HIDE AND SEEK: COSTLY CONSUMER PRIVACY IN A MARKET WITH REPEAT PURCHASES*

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Abstract

When a firm can recognize its previous customers, it may use information about their past purchases in order to price discriminate. We study a model with a monopolist and a continuum of heterogeneous consumers, where consumers have the ability to maintain their anonymity and avoid being identified as past customers, possibly at a cost. When consumers can freely maintain their anonymity, they all individually choose to do so, which results in the highest profit for the monopolist. Increasing the cost of anonymity can benefit consumers, but only up to a point, after which the effect is reversed. We show that if the monopolist or an independent third party controls the cost of anonymity, it often works to the detriment of consumers.

Keywords: Anonymity, Customer Recognition, Price Discrimination, Identity Management.

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1 Introduction

In the modern information economy, firms routinely use consumer data to target their marketing appeals and to price discriminate (Odlyzko, 2003). This practice has, not surprisingly, generated privacy concerns among consumers (Goldfarb and Tucker, 2011). The proliferation of online search and transactions on sites such as Google, Amazon, Groupon, and Netflix, and other interactions on sites such as Facebook, Twitter, and Tumblr, as well as credit card transactions, use of subscription services, and participation in retailer loyalty programs, has brought an unprecedented amount of personal data within reach of marketers. Small bits of self-revelation via past transactions can increasingly be collected and reassembled by computers to help create a detailed picture of an individual's identity.

For example, in 2006, Netflix initiated a contest to analyze the movie rental history of $500,000$ subscribers and improve the predictive accuracy of its recommendation software by at least 10%. Narayanan and Shmatikov (2008) then showed that the customer data released for that contest, despite being stripped of names and other direct identifying information, could often be "de-anonymized" by pinning down an individual's distinctive pattern of movie ratings and recommendations. In 2010, Netflix said that it was shelving plans for a second contest – bowing to concerns raised by the Federal Trade Commission (FTC).

Similarly, Acquisti and Gross (2009) show that information about an individual's place and date of birth can be exploited to predict his or her Social Security number (SSN). Using only publicly available data, they observed a correlation between individuals' SSNs and their birth data and found that for younger cohorts the correlation allows statistical inference of private SSNs. These inferences are made possible by the public availability of the Social Security Administration's Death Master File and the widespread accessibility of personal information from multiple sources, such as data brokers or profiles on social networking sites.

In this paper, we analyze the ability of firms to track individual purchasing patterns in order to practice behavior-based price discrimination (Armstrong, 2006; Fudenberg and Villas-Boas, 2006) and the ability of consumers to block data collection in order to avoid differential pricing. Behavior-based advertising and price discrimination are already ubiquitous (Odlyzko, 2003; Hann et al., 2007). Records containing the sequence of web sites visited and the online purchases made by individuals are regularly used to target tailor-made offers to them (Chen, 2006; Wathieu, 2006; Pancras and Sudhir, 2007; Chen and Zhang, 2008). Similarly, retailer 'loyalty' programs such as supermarket *MVP* cards allow sellers to observe individual purchase histories on which they can condition offers (Larsen and Voronovich, 2004).

Chicago-based electronic coupon seller, Groupon, has recently been accused of price discriminating against consumers who purchased a popular Valentines Day offering for a flower provider. When going through the link provided by the Groupon offering, customers allegedly faced higher prices compared to those who did not.[1] Another – now notorious – example of behavior-based price discrimination is the 2000 incident in which Amazon charged past customers higher prices for DVDs that their purchase histories suggested they would be likely to want.[2]

There are numerous other cases. Examples include Netflix offering a month of free service to new subscribers, AOL offering special "new customer" accounts that can be opened only by revealing credit card numbers that have not been applied before to a similar offer, and credit monitoring agencies offering promotions to new subscribers. The key to these examples is that sellers have difficulty committing to future prices and – in particular – committing not to use information about past purchases when formulating future offers to consumers.

Although technological advances have allowed sellers to track, store, and process individual purchases, consumers – nevertheless – do have some control over allowing sellers to record their behavior. For instance, they can exert effort to understand sellers' privacy disclosures and take actions to circumvent being tracked. Such actions can include erasing or blocking browser cookies, using a temporary email address for creating a new account, maintaining several online identities, paying with a different or "virtual" credit card, and spreading purchases among a variety of unrelated vendors (Low et al., 1994; Chowdhury et al., 2006). In general, these practices, which generally require time, effort, and even money, fall under the rubric of what is known as *identity management strategies* (Acquisti, 2008). Hence, it is appropriate to think of firms and consumers engaged in a complex technological game of *Hide and Seek*, in which sellers attempt to identify individual consumers in order to price discriminate and in which buyers endeavor to conceal their personal data.

To investigate these issues, we construct a model featuring a monopolist who is able to track consumer purchases. Consumers, however, are able to avoid being identified as past customers (or to "anonymize" their identity), possibly at a cost. In our framework, the firm charges past customers more than "new customers" because their past purchases signal a higher willingness to pay.[3] We find that when consumers can freely anonymize, they all individually choose to do so, which – somewhat paradoxically – results in the highest profit for the firm. We show that increasing the cost of obtaining anonymity can benefit consumers, but only up to a point; at that point, the effect is reversed.

[1] `http://www.suntimes.com/business/3822291-417/groupon-offers-refunds-after-complaints-about-flower-deal.html`

[2] `http://www.cnn.com/2005/LAW/06/24/ramasastry.website.prices/`

[3] To focus on a worst-case scenario, and to provide a robust baseline, we leave to future work settings where the seller may want to give discounts to past customers, for instance, due to diminishing marginal utility.

The intuition for this finding is closely related to the celebrated Coase Conjecture (Coase, 1972) and runs as follows. When the cost of maintaining anonymity is high, the seller is better able to recognize past customers and to price discriminate against them. Thus, consumers hesitate to make an initial purchase, knowing this will cause them to pay a premium on future purchases. Anticipating this reluctance by consumers, the seller is forced to offer a lower initial price, and this effect actually dominates the increase in profits arising from price discrimination in future periods. In other words, the seller would prefer to commit itself not to price discriminate based on prior purchases. When the cost of maintaining anonymity is low, consumers – in effect – give the seller this commitment power when they each rationally choose to keep their purchases private.

This paper is related to work in the literatures on intertemporal price discrimination, consumer recognition, and online privacy. Research on intertemporal price discrimination and the "ratchet" effect, where the firm sets higher prices for consumers who signaled higher willingness to pay, dates back to the late 1970's. Stokey (1979) and Salant (1989) show that intertemporal price discrimination is never optimal for a monopolist who can commit to future prices. This is analogous to the fact, mentioned above, that in our model, the monopolist obtains its highest profit when anonymity is costless.[4]

A relatively small literature on consumer recognition and online privacy has begun to develop over the past several years.[5] Contributions by Chen (1997), Fudenberg and Tirole (1998), Fudenberg and Tirole (2000), Villas-Boas (1999), Shaffer and Zhang (2000), Taylor (2003), Chen and Zhang (2008), and Chen and Zhang (2009) developed the notion of consumer recognition and personalized pricing, but did not explicitly consider privacy issues in online environments. Fudenberg and Tirole (1998) explore what happens when the ability to identify consumers varies across goods. They consider a model in which consumers can be anonymous or "semi-anonymous," depending on the good bought. Villas-Boas (1999) and Fudenberg and Tirole (2000) analyze a duopoly model in which consumers have a choice between remaining loyal to a firm and defecting to the competitor, a phenomenon they refer to as "consumer poaching." They show that in the second period a firm always has the incentive to offer discounts to the rival firm's customers who have revealed, through their prior purchase, their preference for the rival firm's product. Such discounts tend to reduce consumer price sensitivity for a firm's product in the first period, as consumers rationally anticipate them, and hence prices rise in the first period thanks to anticipated customer poaching. Taylor (2003) shows that firms can be worse off due to competitive targeted pricing. Chen and Zhang (2008) analyze a "price for information" strategy, where firms price less aggressively in order to

[4]Villas-Boas (2004) shows that committing to future prices can also help in a model with overlapping generations of consumers.

[5]For a general discussion of price discrimination, see Stole (2007). For an analysis of privacy with respect to lawful search and seizure, see Mialon and Mialon (2008).

learn more about their customers. They extend their analysis in Chen and Zhang (2009) to show that price competition can be mitigated by firms vying to distinguish their loyal customers from price sensitive shoppers, actually making firms better off.[6] In contrast to these works, this paper aims to study the social, firm-level, and consumer-level impacts of the price of anonymity, when consumers have the ability to decide whether or not to remain anonymous, even after making purchases.

Closest to our work is an emerging literature on optimal online privacy policies. These were first studied by Taylor (2004), Acquisti and Varian (2005), and Calzolari and Pavan (2006). Fudenberg and Villas-Boas (2006) and Esteves (2010) offer surveys of this literature. Taylor (2004) and Villas-Boas (2004) show how strategic consumers could make a firm worse off in the context of dynamic targeted pricing. The reason is that once consumers anticipate future prices, they may choose to forgo a purchase today to avoid being identified as a past customer and thus be able to purchase at a lower price targeted at new consumers. This strategic "waiting" on the part of consumers can hurt a firm both through reducing sales and diminishing the benefit of price discrimination. Acquisti and Varian (2005) show that it is never profitable for a monopolist to condition its pricing on purchase history, unless a sufficient proportion of consumers are not sophisticated enough to anticipate the firm's pricing strategy or the firm can provide enhanced services to increase consumer valuation in subsequent purchases. Acquisti and Varian (2005) also begin to study consumers' use of anonymizing technologies (so as to circumvent identification by a firm as a past customer) but do not fully study the welfare implications. Calzolari and Pavan (2006) consider the case where two principals sequentially contract with a common agent, and where the upstream principal can sell its information to the downstream principal. They assume that the agent's valuations with the two sellers are perfectly correlated (similar to our setup, where valuations are constant across time). In their setting, the second principal posts its contract after the consumer has decided whether to accept the contract of the first firm. By selling information to the downstream principal, the upstream principal may get some rent. Calzolari and Pavan give conditions under which, if the upstream principal can commit to privacy, it will choose to do so. They identify cases where, given disclosure among the principals, the increase in the rent that has to be assigned to the agent always offsets any potential benefit from the sale of information, as the agent becomes more protective of revealing information about his type. We obtain a similar result in our framework, particularly when comparing the benchmarks of publicly available purchase histories (full recognition) and no purchase histories (no recognition).

The above papers provide important insights regarding the fundamental tensions between con-

[6] Rossi et al. (1996) showed a similar result when consumers are passive recipients of a targeted price and do not react when a firm takes away their surplus.

sumer privacy and price discrimination. This paper considers a richer environment, in which a firm's strategic customers can choose to remain anonymous at some cost.[7] We study how this cost affects equilibrium behavior and welfare. Our model allows us to gain additional insights into the effects of tightening privacy regulation. In particular, we show that while the firm obtains its highest profit when consumers can freely maintain their anonymity, consumers can, under some circumstances, be better off when maintaining anonymity is costly. The reason is that when identity management is very expensive, few consumers will do it. This leaves them with two viable alternatives: buy in the first period and face a high discriminatory price in the future or maintain anonymity by refusing to buy in the first period and receive a low price offer in the future. In order to induce consumers to opt for the first alternative, the firm sets a low *introductory* price in the first period which can, on net, benefit consumers. However, we also identify a range on the cost of becoming anonymous where no consumer maintains their anonymity in equilibrium, yet the firm increases prices. In this region of costs, consumer surplus suffers, as a greater cost of anonymity enables the firm to intensify its price discrimination while offering higher introductory prices. Consequently, we find that facilitating privacy over this region benefits consumers and increases overall welfare.

