td>
</tr>
<tr>
<td>ok</td>
<td>reject</td>
<td>ok $\rightarrow$ accept</td>
</tr>
<tr>
<td>rotten</td>
<td></td>
<td>rotten $\rightarrow$ reject</td>
</tr>
</tbody>
</table>

Classifications ($t \in \hat{T}$): fresh, ok, rotten
First Try: \( M = \text{fresh} \rightarrow \text{accept}, \ \text{ok} \rightarrow \text{accept}, \ \text{rotten} \rightarrow \text{reject} \)
Effort Function $(E : T \times \hat{T} \to \mathbb{R})$: 

<table>
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<th></th>
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<th>ok</th>
<th>rotten</th>
</tr>
</thead>
<tbody>
<tr>
<td>fresh</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ok</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>rotten</td>
<td>30</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

Valuation Function $(V : T \times A \to \mathbb{R})$:

$V(\cdot, \text{accept}) = 20$, $V(\cdot, \text{reject}) = 0$

Mechanism $M : \hat{T} \to A$

First Try: $M = \text{fresh} \to \text{accept}, \text{ok} \to \text{accept}, \text{rotten} \to \text{reject}$

Better: $M^* = \text{fresh} \to \text{accept}, \text{ok} \to \text{reject}, \text{rotten} \to \text{reject}$. 
Comparison With Other Models

Standard Mechanism Design

<table>
<thead>
<tr>
<th>fresh</th>
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Mechanism Design with Partial Verification

<table>
<thead>
<tr>
<th>fresh</th>
<th>0</th>
<th>(\infty)</th>
<th>0</th>
</tr>
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<tbody>
<tr>
<td>ok</td>
<td>(\infty)</td>
<td>0</td>
<td>0</td>
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Mechanism Design with Signaling Costs

<table>
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<tr>
<th>fresh</th>
<th>(\infty)</th>
<th>0</th>
<th>0</th>
</tr>
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<tbody>
<tr>
<td>ok</td>
<td>1.2</td>
<td>5</td>
<td>(-\infty)</td>
</tr>
<tr>
<td>rotten</td>
<td>-3</td>
<td>0</td>
<td>0</td>
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Green and Laffont. Partially verifiable information and mechanism design. RES 1986
Auletta, Penna, Persiano, Ventre. Alternatives to truthfulness are hard to recognize. AAMAS 2011
Question

Given:

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Classifications ($i \in \hat{T}$): fresh, ok, rotten

Effort Function ($E : T \times \hat{T} \to \mathbb{R}$):

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<td>10</td>
<td>0</td>
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Valuation Function ($V : T \times A \to \mathbb{R}$):

$V(\cdot, accept) = 20$, $V(\cdot, reject) = 0$

Then:

Does there exist a Mechanism $M : \hat{T} \to A$ which implements the choice function?

NP-complete!

Auletta, Penna, Persiano, Ventre. Alternatives to truthfulness are hard to recognize. AAMAS 2011
### Results

Non-bolded results are from: Auletta, Penna, Persiano, Ventre. **Alternatives to truthfulness are hard to recognize.** AAMAS 2011

Hardness results fundamentally rely on **revelation principle failing** – conditions under which revelation principle still holds in Green & Laffont ’86 and Yu ’11 (partial verification), and Kephart & C. EC’16 (costly signaling).

<table>
<thead>
<tr>
<th></th>
<th>Transfers (T)</th>
<th>No Transfers (NT)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Two Outcomes (TO)</td>
<td>Injective SCF (FI)</td>
</tr>
<tr>
<td><strong>Free Utilities (FU)</strong></td>
<td><strong>Unrestricted Costs (U)</strong></td>
<td>NP-c</td>
</tr>
<tr>
<td></td>
<td>{0, \infty} Costs (ZI)</td>
<td>NP-c</td>
</tr>
<tr>
<td><strong>Targeted Utilities (TU)</strong></td>
<td><strong>Unrestricted Costs (U)</strong></td>
<td>NP-c</td>
</tr>
<tr>
<td></td>
<td>{0, \infty} Costs (ZI)</td>
<td>NP-c</td>
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with Andrew Kephart (AAMAS 2015)
When Samples Are Strategically Selected

ICML 2019, with

Hanrui Zhang (Duke)

Yu Cheng (Duke → IAS → UIC)

Bob, Professor of Rocket Science

A NEW POSTDOC APPLICANT.