This result is in contrast to the prior literature because it actually agrees with the common intuition that more privacy can be better (even when consumers are strategic). Even more surprising is that this welfare behavior happens in a region of anonymizing costs where no consumer chooses to anonymize. In other words, we find that added privacy can benefit consumers and increase overall surplus, even when no consumer decides to take advantage of it.

We also study two extensions to our base model where the cost of anonymity is determined endogenously. In one, a third party — a privacy gatekeeper — controls the cost of anonymizing. Here, we show that the gatekeeper prefers to bargain with the firm and actually set this cost to zero, which works to the detriment of consumers.[8] In the other, the firm itself has control over the cost of anonymizing. We show that when the firm can set this cost up front, this can also work to the detriment of consumers.

Our analysis also connects with the marketing literature on couponing, market segmentation, and addressability. In our model, the cost of anonymity is, in a sense, a repeat customer's cost of accessing a seller's introductory offers, which are targeted at new customers.[9] Hence, anonymizing costs play an important role in segmenting the consumer population into (at least) 3 segments:

[7] Armstrong et al. (2009) study a related model where consumers can choose to "opt out" of receiving sellers' marketing. They similarly show that each consumer has a private incentive to opt out of intrusive marketing, but when all consumers do this, price competition is relaxed and consumers are harmed. However, they do not study intermediate cases where opting out is costly but not prohibitively so, which are a main focus in this paper.

[8] See Pancras and Sudhir (2007) for a study of an intermediary that stores and sells customer data rather than keep it private.

[9] See Narasimhan (1984) for an analysis of a consumer's decision of whether to exert costly effort in using coupons.

repeat customers who are identified as such, repeat customers who anonymized, and new (potential) customers, where the latter two segments may overlap.[10] When the firm has control over the cost of anonymizing, it would choose to segment the population optimally. We show that this optimal segmentation is obtained when the cost of anonymity is nil and no price discrimination takes place. Shaffer and Zhang (1995, 2002), Chen et al. (2001), and Chen and Iyer (2002) obtain complementary results. They show that price discrimination can lead to intense price competition where firms may have an incentive to (i) decrease the level of accuracy of targeted promotions, (ii) differentially invest in customer addressability, and (iii) seek commitment mechanisms not to price discriminate. In our context, the ability to anonymize at a cost of the firm's choosing facilitates such a commitment mechanism. In this regard, our analysis is also related to Hermalin and Katz (2006) who evaluate the efficiency of various privacy policies. Complementary to their work, we show that a firm's ability to set the cost of anonymity up front enables it to commit to prices before consumers make purchasing decisions. Hence, in setting this cost, the firm is able to control the extent of private information that purchasing consumers disclose.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 gives two benchmarks: when there is no customer recognition, and when consumers cannot anonymize their identities after purchasing. The equilibrium of the game with the ability to anonymize is derived in Section 4 along with comparative statics. Section 5 extends the base model to allow for an endogenous cost of anonymity, and Section 6 concludes. The proofs are relegated to an appendix.

2 Model

2.1 The Consumers

There is a continuum of consumers with total mass normalized to one. All consumers are risk-neutral, possess a common discount factor $\delta \in (0, 1]$, and maximize their present expected utilities.[11] Each consumer demands at most one unit of a non-durable, indivisible good in each of two periods. Consumer i's valuation for the good is the same in each period and is determined by the realization of a random variable v_i with support normalized to be the unit interval. Consumer valuations are independently and identically distributed according to a cumulative distribution function $F(v)$ with density $f(v)$, which is strictly positive on $(0, 1)$. Consumer i's valuation v_i is initially private

[10] See Gerstner and Holthausen (1986) for an early analysis of a firm's pricing strategies when market segments overlap.

[11] The role of the discount factor δ will be in breaking the indifference of non-marginal types, i.e., tipping them in favor of buying or not buying in the first period.

information.

2.2 The Firm

There is a monopolist that produces and sells the good in each period. The firm's production cost is normalized to zero, it possesses the same discount factor δ as consumers, and it maximizes its discounted expected profit. It does not observe consumer valuations directly but maintains a database containing purchasing histories. Each consumer is either *anonymous* or *identifiable*. If a consumer is anonymous, then there is no record of any prior purchases by her; i.e., she is not in the database. If she is identifiable, then in the second period the firm knows the purchasing decision that she made in the first period. We emphasize that the firm has no commitment power, i.e., the firm is unable to set and commit to second-period prices in the first period.[12] Because there is a continuum of consumers, each of them realizes that her first-period purchasing decision alone does not affect the prices charged by the firm in the next period.[13]

2.3 The Game

All aspects of the environment, including the distribution of valuations $F(v)$, are common knowledge. At the beginning of the game all consumers are anonymous. Hence, the firm offers the same first-period price p_1 to all of them.[14] Next, each consumer decides whether to buy the good in the first period, $q_1^i = 1$, or not to buy it, $q_1^i = 0$. Consumers who elect to buy the good also decide whether to let the firm keep a record of the transaction ($r^i = 1$) or to *anonymize* their identity with the firm by deleting the record of the sale ($r^i = 0$). The cost to any consumer who anonymizes is $c \geq 0$,[15] and we without loss of generality assume that this cost is expended in the second period (i.e., it is discounted by δ). This cost represents the time, effort, and any monetary expense of maintaining anonymity. We also allow consumers who purchase to randomize between

[12]In a world where the monopolist can commit to its future prices, our analysis of privacy is uninteresting, for no user would pay the cost for remaining anonymous.

[13]The results hold when there is a finite number of consumers, provided that we add the following assumption: the firm cannot update its beliefs over how many consumers anonymized based on an inventory count. Without this assumption, the firm could infer how many consumers anonymized based on the inventory count. With a continuum of (massless) consumers, inventory expectations on the equilibrium path are confirmed even if a single agent deviates.

[14]We note that given our setup, the firm cannot do any better by using mechanisms that are more sophisticated than simply posting prices.

[15]The qualitative nature of the results still holds under certain conditions when the cost of maintaining anonymity varies across consumers, or when it is correlated with their valuations. It is also possible to extend the analysis to the case where privacy is a continuous choice (e.g., with a convex cost and a concave benefit), but obtaining insightful comparative statics while doing so requires a parameterized cost function, bringing us to a similar, albeit more complicated setup. In order to simplify the analysis, we model privacy as a binary choice (though allow randomization) and assume that consumers incur the same cost of maintaining their anonymity. See Taylor (2004) for a model with correlated valuations, and Acquisti and Varian (2005) for a model with varying levels of consumer sophistication.

anonymizing and staying identified. A consumer who does not purchase the good continues to be anonymous and is thus pooled with the buyers who anonymized, from the firm's perspective. At the beginning of period two, the firm posts a price p_2^0 to the unidentified (anonymous) consumers and a price p_2^1 to the identified ones. [16] Consumers can buy the good only at the price offered to them, $q_2^i \in \{0, 1\}$; i.e., no arbitrage is possible. Hence, a consumer i with valuation v_i who purchases in both periods has (present discounted) utility $v_i - p_1 + \delta(v_i - p_2^1)$ if he does not anonymize, and utility $v_i - p_1 + \delta(v_i - p_2^0 - c)$ if he does. Figure 1 summarizes the timeline of the game.

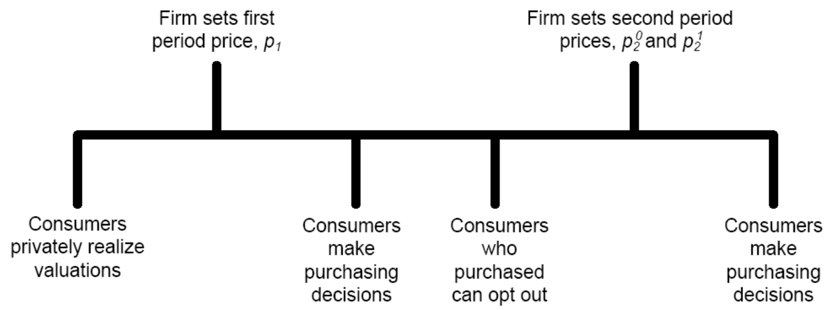


Figure 1: Timeline of the game.

The solution concept we use is Perfect Bayesian Nash Equilibrium (PBE). A PBE here consists of the firm's strategy (composed of first-period price p_1 and second-period prices p_2^0 and p_2^1, corresponding to the firm's two information sets in the second period[17]); the consumers' strategies (composed of first-period purchasing decisions q_1^i and anonymizing decision $r^i \in \{0, 1\}$ as a function of p_1 and v_i, and second-period purchasing decision q_2^i as a function of[18] v_i and p_2^r); and the firm's beliefs about consumers' valuations given their identification status (F^1 and F^0 for identified and anonymous consumers, respectively[19]). These constitute a PBE if all strategies are sequentially rational given the beliefs and the beliefs are consistent given the strategies.

We assume that $p(1 - F(p))$ is concave, i.e., the firm's marginal revenue in a single period game is decreasing in p (or $-2f(p) - pf'(p) \leq 0$); and that marginal revenue is concave (or $-3f'(p) - f''(p)p \leq 0$). We denote the firm's optimal price in a one-shot version of the game by $p^\star$, i.e., $p^\star = \arg\max_p p(1 - F(p))$.

[16] Subsection 4.1 considers the case where the firm sets second-period prices *before* consumers decide on whether or not to anonymize.

[17] Technically, the strategy should also specify what the firm would have charged in the second period if it had made a different pricing decision in the first period, but we omit this for notational simplicity.

[18] In principle, the second-period decision can also directly depend on the first-period price, but in equilibrium it will only depend on v_i and p_2^r.

[19] The firm will also have beliefs about what actions an anonymous agent took in the first period (did the agent purchase and anonymize or not purchase at all), but this will not affect the analysis.

3 Benchmarks with Exogenous Privacy

3.1 No Recognition

First, consider as a benchmark the case where there is no consumer recognition, so that the firm cannot price discriminate in the second period between consumers that bought and did not buy in the first period. In other words, data on past purchases is exogenously unavailable. Since the firm does not price discriminate based on purchasing history, a consumer will buy the good in each period in which his valuation exceeds the price. Thus, the firm sets the same price in each period, $p^\star = \arg\max_p p(1 - F(p))$, generating a per-period profit of $p^\star(1 - F(p^\star))$. Consumer surplus in each period is given by $\int_{p^\star}^1 (v - p^\star)\, \mathrm{d}F(v)$.

3.2 Full Recognition

Consider now the opposite extreme in which data on past transactions is public information. The firm is then able to recognize its previous customers and consumers are unable to maintain their anonymity at any cost (as in Hart and Tirole (1988), Schmidt (1993), Villas-Boas (2004), Taylor (2004), and Fudenberg and Villas-Boas (2006)). In this setting, the firm can discriminate between two different groups of consumers in the second period: identified consumers who purchased in the first period, and unidentified consumers who did not. The firm consequently sets two different prices in the second period, p_2^1 to identified consumers and p_2^0 to unidentified consumers. (We emphasize again that the firm has no commitment power.)