SHE HAS 15 PAPERS AND I ONLY WANT TO READ 3.
Academic hiring…

Charlie, Bob’s student

GIVE ME 3 PAPERS BY ALICE THAT I NEED TO READ.

SURE.

CHARLIE IS EXCITED ABOUT HIRING ALICE

www.phdcomics.com
I need to choose the best 3 papers to convince Bob, so that he will hire Alice.

Charlie will definitely pick the best 3 papers by Alice, and I need to calibrate for that.
The general problem

A distribution (Alice) over paper qualities $\theta \in \{g, b\}$ arrives, which can be either a good one ($\theta = g$) or a bad one ($\theta = b$)
The general problem

The principal (Bob) announces a policy, according to which he decides, based on the report of the agent (Charlie), whether to accept $\theta$ (hire Alice)

I WILL HIRE ALICE IF YOU GIVE ME 3 GOOD PAPERS, OR 2 EXCELLENT PAPERS.

AND I WANT ALICE TO BE FIRST AUTHOR ON AT LEAST 2 OF THEM.
The general problem

The agent (Charlie) has access to \( n(=15) \) iid samples (papers) from \( \theta \) (Alice), from which he chooses \( m(=3) \) as his report.
The general problem

The agent (Charlie) sends his report to the principal, aiming to convince the principal (Bob) to accept $\theta$ (Alice)
The general problem

The principal (Bob) observes the report of the agent (Charlie), and makes the decision according to the policy announced.

I read the 3 papers you sent me.

One is not so good, but the other two are incredible.

It looks like Alice is doing good work, so let's hire her.
Questions

How does strategic selection affect the principal’s policy? Is it easier or harder to classify based on strategic samples, compared to when the principal has access to iid samples? Should the principal ever have a diversity requirement (e.g., at least 1 mathematical paper and at least 1 experimental paper), or only go by total quality according to a single metric?
Agent’s problem:
  • “How do I distinguish myself from other types?”
  • “How many samples do I need for that?”

Principal’s problem:
  • “How do I tell ML-flexible agents from others?”
  • “At what point in their career can I reliably do that?”
One good and one bad distribution

Pick a subset of the right-hand side (to accept) that maximizes \((\text{green mass covered} - \text{black mass covered})\)

If positive, can (eventually) distinguish; otherwise not. NP-hard in general.

This subset covers \(.5 + .2 = .7\) good mass and \(.4 + .3 = .7\) bad mass, so it doesn’t work. (What does?)
But if we know the strategy for the good distribution (revelation principle holds):

\begin{align*}
\text{samples} & \quad \text{signals} \\
.4 & \quad .8 \\
.3 & \quad .2 \\
.3 & \quad 0
\end{align*}

Can place good mass on the signals side because we know the strategy.

Solve as maximum flow/matching from left to right with capacities on vertices. Duality gives set of signals to accept (\sim Hall’s marriage theorem).
Optimization: reduction to min cut

-3

In sampling case, can check existence of edges with previous technique

Values are $P(\text{type}) \times \text{value}(\text{type})$

Can be generalized to more outcomes than accept/reject, if types have the same utility over them.

(vertices; edges imply ability to (cost-effectively) misreport)

(edges between types have capacity $\infty$

(when revelation principle holds)
Conclusion

First part:
When considering correlation, small changes can have a huge effect. Automatically designing robust mechanisms addresses this. Combines well with learning (under some conditions).

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v.

<table>
<thead>
<tr>
<th>0.251001</th>
<th>0.249999</th>
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<tr>
<td>0.249999</td>
<td>0.249001</td>
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Second part:
With costly or limited misreporting, revelation principle can fail. Causes computational hardness in general. Sometimes agents report based on their samples. Some efficient algorithms for the infinite limit case; sample bounds.

Thank you for your attention!