Proposition 1 (Fudenberg & Villas-Boas 2006)**.** *Assume the history of purchases is publicly observable. Then,*
(i) There exists essentially a unique equilibrium that is characterized by prices p_1, p_2^0, p_2^1, and a threshold $\tilde{v}$, such that consumers with valuations $v \in [\tilde{v}, 1]$ purchase in both periods; consumers with valuations $v \in [p_2^0, \tilde{v}]$ purchase only in the second period. The cutoff type $\tilde{v}$ satisfies $\tilde{v} \geq p^\star$.
(ii) The equilibrium prices and the threshold $\tilde{v}$ satisfy $p_1 = (1 - \delta)\tilde{v} + \delta p_2^0$ and $p_2^1 = \tilde{v}$ with $\tilde{v}$ and p_2^0 jointly determined from $F(p_2^0) + f(p_2^0)p_2^0 = F(\tilde{v})$ and $\tilde{v} = (1 + \delta \frac{\partial p_2^0}{\partial \tilde{v}})\frac{1-F(\tilde{v})}{f(\tilde{v})}$. The first-period price p_1 is then determined from $p_1 = \tilde{v}(1 - \delta) + \delta p_2^0$.

Let us consider the monopolist's pricing strategy towards identified consumers in the second period. If the cutoff type for identified consumers (those who purchase in the first period) $\tilde{v}$ satisfies $\tilde{v} \geq p^\star$, then the monopolist sets $p_2^1 = \tilde{v}$. If, on the other hand, $\tilde{v} < p^\star$, the monopolist sets $p_2^1 = p^\star$. That is, $p_2^1 = \max\{\tilde{v}, p^\star\}$. From Proposition 1, since $\tilde{v} \geq p^\star$ holds on the path of play of the full-recognition equilibrium, $p_2^1 = \max\{\tilde{v}, p^\star\} = \tilde{v}$. Hence, the marginal consumer who buys in the first period—the one with valuation $\tilde{v}$—gets no surplus in the second period. This is the *ratchet effect* of consumers who reveal their types (Freixas et al., 1985; Laffont and Tirole, 1988). The proof of Proposition 1 is in the appendix.

Paradoxically, the full-recognition case can result in higher consumer surplus than the no-recognition case, because the firm will need to set p_1 lower to attract consumers in the first period. Correspondingly, in the model with the ability to anonymize, we will show that a low cost of anonymizing one's identity can lead to a Prisoner's Dilemma situation where consumers maintain anonymity but collectively suffer as a result, in comparison to the situation where anonymity is prohibitively costly and consumers face the ratchet effect.

By definition, a consumer with valuation $\tilde{v}$ is indifferent between purchasing in both periods and purchasing only in the second period. It follows that the indifference condition that characterizes $\tilde{v}$ is given by $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$. Hence, $p_1 = \tilde{v} - \delta(\tilde{v} - p_2^0)$. Using $p_1 = \tilde{v} - \delta(\tilde{v} - p_2^0)$ and $p_2^1 = \tilde{v}$, one can simplify the firm's present discounted profit to obtain

$$\tilde{v}(1 - F(\tilde{v})) + \delta p_2^0(1 - F(p_2^0))$$

For $\delta > 0$, since $p^\star$ uniquely maximizes $p(1 - F(p))$ and $\tilde{v} \geq p^\star$, we have

$$(1 + \delta)p^\star(1 - F(p^\star)) \geq \tilde{v}(1 - F(\tilde{v})) + \delta p_2^0(1 - F(p_2^0))$$

Hence, the firm's profit under full recognition is lower than its profit under no recognition. The intuition is that some consumers refrain from purchasing in the first period because they anticipate a lower price in the next as a result, and the firm is unable to fully recoup the loss in first-period profit by price discriminating in the second period. Hart and Tirole (1988) and Fudenberg and Villas-Boas (2006) show that if the firm is able to commit to second-period prices in the first period, it would set $p_2^1 = p_2^0 = p^\star$, a result which extends to our framework. Hence, the firm's profit under commitment coincides with its profit in the no-recognition equilibrium, i.e., when the firm does not have the ability to track consumers' purchases.

4 Equilibrium with Endogenous Privacy

We now consider the setting in which consumers who purchase in the first period can anonymize at a cost of c. Consumers who purchase in the first period and do not anonymize are identified by the firm in the second period (the firm recognizes that they purchased at a price p_1 and did not anonymize) and will be offered price p_2^1 in period 2. All other consumers are offered p_2^0 in period 2. As above, let $\tilde{v}$ denote the lowest consumer type to purchase in the first period. We note that, given that a consumer with valuation $\tilde{v}$ prefers to buy in the first period, i.e., $\tilde{v}-p_1+\delta\max\{\tilde{v}-p_2^1, \tilde{v}-p_2^0-c, 0\} \geq \delta\max\{\tilde{v}-p_2^0, 0\}$, then all consumers with valuations $v \geq \tilde{v}$ do as well. Denote by $\alpha(v)$ the probability that a consumer of type $v \in [\tilde{v}, 1]$ anonymizes after purchasing. Then the distribution of valuations among anonymous consumers is

$$F^0(v) = \begin{cases} \frac{F(v)}{F(\tilde{v})+\int_{\tilde{v}}^1 \alpha(x)f(x)\,\mathrm{d}x} & \text{if } v \leq \tilde{v} \\ \\ \frac{F(\tilde{v})+\int_{\tilde{v}}^v \alpha(x)f(x)\,\mathrm{d}x}{F(\tilde{v})+\int_{\tilde{v}}^1 \alpha(x)f(x)\,\mathrm{d}x} & \text{if } v > \tilde{v} \end{cases}$$

and the distribution of valuations among identifiable consumers (for $v \geq \tilde{v}$) is given by

$$F^1(v) = \frac{\int_{\tilde{v}}^v (1-\alpha(x))f(x)\,\mathrm{d}x}{\int_{\tilde{v}}^1 (1-\alpha(x))f(x)\,\mathrm{d}x}$$

where $F^1(v) = 0$ if $v < \tilde{v}$.

4.1 Partial Commitment

Let us briefly consider the case where the firm posts second-period prices *before* consumers choose whether or not to anonymize. In effect, the firm has some degree of commitment power, as it is able to commit to second-period prices after first-period purchases take place. As the following lemma shows, a consequence of this timing is that, reminiscent of the full-recognition benchmark, no consumer anonymizes in equilibrium.

Lemma 1. *For any $c > 0$, if the firm posts second-period prices* before *consumers choose whether to anonymize, prices satisfy $p_2^1 - p_2^0 \leq c$ — that is, the benefit of anonymizing is smaller than its cost, so no consumer anonymizes in equilibrium.*

From Lemma 1, it follows that the firm's second-period problem here is to choose prices subject to $p_2^1 - p_2^0 \leq c$. Letting $p_2^{0,FR}$ and $p_2^{1,FR}$ denote second-period prices in the full-recognition benchmark, if $c \geq p_2^{1,FR} - p_2^{0,FR}$, it is straightforward to see that the constraint is non-binding and full-recognition outcome results. For $c < p_2^{1,FR} - p_2^{0,FR}$, however, the constraint is binding, which can work to the firm's advantage. In particular, the constraint $p_2^1 - p_2^0 \leq c$ enables the firm to commit not to price discriminate against identified customers by an amount larger than c.

In either case, given that the firm posts second-period prices prior to consumers' anonymity decisions, no consumer ends up anonymizing. In effect, the firm has the power to dissuade consumers from taking wasteful anonymizing actions. As will become clear from the following analysis, this limited commitment power benefits the firm relative to the case where prices are posted *after* anonymity decisions.

4.2 Costless Anonymity

Let us now return to the timing in our model, where consumers first choose whether or not to anonymize and the firm *then* posts second-period prices. As a starting point, we first consider the case where $c = 0$. Here, we show that the equilibrium is essentially unique and corresponds to the no-recognition benchmark.

Proposition 2. *When anonymity is costless ($c = 0$), every*[20] *PBE satisfies (and a PBE exists that satisfies):*
(i) The firm sets the one-shot monopoly prices $p_1 = p_2^0 = p^\star$ and $p_2^1 \geq p^\star$.
(ii) Consumers with valuations $v \in [p^\star, 1]$ purchase in both periods and anonymize.
(iii) The no-recognition benchmark outcome is obtained.

This result says that if the cost of maintaining anonymity is nil, then it is in the best interest of every individual who purchases the good in the first period to maintain her anonymity, effectively resulting in the no-recognition outcome from Subsection 3.1. However, as indicated in Subsection 3.2, this turns out to be exactly what the firm wants.

From the perspective of consumers, in Subsection 4.5, we show that this outcome is a Prisoner's Dilemma situation: individually, each consumer chooses to maintain her anonymity; as a result, however, consumer surplus ends up being lower due to there being no price discrimination. In fact, relative to the case where anonymity is costly, every consumer ends up being (weakly) worse off overall when there is no cost associated with anonymizing. In other words, by maintaining anonymity, consumers impose a negative externality on other consumers. Below we study how firm profit and consumer surplus are affected by the cost c of maintaining anonymity.

4.3 Costly Anonymity

We now move on to the general case in which there is some cost $c \geq 0$. We will restrict our attention to PBEs in which the following holds: all consumers who purchase the good in the first period anonymize with the same probability α (alternatively, α can be thought of as the proportion of consumers who anonymize, i.e., the fraction of purchasing consumers who take actions to maintain their anonymity). This restriction is motivated by the fact that all consumers who purchased in the first period face the same tradeoff when deciding to anonymize: either pay p_2^1, or pay $p_2^0 + c$.

[20] This is excluding the possibility of a PBE in which a measure zero subset of the consumers uses a different strategy.

(In equilibrium, all first-period buyers will buy again in the second period.) Hence, the benefits of anonymizing (or not) are a function of the prices and the cost of anonymizing. A consumer's level of v only determines whether the consumer is in the group that buys in both periods or only in the second period.[21] We refer to such an equilibrium as a *pooling equilibrium*.[22, 23]

The firm's second-period beliefs over valuations in a pooling equilibrium are given by

$$F^0(v) = \begin{cases} \frac{F(v)}{F(\tilde{v})+\alpha(1-F(\tilde{v}))} & \text{if } v \leq \tilde{v} \\ \\ \frac{F(\tilde{v})+\alpha(F(v)-F(\tilde{v}))}{F(\tilde{v})+\alpha(1-F(\tilde{v}))} & \text{if } v > \tilde{v} \end{cases} \tag{1}$$

and

$$F^1(v) = \begin{cases} 0 & \text{if } v \leq \tilde{v} \\ \\ \frac{F(v)-F(\tilde{v})}{1-F(\tilde{v})} & \text{if } v > \tilde{v} \end{cases} \tag{2}$$

In the second period, the firm chooses its prices to maximize profit according to

$$\max_{p_2^r} \, (1 - F^r(p_2^r))p_2^r \quad \text{for} \quad r = 0, 1 \tag{3}$$

The following lemma shows that when $c > 0$, there does not exist a PBE in which all of the consumers who purchased in the first period anonymize. The intuition is that when $c > 0$, it is optimal for the firm to lower the first-period price in order to attract more customers in the first period. For some of these customers, anonymizing is not a best response unless some other consumers purchase and stay identified.

Lemma 2. *For $c > 0$, there does not exist a PBE in which all first-period customers anonymize.*

The intuition for Lemma 2 is rooted in the commitment problem of the seller. The seller is

[21] We thank an anonymous referee for helping us clarify this argument.

[22] The restriction to pooling equilibria can also be justified using a purification argument. Suppose that, instead of all agents facing the same cost of anonymizing, each agent's cost is drawn from a commonly known distribution. Furthermore suppose these costs are drawn i.i.d. across agents, and are independent of the agent's valuation. Let d_i, $i \in \mathbb{N}$, denote a sequence of continuous distributions over the anonymizing cost that an individual agent faces such that $\lim_{i\to\infty} d_i$ is the degenerate distribution on c (where c is the cost in the original game G). Let G^{d_i} denote the cost-perturbed game where each consumer's cost of maintaining anonymity is realized, for simplicity, immediately after the first-period purchasing decisions according to d_i (e.g., due to uncertainties about which information the firm actually tracks; due to individual shocks regarding opportunity costs; and/or due to time delays prior to the availability of anonymizing actions). It can be shown that the pooling equilibrium we characterize is the unique equilibrium that results from taking the limit of the equilibria of G^{d_i} when $i \to \infty$. Appendix B shows the possibility of an equilibrium in pure strategies.

[23] A pooling equilibrium exists for the following reason. By setting $p_1 \geq 1$, the firm reaps no profits in the first period; similarly, when setting $p_1 = 0$, the firm's first-period profit is 0. In either case, present-discounted second-period profit is given by $\delta p^\star(1 - F(p^\star))$. The firm can thus perform strictly better by setting $p_1 \in (0, 1)$ and still reap the same second-period profit by setting $p_2^1 = p_2^0 = p^\star$. Since the firm's objective is continuous in p_1 and since the firm's problem is defined on a compact set, it follows that a solution exists in the interior, $p_1 \in (0, 1)$.

unable to directly commit not to price discriminate in the second period, but is able to influence consumers' decisions to become anonymous in the second period by raising the first-period price. However, doing so results in a loss of profit, both due to not capturing the cost consumers expend on anonymizing and due to lower revenues in the first period. Hence, the seller ends up choosing to set a lower first-period price and not all consumers anonymize.

The next lemma provides a useful ordering of the equilibrium prices and cutoff type, and proves that anonymous consumers pay a discounted price in the second period. The key drivers for the result are the seller's inability to commit to second-period prices and consumers' strategic prediction of future prices in the first period.

Lemma 3. *For $c > 0$, if $\tilde{v} \geq p^\star$, then $p_2^0 \leq p_1 \leq \tilde{v} = p_2^1$ holds on the path of play of any pooling eq. — that is, new and anonymous consumers pay discounted prices relative to identified consumers.*

The following lemma characterizes the pooling equilibrium for sufficiently small values of the cost of anonymizing, c. (Proposition 3, which follows, gives the relevant range on c.)

Lemma 4 (Pooling equilibrium)**.** *For sufficiently small $c > 0$, every pooling equilibrium satisfies:*
(i) Consumers with valuations $v \in [\tilde{v}, 1]$ purchase in both periods and anonymize with probability α; consumers with valuations $v \in [\tilde{v} - c, \tilde{v}]$ purchase only in the second period; and $\tilde{v} \geq p^\star$.
(ii) Prices satisfy $p_1 = \tilde{v} - \delta c$, $p_2^1 = \tilde{v}$, and $p_2^0 = \tilde{v} - c$. The firm's beliefs about anonymous and identified consumers' valuations are given by (1) and (2).
(iii) For $h(v) = F(v) + vf(v)$ and $h'(v) = 2f(v) + vf'(v)$, the cutoff type $\tilde{v}$ and anonymizing probability α are determined from:

$$\tilde{v} = \delta c + \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \delta \frac{1 - h(\tilde{v} - c) - ch'(\tilde{v} - c)}{f(\tilde{v})} \tag{4}$$

$$\alpha = \frac{h(\tilde{v} - c) - F(\tilde{v})}{1 - F(\tilde{v})} \tag{5}$$

Excluding deviations of measure 0, Lemma 4 uniquely determines the behavior on the path of play. The resulting equilibrium, with prices $p_1 = \tilde{v} - \delta c$, $p_2^0 = \tilde{v} - c$, and $p_2^1 = \tilde{v}$, has the following properties. A consumer with valuation at least $\tilde{v}$ will purchase in the first period as well as in the second period, and be indifferent between anonymizing and staying identified. A consumer with valuation $\tilde{v}$ will be indifferent among only buying in the first period, only buying in the second period, and buying in both periods. A consumer with valuation at most $\tilde{v}$ will not purchase in the first period, and purchase in the second period if and only if her valuation is at least $\tilde{v} - c$. Essentially, the firm offers anonymous customers "introductory" prices in each period (though the introductory price is more attractive in the second period).

We now move on to general values of c (not necessarily small). Let $\alpha(c)$ denote the probability that a consumer who purchased in the first period maintains anonymity, when the cost of doing so is c. (In a pooling equilibrium, by definition, this probability is the same for all agents who purchase in the first period.) Also let $p_2^{1,FR}$ and $p_2^{0,FR}$ denote the second period full-recognition benchmark prices, and finally let $\bar{c} = p_2^{1,FR} - p_2^{0,FR}$.

We proceed with several lemmas that address higher level of costs, which we then integrate to give a more general characterization of equilibrium. The first lemma addresses the case where the cost of anonymizing is prohibitively high.

Lemma 5. *For anonymizing costs that are prohibitively high, $c \geq \bar{c}$, the outcome from any pooling equilibrium coincides with the full-recognition benchmark outcome.*

The intuition for this result is the following. For prohibitively high costs of maintaining anonymity, the full-recognition benchmark outcome is obtainable by the firm. In fact, the firm's problem is a constrained version of its counterpart in the full-recognition benchmark, where its optimal strategy and corresponding outcome in the latter is feasible. In essence, since the cost of anonymizing is high, the ability to do so does little to help the firm's profit in terms of encouraging more consumers to purchase in the first period, but more to hurt the firm's profit by failing to capture a significant part of the cost that anonymizing consumers incur. Consequently, the firm sets prices that encourage consumers to stay identified in equilibrium.

The next lemma addresses the region of cost where consumers no longer anonymize.

Lemma 6. *Let $\hat{c}$ denote the smallest cost such that $\alpha(\hat{c}) = 0$. Then $\hat{c} < \bar{c}$.*

The intuition for Lemma 6 relates to that of Lemma 5. In particular, the firm works to mitigate its loss of potential profit from consumers anonymizing by reducing consumers' incentive to anonymize. According to Lemma 6, the firm begins to do so for costs smaller than $\bar{c}$.

The following Lemma shows that indeed no consumer anonymizes in equilibrium for all $c \geq \hat{c}$.

Lemma 7. *For all $c \geq \hat{c}$, $\alpha(c) = 0$ — that is, given anonymizing costs that exceed the threshold $\hat{c}$, consumers do not anonymize in equilibrium.*

The intuition for this result is that the firm maintains the status quo in terms of anonymizing behavior once it is able to eliminate consumers' incentives to do so at $\hat{c}$. The firm achieves this by setting a first-period price that is sufficiently low (with a corresponding cutoff type $\tilde{v}$ that is also sufficiently low).

Proposition 3 characterizes the pooling equilibrium across different values of c.

Proposition 3. *Let $p_2^{1,FR}$ and $p_2^{0,FR}$ denote second-period prices in the full-recognition benchmark, and let $\bar{c} = p_2^{1,FR} - p_2^{0,FR}$. A pooling equilibrium exists, and any pooling equilibrium satisfies the following properties on the path of play:*

1. *There exists a threshold anonymizing cost beyond which no consumer anonymizes, given by $\hat{c} \in (0, \bar{c})$, such that $\alpha(c) \in (0, 1)$ for all $c \in [0, \hat{c})$ and $\alpha(c) = 0$ for all $c \geq \hat{c}$.*

2. *For $c \in (0, \hat{c}]$, the unique pooling equilibrium outcome is characterized by Lemma 4.*

3. *For $c \in (\hat{c}, \bar{c})$, no consumer anonymizes. Let $\bar{v}$ be defined by $F(\bar{v}-c)+(\bar{v}-c)f(\bar{v}-c) = F(\bar{v})$. In eq., $\tilde{v}$ is non-decreasing in c, with $\tilde{v}(\hat{c}) < \tilde{v}(\bar{c})$, that is, fewer consumers purchase in the first period as c increases. Here, $\tilde{v}$ is set to maximize $\tilde{v}(1 - F(\tilde{v}))(1 + \delta) + \delta p_2^0(F(\tilde{v}) - F(p_2^0))$, subject to $\tilde{v} \leq \bar{v}$, $F(p_2^0) + f(p_2^0)p_2^0 = F(\tilde{v})$, and $p_1 = (1 - \delta)\tilde{v} - \delta p_2^0$.*

4. *For $c \geq \bar{c}$, the outcome from any pooling eq. coincides with the full-recognition outcome.*

Figure 2 shows how the probability of maintaining anonymity is affected by the cost of anonymizing, c. The region $[\hat{c}, \bar{c}]$ is of particular interest: as we will show shortly, consumer and social surplus (weakly) decrease in this region, the firm's profit (weakly) increases, but the probability of maintaining anonymity is fixed at 0. The various regions can be explained as follows. First, the firm loses

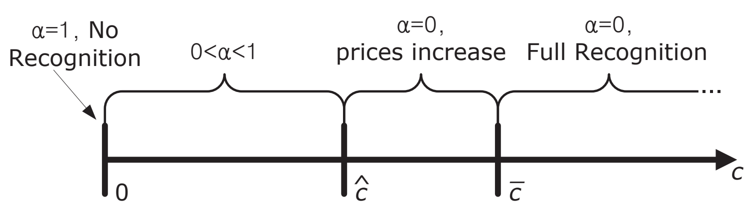


Figure 2: Eq. probability α of maintaining anonymity as a function of the anonymizing cost, c, on the path of play.

profit when consumers maintain their anonymity. When a consumer chooses to anonymize, because the firm's second-period price for anonymous consumers is c lower than for identified consumers (to keep consumers indifferent between anonymizing and staying identified), this effectively costs the firm c. That is, the cost of maintaining anonymity is passed on to the firm. Second, since some consumers anonymize, the second-period price to anonymous consumers, p_2^0, targets both first-time buyers and repeat customers who anonymized. Hence, (anonymous) repeat customers are interfering with the firm's ability to capture more first-time buyers in the second period, lowering the firm's profit. On the other hand, because consumers can anonymize, more consumers decide to buy in the first period, which helps the firm's profit. This latter effect dominates when c is low, but is overcome by the former two effects as c grows larger – to the point (at $c = \hat{c}$) where it pays off for the firm to lower the first price sufficiently so that no consumer anonymizes. Once the cost reaches $\hat{c}$, no consumer will anonymize, allowing the firm to more easily price-discriminate as c increases.

When valuations are uniformly distributed, Proposition 3 can be used to more precisely characterize the equilibrium outcome. In particular, we have $\hat{c} = \frac{1+\delta}{4+3\delta}$, and $\bar{c} = \frac{2+\delta}{8+2\delta}$; consumers with $v \in [\tilde{v}, 1]$ purchase in the first period and anonymize with probability α, where

$$\tilde{v} = \begin{cases} \frac{1+\delta-\delta c(1+2\delta)}{2(1+\delta)} + \delta c & \text{if} \quad c \leq \hat{c} \\ \min\{2c, 2\bar{c}\} & \text{if} \quad c > \hat{c}, \end{cases} \tag{6}$$

$$\alpha = \begin{cases} \frac{1+\delta-(4+3\delta)c}{1+\delta(1-c)} & \text{if} \quad c \leq \hat{c} \\ 0 & \text{if} \quad c > \hat{c}, \end{cases} \tag{7}$$

The firm sets the first-period price

$$p_1 = \begin{cases} \frac{1+\delta-\delta c(1+2\delta)}{2(1+\delta)} & \text{if} \quad c \leq \hat{c} \\ \min\{(2-\delta)c, (2-\delta)\bar{c}\} & \text{if} \quad c > \hat{c}, \end{cases} \tag{8}$$

Second-period prices satisfy $p_2^0 = \frac{1}{2}(\tilde{v} + \alpha(1-\tilde{v}))$, and $p_2^1 = \tilde{v}$. The firm's beliefs about anonymous and identified consumers' valuations are given by (1) and (2).

4.4 Firm Profit

For $c \in [0, \hat{c}]$ and for $c = \bar{c}$, the firm's present-discounted profit is given by

$$\Pi(c) = (\tilde{v}(c) - \delta\alpha(c)c)(1 - F(\tilde{v}(c))) + \delta p_2^0(1 - F(p_2^0)) \tag{9}$$

At $c = 0$, the firm obtains the no-recognition benchmark profit (where all consumers anonymize), given by $(1+\delta)p^\star(1 - F(p^\star))$. This profit was shown to exceed the one in the full-recognition benchmark in Section 3.2. The following lemma shows that the firm's profit for $c \in (\hat{c}, \bar{c})$ is even lower than at $c = \bar{c}$.[24]

Lemma 8. *The firm's profit is nondecreasing in c over $[\hat{c}, \bar{c}]$ and is strictly higher for $c \geq \bar{c}$ than $\hat{c}$.*

Lemma 8 and its preceding discussion indicate that the firm's profit is non-monotonic in the cost of anonymity: It is higher at $c = 0$ than at either $\hat{c}$ or $\bar{c}$, but lower at $\hat{c}$ than at $\bar{c}$. Proposition 4 summarizes the above findings. We include more precise profit characterization for the special case where valuations are uniformly distributed.

[24] For all $c \geq \bar{c}$, the firm obtains the full-recognition equilibrium profit.

Proposition 4 (Firm profit)**.** *(i) The firm's profits,* $\Pi(c)$*, are non-monotone in* c*. They reach a global maximum at* $c = 0$ *and are nondecreasing over* $[\hat{c}, \bar{c}]$*, with* $\Pi(c) = \Pi(\bar{c}) > \Pi(\hat{c})$ *for all* $c \geq \bar{c}$*. (ii) In the case where valuations are uniformly distributed, there exists a threshold* $c^{\#} < \hat{c}$ *such that* $\Pi(c)$ *is strictly decreasing over* $[0, c^{\#}]$*, strictly increasing over* $[c^{\#}, \hat{c}]$*, and constant over* $[\bar{c}, \infty)$*.*

4.5 Uniformly Distributed Valuations

When valuations are uniformly distributed, $p^\star = \arg\max_p p(1 - F(p)) = 1/2$. The solution to the firm's second period problem is given by $p_2^0 = \frac{1}{2}\left(\tilde{v} + \alpha(1 - \tilde{v})\right)$ and $p_2^1 = \tilde{v}$. When consumers anonymize (i.e., $\alpha > 0$), we also have $p_2^1 - p_2^0 = c$. Substituting for p_2^0 and p_2^1, we can obtain $\alpha = \max\{\frac{\tilde{v}-2c}{1-\tilde{v}}, 0\}$. From the indifference condition for the marginal type $\tilde{v}$, we have

$$\tilde{v} - p_1 + \alpha\delta(\tilde{v} - p_2^0 - c) + (1 - \alpha)\delta(\tilde{v} - p_2^1) = \delta(\tilde{v} - p_2^0)$$

Substituting for second-period prices and rearranging gives $\tilde{v} = p_1 + \delta c$ when $\alpha > 0$ and $\tilde{v} = \frac{2p_1}{2-\delta}$ when $\alpha = 0$. Thus, we have

$$\alpha = \max\{\frac{p_1 - (2 - \delta)c}{1 - p_1 - \delta c}, 0\} \tag{10}$$

and

$$\tilde{v}(p_1) = \begin{cases} p_1 + \delta c & \text{if} \quad p_1 \geq (2 - \delta)c \\ \\ \frac{2p_1}{2-\delta} & \text{if} \quad p_1 < (2 - \delta)c \end{cases} \tag{11}$$

Given $c \geq 0$, the firm's first-period problem is then obtained by solving:

$$\max_{p_1(c)} (1 - \tilde{v})(p_1 + \delta(1 - \alpha)p_2^1 + \delta\alpha p_2^0) + \delta(\tilde{v} - p_2^0)p_2^0 \tag{12}$$

subject to $p_2^0 = \frac{1}{2}\left(\tilde{v} + \alpha(1 - \tilde{v})\right)$, $p_2^1 = \tilde{v}$, (10), and (11). Letting $\hat{c} = \frac{1+\delta}{4+3\delta}$ and $\bar{c} = \frac{2+\delta}{8+2\delta}$, the equilibrium first-period price and marginal type are characterized by the following.

$$p_1(c) = \begin{cases} \frac{1+\delta-\delta c(1+2\delta)}{2(1+\delta)} & \text{if} \quad c \leq \hat{c} \\ \\ \min\{(2 - \delta)c, (2 - \delta)\bar{c}\} & \text{if} \quad c > \hat{c}, \end{cases} \tag{13}$$

$$\tilde{v}(c) = \begin{cases} \frac{1+\delta-\delta c(1+2\delta)}{2(1+\delta)} + \delta c & \text{if} \quad c \leq \hat{c} \\ \\ \min\{2c, 2\bar{c}\} & \text{if} \quad c > \hat{c}, \end{cases} \tag{14}$$

Figures 3(a)-(c) show the specific case of $\delta = 1$.[25] For $c = 0$, the outcome results in no recognition, with the firm setting the monopoly price in each period, 50% of consumers purchasing in each period, and all of them anonymizing. At the other extreme, if $c \geq \bar{c} = .3$, the full-recognition outcome obtains in which 40% of consumers purchase in the first period, 70% purchase in the second period, and no consumer anonymizes. Comparative statics over the range of costs slightly greater than $\hat{c} = 2/7$ are particularly interesting. As Figures 3(a) and (b) show, there is a steep increase in the first-period price and a steep decline in first-period purchases. The reason can be gleaned from Figure 3(c): once no consumer anonymizes (at $c = \hat{c}$), as c increases, the firm raises its profit by intensifying price discrimination. It does so by raising the first-period price, which, as the next subsection showcases, is to the detriment of consumers.

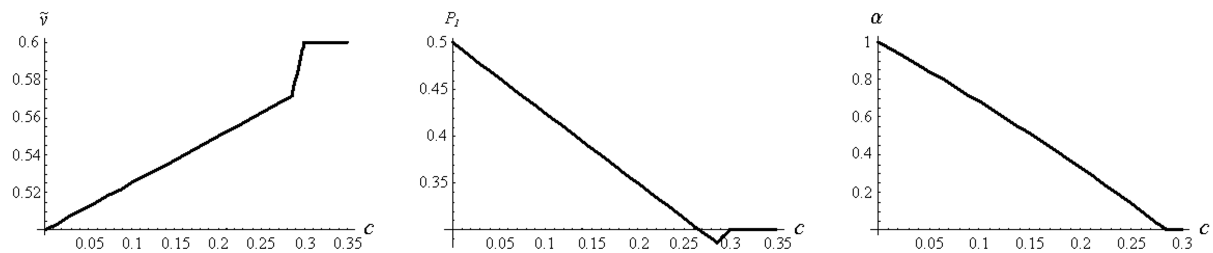


Figure 3: Comparative statics in the case with uniform valuations and $\delta = 1$.

4.6 Consumer Surplus

We now turn to consumer surplus. Proposition 4 shows that the firm obtains its highest profit when consumers can costlessly maintain their anonymity. However, this does not immediately imply that consumer surplus is at its lowest in this case, because the total surplus may vary depending on the cost of anonymizing. Specifically, since there is no cost to production, the efficient outcome in this model – the first best – would be for every consumer to obtain the good in each period. Hence, the efficient outcome is not obtained for any $c \geq 0$, since some consumers do not purchase.

For $c \in [0, \hat{c}]$ and $c \geq \bar{c}$, consumer surplus as a function of c, $CS(c)$, is given by[26]

$$\underbrace{\int_{\tilde{v}}^{1} vf(v)dv - (1 - F(\tilde{v}))(\tilde{v} - \delta c)}_{(\star)} + \underbrace{\delta\Big(\int_{\tilde{v}-c}^{1} vf(v)dv - (1 - F(\tilde{v} - c))\tilde{v} + (F(\tilde{v}) - F(\tilde{v} - c))c\Big)}_{(\star\star)} \quad (15)$$

In (15), $(\star)$ is consumer surplus from first period transactions: consumers with valuations $v \in [\tilde{v}, 1]$ purchase the good and pay a price $p_1 = \tilde{v} - \delta c$; $(\star\star)$ is consumer surplus from period two transactions:

[25] Given $\delta = 1$, we have $\hat{c} = 2/7$ and $\bar{c} = 3/10$. For $c \in [0, 2/7]$, we have $p_1 = \frac{1}{4}(2 - 3c)$, $\tilde{v} = \frac{2+c}{4}$, and $\alpha = \frac{2-7c}{2-c}$. For $c \in [2/7, 3/10]$, we have $p_1 = c$, $\tilde{v} = 2c$, and $\alpha = 0$.

[26] Proposition 5 shows that the expression in (15), evaluated at $c = \bar{c}$, gives a lower bound on consumer surplus for $c \in (\hat{c}, \bar{c})$. Consumer surplus for $c \geq \bar{c}$ equals consumer surplus at $c = \bar{c}$, since the equilibrium outcome is unchanged.

consumers with valuations $v \in [\tilde{v}, 1]$ are repeat customers and end up expending $\tilde{v}$ (factoring in the cost c), and consumers with valuations $v \in [\tilde{v} - c, \tilde{v}]$ are first-time customers who receive a price discount of c.

Proposition 5 (Consumer surplus)**.** *(i) Consumer surplus, $CS(c)$, is non-increasing over $[\hat{c}, \bar{c}]$, with $CS(c) = CS(\bar{c}) < CS(\hat{c})$ for all $c \geq \bar{c}$. (ii) In the case where valuations are uniformly distributed, CS is non-monotone in c, reaching a global minimum at $c = 0$, a global maximum at $\hat{c}$, strictly increasing in c over $[0, \hat{c})$, and strictly decreasing over $[\hat{c}, \bar{c}]$. Moreover, each consumer is individually (weakly) better off under $c > 0$ than at $c = 0$.*

Surprisingly, Proposition 5 shows that consumers can actually be worse off as the cost of anonymizing increases. In other words, facilitating privacy can actually improve consumers' welfare. The reason is the following. First, for costs $c \in [\hat{c}, \bar{c}]$, no consumer anonymizes. Second, as c rises, the fraction of the market that buys in the first period decreases as $\tilde{v}(c)$ is non-decreasing; moreover, the firm's optimal prices, which are weakly increasing in c, also change to the detriment of consumers. Hence, the cost anonymizing has a passive effect on the market outcome, even if this cost is never incurred in equilibrium.[27]

This finding extends the previous literature, which finds that strategic consumers are better off under full recognition compared to no recognition. The above result brings forth the observation that although consumers may not anonymize in some region of cost (i.e., for $c \in [\hat{c}, \bar{c}]$), they could still be better off when privacy is more accessible, even if they choose not to anonymize. Said another way, there are different shades of "full recognition," and as far as consumers (and overall welfare) are concerned, some are better than others — namely the ones where privacy is more accessible. In this region of costs, consumer surplus suffers, as a greater cost of anonymity enables the firm to intensify its price discrimination while offering higher introductory prices.

The last part of Proposition 5 clearly demonstrates the Prisoner's Dilemma nature of the outcome: when $c = 0$, individually, each consumer chooses to maintain her anonymity; however, every consumer ends up being (weakly) worse off as as a result of there being no price discrimination. The reason is that by anonymizing, consumers impose a negative externality on other consumers; a non-zero cost of maintaining anonymity serves to at least partially alleviate this negative effect by diminishing the incentive to anonymize. We relegate intuition for the rest of Proposition 5 to the next subsection, where we also address social (total) surplus.

[27]We thank an anonymous referee for crystalizing this intuition.

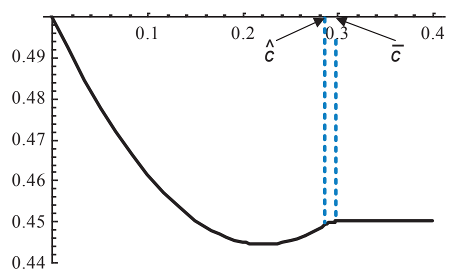
4.7 Social Surplus

Social welfare in our framework can be interpreted in two different ways, depending on whether the cost of anonymizing is deadweight loss, or collected as a fee by a third party (for example, one can rent an anonymous postal box for a fee). In the former case, social surplus equals firm profit plus consumer surplus; in the latter, social surplus is higher than this sum if consumers anonymize at positive costs.

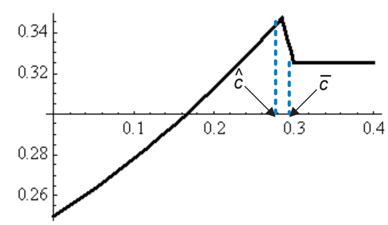
Over the region of anonymizing costs $[\hat{c}, \bar{c}]$, since no consumers choose to maintain their anonymity, social surplus is simply a function of how many consumers purchase in equilibrium. We thus have the following result.

Proposition 6. *(i) Social surplus, $SS(c)$, is non-increasing over $[\hat{c}, \bar{c}]$, with $SS(c) = SS(\bar{c}) < SS(\hat{c})$ for all $c \geq \bar{c}$. (ii,a) With uniform valuations, if the cost c is deadweight loss, SS is non-monotone in c, reaching a global minimum at $c^{\dagger} \in (0, \hat{c})$, a global max at $\hat{c}$, strictly decreasing over $[0, c^{\dagger})$, strictly increasing over $[c^{\dagger}, \hat{c}]$, and strictly decreasing over $[\hat{c}, \bar{c}]$; (ii,b) if c is not wasted, SS differs from (ii,a) by having a global minimum at $c = 0$ and strictly increasing over $[0, \hat{c}]$.*

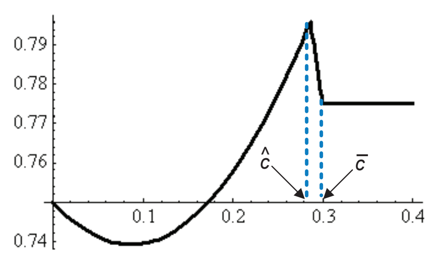
Figures 4(a)-(d) show comparative statics for the uniform case when $\delta = 1$.



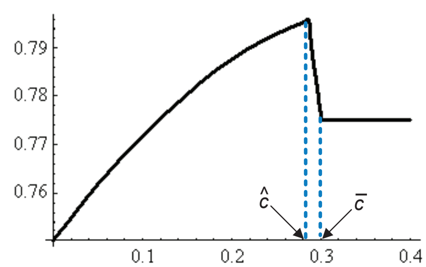
(a) Firm profit



(b) Consumer surplus



(c) Social surplus, c is deadweight loss



(d) Social surplus, c is a fee

Figure 4: Comparative statics when valuations are uniformly distributed and $\delta = 1$ as a function of c.

The intuition is as follows: when $c = 0$, all consumers who purchase in the first period choose to maintain their anonymity. As c begins to rise, consumers must pay a non-trivial cost in order to anonymize. The firm resorts to reducing the first-period price to counteract the negative effect on consumers' buying incentives: consumers are reluctant to purchase in the first period because of the cost they will incur in the second period either from anonymizing or from paying a high price. This results in a lower profit for the firm than it would have had with a lower c, but in higher consumer surplus. In the case where c is deadweight loss, this also results in lower social surplus because many consumers anonymize in this region of cost.

As c approaches $\hat{c}$, fewer consumers anonymize. This gives the firm more flexibility in setting second-period prices, allowing it to better price discriminate which leads to a slight increase in profit. The firm continues to depress the first-period price over this range; additionally, better price discrimination allows the firm to target more low valuation consumers in the second period. This results in higher consumer surplus. Hence, as c approaches $\hat{c}$, both profit and consumer surplus are increasing so that social surplus is increasing when the cost c is wasted. When it is not wasted, social surplus is increasing but not as steeply because there is no surplus recovered directly from fewer consumers anonymizing (we recall that in this case, social surplus equals the sum of profit, consumer surplus, and the total cost of anonymizing). When c is in $[\hat{c}, \bar{c}]$, no consumer anonymizes. The firm increases prices in this range in order to better price discriminate in the second period, which results in fewer consumers purchasing and lower surplus overall.

5 An Endogenous Cost of Anonymity

In this section, we consider two simple extensions of our model. In the first, the firm itself is able to directly control the cost of maintaining anonymity, with the possibility of the firm collecting this cost as a fee. In the second, a third party — a privacy gatekeeper — enables consumers to shop anonymously, for a fee of its choosing.

5.1 In-House Privacy

In the case where the firm sets the cost of anonymizing, c, there are several possibilities to consider. First, the firm may be able to set c at the beginning of the game; alternatively, the firm may not be able to commit *ex ante* to a particular level of c, whereby c is only determined after first-period purchases take place. Second, the firm may also be able to collect c as a fee. The following result characterizes the firm's behavior.

Proposition 7 (In-house privacy)**.** *If the firm can set the cost of anonymizing, c, (i) prior to first-period purchases, it would set $c = 0$, and the no-recognition outcome results; (ii) after first-period purchases, it would set $c \geq \bar{c}$, resulting in full recognition. The firm would make the same choices were it to collect c as a fee.*

Figure 5 depicts the firm's profit when it does and does not collect the anonymizing cost c as a fee for the case of uniform valuations and $\delta = 1$, where c is set prior to first-period purchases.

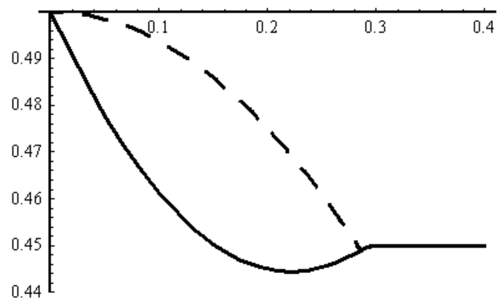


Figure 5: Firm's profit as a function of c. Dashed line: c is collected as a fee; Solid line: c is not collected.

While the firm's ability to set the cost of anonymizing up front gives it the power to commit to having the same prices in both periods, perhaps more interesting is that the firm can do no better than the no-recognition outcome, even when it can collect anonymity payments as fees. We also observe that if the cost of maintaining anonymity were constrained to be above a relatively high threshold (e.g., a threshold $c \in [\hat{c}, \bar{c})$ resulting from a minimum level of anonymizing effort), it is in the firm's best interest to make it costlier for consumers to anonymize. In fact, the firm's profit is maximized when it increases the cost of anonymizing to a prohibitively high level. Therefore, the firm's ability to set the cost of anonymizing up front tends to work to the detriment of its customer base.[28]

When the firm cannot commit to the cost of anonymizing up front, it would choose to set it at a prohibitively high level, which results in the full-recognition outcome. Hence, the firm's lack of commitment regarding anonymizing costs can work to the benefit of consumers (e.g., relative to the case where the cost of anonymizing is low). It is interesting that in this case as well, the firm effectively chooses not to collect any anonymity payments (when it has the ability to do so) by making these payments prohibitively large.

5.2 Privacy Gatekeeper

Let us now suppose that the firm does not control the cost of maintaining anonymity. Instead, there is a third party — a privacy gatekeeper of sorts, that is able to set and commit to a fee for

[28]In our base model, the firm essentially subsidizes anonymity costs by offering a discounted first-period price. However, this is not a "true" subsidy in the sense that when consumers face their anonymizing decisions, they take into account the full cost, c. Indeed, in the case with uniformly distributed valuations, the firm has an incentive to subsidize anonymizing costs up front on a range of costs (specifically, lower costs), and in some cases (higher costs) it would prefer to raise the cost of anonymizing. We thank an anonymous referee for bringing this to our attention.

anonymizing at the beginning of the game.[29, 30] We also consider a second possibility, where the gatekeeper can concurrently negotiate with the firm for setting the anonymizing fee at a particular level. This can be modeled most simply with a take-it-or-leave-it offer (e.g., the gatekeeper offers the firm terms given by a price schedule $x, y > 0$, where the firm must pay an amount x for setting $c = y$). We assume that the gatekeeper can commit to a price schedule up front.[31] The following proposition characterizes the gatekeeper's behavior in equilibrium.

Proposition 8 (Privacy gatekeeper)**.** *If a privacy gatekeeper sets the cost of anonymizing* ex ante *and can (i) only charge consumers, it sets $c = c^\star \in (0, \hat{c})$, where $c^\star = \arg\max_c \alpha(c)(1 - F(\tilde{v}(c)))c$ — that is, some consumers pay a fee to the gatekeeper in order to anonymize; (ii) charge consumers* and *negotiate terms with the firm for setting c, it sets $c = 0$ — that is, all purchasing consumers anonymize at no cost and the gatekeeper is paid by the firm.*

The intuition for this result is as follows. When anonymizing is costly, the firm offers a discounted price to consumers who purchase in the first period. These discounts are enjoyed by consumers who subsequently anonymize as well as by those who remain identified. The gatekeeper's revenues from anonymizing fees thus only partially capture profit that is lost to the firm. Hence, if negotiation with the firm is possible, there is always a positive surplus associated with an agreement between the gatekeeper and the firm to make anonymizing costless to consumers.

6 Conclusions

We studied a model in which a firm is able to recognize and price discriminate against its previous customers, while consumers can maintain their anonymity at some cost. We showed that the firm obtains its highest profit when consumers can costlessly maintain their anonymity, but consumers can be better off when maintaining anonymity is costly, though only up to a point. We then considered two extensions to the base model. In the first, the firm controlled the cost of anonymizing, which worked to the detriment of consumers when the firm could set this cost up front. In the second, a third party — a privacy gatekeeper — controlled this cost. Here, we showed that the

[29] For instance, a consumer could rent a postal box from UPS so as not to disclose her home address. However, UPS could make such an "anonymizing service" much simpler: it could provide customers with individualized one-time codes. Consumers would give these codes to sellers instead of their home addresses. When sellers ship purchases via UPS, they would print the corresponding code on the label, and UPS would use this code to determine the customer's address. Google Checkout provides a related service to mask a consumer's email address in online transactions (at no charge to consumers).

[30] If the fee for anonymizing is charged in the first period, the gatekeeper sets c/δ, which corresponds to setting c in the base model.

[31] The results here readily extend to the more general case where the outcome of a negotiation between the gatekeeper and the firm is obtained using a Nash bargaining solution with arbitrary bargaining powers.

gatekeeper preferred to bargain with the firm and actually set the cost of anonymizing to zero, which also worked to the detriment of consumers.

This paper suggests that certain aspects of consumer privacy may be misjudged by policymakers, firms, and consumer advocacy groups. In particular, facilitating privacy can work to reduce consumer and social surplus when the cost of maintaining anonymity is already low, although the opposite (or a neutral) effect takes place at higher costs. Of course, in practice, many other considerations need to be taken into account that are not in the scope of our model, such as the intrinsic value of privacy, as well as the (possibly accidental) release of sensitive information and corresponding spillover effects—for example, the release of an individual's medical records to her employer.

Our model can, in principle, be extended to take certain other economic considerations into account. An obvious direction is to study a setting with competition. Indeed, we have some initial findings in a similar model with two firms selling differentiated products; these findings suggest that phenomena similar to those identified above continue to occur. Another direction for further study is to consider settings where consumers obtain some benefit from being identified, such as smaller search costs or better technical support. Our preliminary finding here is that the results from the basic model carry through, except the relevant range of costs grows larger.[32] One can also consider an opt-*in* policy. For instance, the firm could pay consumers to be identified (as in the case of membership programs that offer discounts).[33] Another direction is to increase consumer heterogeneity by, for instance, allowing each consumer to have a different, possibly correlated cost for anonymizing. One can also enrich the model by studying consumers with diminishing marginal utility for future units of the product. Finally, it would be interesting to study the steady-state equilibrium of an infinite-horizon version of our model with overlapping generations of consumers.

There are, in fact, a multitude of questions concerning issues of privacy that are both interesting and potentially important. A primary message of this paper is that the answers to such questions may not be as obvious as they first appear.

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[32]Notably, in such environments, consumers could benefit from having multiple accounts: one account to use for obtaining these benefits, and another account to use for potential access to lower prices.

[33]See Campbell et al. (2011) for a recent study of the effects consumers' costs of opting in may have on innovation.

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Appendix

A Omitted Proofs

This section of the appendix contains all of the proofs omitted in the text.

Proposition 1

Proof: Let $\tilde{v}$ denote the cutoff type of the marginal consumer who purchases in the first period, such that $\tilde{v} - p_1 + \delta(\tilde{v} - p_2^1) = \delta(\tilde{v} - p_2^0)$. From this indifference, it follows that the equilibrium is necessarily in threshold strategies, as any types $v > \tilde{v}$ will strictly prefer to purchase in the first period.

Let us initially assume that $\tilde{v} \geq p^\star$. The firm's second-period problems towards identified and anonymous consumers are then given by $p_2^1 = \arg\max_{p \geq \tilde{v}} p(1 - F(p))$ and $p_2^0 = \arg\max_p p(F(\tilde{v}) - F(p))$, respectively. From concavity of $v(1 - F(v))$, it follows that $p_2^1 = \max\{\tilde{v}, p^\star\}$ and p_2^0 satisfies $F(p_2^0) + p_2^0 f(p_2^0) = F(\tilde{v})$. Since $\tilde{v} \geq p^\star$, we have $p_2^1 = \tilde{v}$. Thus, the marginal consumer to purchase in the first period obtains zero payoff in the second. From the indifference equality for the marginal type, we have $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$. Hence, $p_1 = \tilde{v}(1-\delta) + \delta p_2^0$. Implicitly differentiating this expression with respect to p_1, we obtain $\partial\tilde{v}/\partial p_1 - 1 = \delta(\partial\tilde{v}/\partial p_1 - p_2^{0'}\partial\tilde{v}/\partial p_1)$. Thus, $\partial\tilde{v}/\partial p_1 = (1 - \delta(1 - p_2^{0'}))^{-1}$ and equivalently, $\partial p_1/\partial\tilde{v} = (1 - \delta(1 - p_2^{0'}))$. Substituting for p_1 using the above, the firm's first-period problem in this case is given by

$$\max_{\tilde{v}} (\tilde{v}(1 - \delta) + \delta p_2^0)(1 - F(\tilde{v})) + \delta\tilde{v}(1 - F(\tilde{v})) + \delta p_2^0(F(\tilde{v}) - F(p_2^0))$$

The first-order condition with some re-arranging gives

$$1 - F(\tilde{v}) - \tilde{v} f(\tilde{v}) + \delta p_2^{0'}(1 - F(p_2^0) - p_2^0 f(p_2^0)) = 0$$

Simplifying using $F(p_2^0) + p_2^0 f(p_2^0) = F(\tilde{v})$, we have

$$\tilde{v} = (1 + \delta p_2^{0'})\frac{1 - F(\tilde{v})}{f(\tilde{v})} \tag{16}$$

Now, to consider the alternative case, let us assume that $\tilde{v} < p^\star$. Since $p^\star = \frac{1-F(p^\star)}{f(p^\star)}$ and $\tilde{v} < \frac{1-F(\tilde{v})}{f(\tilde{v})}$ for $\tilde{v} < p^\star$, it follows from (16) that $p_2^{0'} \leq 0$. From $\tilde{v} = p^\star < 1$, concavity of $p(1 - F(p))$, and from the fact that p_2^0 satisfies $F(p_2^0) + p_2^0 f(p_2^0) = F(\tilde{v})$, it follows that $p_2^{0'} > 0$, a contradiction. Thus, $\tilde{v} > p^\star$. ■

Lemma 1

Proof: Suppose, for the sake of contradiction, that $p_2^1 - p_2^0 > c$ holds in equilibrium. Then a consumer who purchased in the first period would always anonymize. However, the firm then possesses a profitable deviation by setting $p_2^1 = p_2^0 + c - \epsilon$, for $\epsilon \in (0, c)$. This is because no fewer consumers would purchase in the first period, and no consumer would anonymize, increasing the firm's second-period profit from each of its original repeat customers by $c - \epsilon$. Thus, $p_2^1 - p_2^0 \leq c$. Now, assume on the contrary that a positive measure of consumers anonymizes in equilibrium. Then $p_2^1 - p_2^0 = c$ must hold. The firm then possesses a similar profitable deviation by setting $p_2^1 = p_2^0 + c - \epsilon$ for $\epsilon > 0$ small. ■

Proposition 2

Proof: For any p_1, since it is costless to anonymize, all consumers with valuations $v \geq p_1$ will purchase the good in the first period. If $p_2^1 < p_2^0$, no purchasing consumers will anonymize. In this case, however, since the firm sets period 2 prices after consumers decide whether or not to maintain their anonymity, p_2^1 targets identified consumers in $[\underline{v}, 1]$, where $\underline{v} \leq p_1$ (some range of additional consumers $[\underline{v}, p_1]$ will decide to purchase in the first period in order to obtain the discount in the second period). On the other hand, p_2^0 targets the anonymous consumers in $[0, \underline{v}]$ (and there will be at least some consumers in this interval, because f is positive on $(0, 1)$). Hence, setting $p_2^1 < p_2^0$ cannot be a best response for the firm. Thus, in every equilibrium, $p_2^1 \geq p_2^0$.

Since consumers anticipate that $p_2^1 \geq p_2^0$, it is a best response for consumers who purchased in the first period to anonymize. We now show that there is a PBE where all of them use this best response; moreover, we characterize all the PBEs in which this is the case. If all of them anonymize, then all consumers are anonymous in the second period, and the firm sets $p_2^0 = p^\star$ to maximize period 2 profit. Moreover, given that in the second period everyone will be anonymous, only consumers with valuations $v \in [p_1, 1]$ purchase in the first period, so that $\tilde{v} = p_1$. Hence, the firm's first-period problem is to choose p_1 to maximize $(1 - F(p_1))p_1 + \delta(1 - F(p^\star))p^\star$, which results in $p_1 = p^\star$. Since no consumer is identified in period 2, the firm's beliefs about identified consumers' valuations are off equilibrium. Consistent off-equilibrium beliefs here are, for example, for the firm to believe identified consumers' valuations to be at least $\tilde{v} = p_1 = p^\star$, so that setting $p_2^1 \geq p^\star$ is a best response.

Now, suppose, for the sake of contradiction, that there are equilibria in which some consumers do *not* anonymize on the path of play. For this to be the case in equilibrium, since $p_2^1 \geq p_2^0$, we must have $p_2^1 = p_2^0$, otherwise no consumer would choose to stay identified. Let $p_2^1 = p_2^0 = \tilde{p}$. First, we will show $\tilde{p} = p^\star$. For the sake of contradiction, suppose not, that is $\tilde{p} \neq p^\star$. Then in period 2, only consumers with valuations in $[\tilde{p}, 1]$ purchase, and the firm's period 2 profit is given by $\tilde{p}(1 - F(\tilde{p}))$. However, if the firm sets $p_2^1 = p_2^0 = p^\star$ in the second period, consumers with valuations in $[p^\star, 1]$ would buy, resulting in period 2 profit of $p^\star(1 - F(p^\star))$ — which is strictly higher since $p^\star$ uniquely maximizes $p(1-F(p))$. Hence, $p_2^1 = p_2^0 = p^\star$ holds in this case, and the firm's period 2 profit is given by $p^\star(1 - F(p^\star))$ for any p_1. Thus, it is profit maximizing for the firm to set $p_1 = p_2^0 = p_2^1 = p^\star$.

Assume now that some positive mass of consumers who purchased in period 1 stays identified. Let G denote the (un-normalized) distribution of anonymous consumers in period 2, so that $\int_0^1 \mathrm{d}G(p) < 1$. $G(p)$ coincides with $F(p)$ up to $p = p^\star$ because identified consumers can only be in $[p_1, 1] = [p^\star, 1]$. Thus, $F(p^\star) = G(p^\star)$. Let $g^-(p^\star) = \lim_{p \to p^\star} dG(p)/dp$. $g^-(p^\star)$ exists and satisfies $g^-(p^\star) = f(p^\star)$ because F is twice differentiable.[34] In period 2, the firm sets prices optimally to each group of consumers, and from our above observations, this has to result in $p_2^0 = p^\star$ in a PBE. Since $p(1 - F(p))$ is concave, the first-order condition that gives $p^\star$ is $1 - F(p^\star) - p^\star f(p^\star) = 0$. However, the first-order condition of the firm's problem in pricing towards anonymous consumers, evaluated at $p_2^0 = p^\star$, gives $\int_0^1 \mathrm{d}G(p) - G(p^\star) - p^\star g^-(p^\star) = \int_0^1 \mathrm{d}G(p) - F(p^\star) - p^\star f(p^\star) < 0$ since $\int_0^1 \mathrm{d}G(p) < 1$. Hence, for some $\epsilon > 0$, the seller is better off setting $p_2^0 = p^\star - \epsilon$ than setting $p_2^0 = p^\star$, resulting in the desired contradiction. ■

Lemma 2

Proof: Assume on the contrary that there exists an equilibrium in which all consumers anonymize. Since all consumers are anonymous in the second period, the seller sets $p_2^0 = p^\star$. Since consumers must find maintaining anonymity to be a best response, we have $p_2^1 \geq p_2^0 + c = p^\star + c$; in addition,

[34] We note that if f is discontinuous, then PBEs in which some consumers stay identified do exist.

we have $\tilde{v} \geq p^\star + c$, else, a positive mass of consumers would not purchase in the second period and thus would not anonymize. The resulting second-period profit for the firm is $p^\star(1 - F(p^\star))$.

The indifference condition for the cutoff type $\tilde{v}$ is thus given by $\tilde{v} - p_1 + \delta(\tilde{v} - c - p^\star) = \delta(\tilde{v} - p^\star)$, which gives $p_1 = \tilde{v} - \delta c$. Combined with $\tilde{v} \geq p^\star + c$, we have $p_1 \geq p^\star + (1 - \delta)c$.

The firm sets p_1 to maximize present-discounted profits, given by $(1 - F(p_1 + \delta c))p_1 + \delta p^\star(1 - F(p^\star))$. The first-order condition gives $p_1 = \frac{1-F(p_1+\delta c)}{f(p_1+\delta c)}$. From concavity, $\frac{1-F(p)}{f(p)}$ is decreasing in p at $p^\star$. Thus, $p_1 = \frac{1-F(p_1+\delta c)}{f(p_1+\delta c)} \leq \frac{1-F(p^\star+c)}{f(p^\star+c)} < \frac{1-F(p^\star)}{f(p^\star)} = p^\star$, a contradiction. ■

Lemma 3

Proof: From Lemma 2, $\alpha < 1$, i.e., some consumers do not anonymize. From the seller's second period maximization problem it then follows that $p_2^1 = \max\{p^\star, \tilde{v}\} = \tilde{v}$. The seller's second period prices p_2^1 and p_2^0 both target the range of consumers in $[\tilde{v}, 1]$ (technically, this holds for p_2^0 only when $\alpha > 0$). However, p_2^0 also targets consumers in $[0, \tilde{v}]$ (for any α). It directly follows from the seller's maximization problem (3) that $p_2^0 \leq p_2^1$. Moreover, since consumers anticipate in the first period that $p_2^0 \leq p_2^1$, no consumer would purchase in the first period if the price p_1 exceeds her valuation. Thus, $p_1 \leq \tilde{v}$.

Now, assume on the contrary that $p_2^0 > p_1$ is part of an equilibrium; then no consumer skips purchasing in the first period to purchase in the second (in other words, there is no point to delaying a purchase in hopes of a cheaper price). Consequently, all consumers with valuations $v \geq p_1$ purchase in the first period, giving $\tilde{v} \leq p_1$. It follows that $p_2^0 > \tilde{v}$. But since $p_2^1 = \tilde{v}$, we have $p_2^0 > \tilde{v} = p_2^1$, a contradiction. ■

Lemma 4

Proof: Assume $\tilde{v} \geq p^\star$ holds on the path of play. By Lemma 2, $p_2^1 = \max\{p^\star, \tilde{v}\} = \tilde{v}$. (We show below that $\tilde{v} \geq p^\star$ is indeed satisfied.) From Lemma 3, the solution to the firm's period 2 problem for anonymous consumers satisfies the first-order condition of (3):

$$F(p_2^0(\tilde{v})) + f(p_2^0(\tilde{v}))p_2^0(\tilde{v}) = F(\tilde{v})(1 - \alpha) + \alpha \tag{17}$$

If $\alpha > 0$, in order for consumers who mix between anonymizing and not, $p_2^0 + c = p_2^1$ must hold. Since $p_2^1 = \tilde{v}$, we therefore have $p_2^0 = \tilde{v} - c$. Combining this observation with (17), we obtain:

$$\alpha = \frac{(\tilde{v} - c)f(\tilde{v} - c) + F(\tilde{v} - c) - F(\tilde{v})}{1 - F(\tilde{v})} \tag{18}$$

Substituting using $h(v) = F(v) + vf(v)$ in the above, we immediately obtain (5).

From $p_2^1 = p_2^0 + c = \tilde{v}$, it follows that a consumer with valuation $\tilde{v}$ obtains a payoff of zero in period 2. Moreover, since a consumer with valuation $\tilde{v}$ is indifferent between purchasing now and possibly later and purchasing only later, she is overall indifferent between purchasing only now, purchasing only later, and purchasing now and later (with and without anonymizing). Since this consumer receives zero payoff in period 2, the following equality holds: $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$, i.e., the consumer is indifferent between purchasing only in the first period and only in the second period. Substituting using $p_2^0 = \tilde{v} - c$, we obtain $\tilde{v} = p_1 + \delta c$. Hence, if $\tilde{v} \geq p^\star$, the firm's first-period problem of choosing p_1 is equivalent to choosing $\tilde{v}$ such that $p_1 = \tilde{v} - \delta c$ and $p_2^0 = \tilde{v} - c$. The firm's first-period problem is to choose p_1 to maximize its present-discounted profit:

$$\max_{p_1} \, (1 - F(\tilde{v}(p_1)))p_1 + \delta\big((1 - \alpha)(1 - F(\tilde{v}(p_1)))\tilde{v}(p_1) +$$

$$+(F(\tilde{v}(p_1)) + \alpha(1 - F(\tilde{v}(p_1))) - F(p_2^0(\tilde{v}(p_1))))p_2^0(\tilde{v}(p_1)))$$

Using the above observations, the firm's first-period problem is reduced to

$$\max_{\tilde{v}} \ (\tilde{v} - \alpha\delta c)(1 - F(\tilde{v})) + \delta(\tilde{v} - c)(1 - F(\tilde{v} - c)) \tag{19}$$

Substituting for α using (18) in the above, we obtain

$$\max_{\tilde{v}} \ \tilde{v}(1 - F(\tilde{v})) + \delta(\tilde{v} - c)(1 - F(\tilde{v} - c))) - \delta c(F(\tilde{v} - c) + (\tilde{v} - c)f(\tilde{v} - c) - F(\tilde{v}))$$

Letting $h(v) = F(v) + vf(v)$ and $h'(v) = \partial h(v)/\partial v = 2f(v) + vf'(v)$, the first-order condition is

$$1 - F(\tilde{v}) - f(\tilde{v})\tilde{v} + \delta(1 - h(\tilde{v} - c) - ch'(\tilde{v} - c)) + \delta c f(\tilde{v}) = 0 \tag{20}$$

Rearranging (20), we obtain

$$\delta c + \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \delta \frac{1 - h(\tilde{v} - c) - ch'(\tilde{v} - c)}{f(\tilde{v})} - \tilde{v} = 0 \tag{21}$$

Given a sufficiently small c, (21) can be used to solve for $\tilde{v}$ (we recall $\delta > 0$ is assumed throughout).

To show that $\tilde{v} \geq p^\star$ is indeed satisfied, substitute $\tilde{v} = p^\star$ into (20). Using the fact that $1 - F(p^\star) - p^\star f(p^\star) = 0$ and dividing by δ gives $1 - h(p^\star - c) - c(h'(p^\star - c) - f(p^\star))$. First, it is easy to see that this expression equals 0 when $c = 0$, since $h(p^\star) = 1$. Differentiating this expression with respect to c gives $f(p^\star) + ch''(p^\star - c)$, or alternatively, $f(p^\star) + c(3f'(p^\star - c) + (p^\star - c)f''(p^\star - c))$. Since $3f'(v) + (v)f''(v)$ is assumed non-negative for $v \in [0, 1]$ and $f(p^\star) > 0$, this derivative is positive. Hence, the first-order condition in (21) evaluated at $\tilde{v} = p^\star$ is nonnegative and strictly positive for $c > 0$. It follows that $\tilde{v} \geq p^\star$.

Rearranging (21) gives

$$1 + \delta(1 - h(\tilde{v} - c) - ch'(\tilde{v} - c)) = (\tilde{v} - \delta c)f(\tilde{v}) + F(\tilde{v}) \tag{22}$$

By assumption, $v(1 - F(v))$ and its derivative are concave. It follows that the left-hand side of (22) is decreasing in $\tilde{v}$ while the right-hand side is increasing. Hence, a unique $\tilde{v}$ satisfies the first-order condition, giving a unique behavior in a pooling equilibrium on the path of play.

The firm's problem is defined over a compact interval, where profit is not maximized at neither boundary $\tilde{v} = 0$ nor $\tilde{v} = 1$. Moreover, the firm's first and second-period problems are well defined given $\tilde{v} \geq p^\star$ and the solution to the firm's problem indeed satisfies $\tilde{v} \geq p^\star$. It follows that a pooling equilibrium exists.

We note that equilibrium behavior off the path of play has not been specified. In the first period, for instance, if the firm sets $p_1 = 0$, there exists an equilibrium of the continuation game in which all consumers buy. If no consumer anonymizes, the firm does not learn anything about identified consumers. In this case, the firm sets $p_2^1 = p^\star$, while any $p_2^0 \geq p^\star$ can be sustained since beliefs about anonymous consumers are off path.

In the second period, per Lemma 2, for all $c > 0$, there is a positive mass of both identified and anonymous consumers on the path of play. Aside for the case of $c = 0$, addressed in Proposition 2, and the case of $p_1 \geq p^\star + (1 - \delta)c$, addressed in Lemma 2, there are no other non-trivial[35]

[35] There are off-path situations in which consumers behave in a non-utility maximizing way, but such individual behavior does not affect the prices offered by the firm nor the firm's beliefs. Similarly, off-path situations following $p_1 = 1$ (no consumer purchases in the first period) or $p_1 = 0$ (all consumers purchase in the first period) are obvious. These prices cannot be sustained in equilibrium: the firm possesses profitable deviations in the first period by setting

off-equilibrium beliefs on the continuation game that follows the first period. ■

Lemma 5

Proof: We have shown in Proposition 2 that the pooling equilibrium outcome coincides with the no-recognition benchmark outcome when $c = 0$. Let p_1^{FR}, $p_2^{0,FR}$, and $p_2^{1,FR}$ denote the first and second-period prices in the full-recognition benchmark outcome, respectively. Let $\hat{c}$ denote the smallest $c > 0$ such that $\alpha(\hat{c}) = 0$, and let $\bar{c} = p_2^{1,FR} - p_2^{0,FR}$.

It is straightforward to see that in the model with anonymizing, given cost $\bar{c}$, the full-recognition outcome is obtainable by the seller. Namely, if the seller sets $p_1 = p_1^{FR}$, then indeed $p_2^1 = p_2^{1,FR}$, $p_2^0 = p_2^{0,FR}$, and $\alpha(\bar{c}) = 0$ are part of a pooling equilibrium.

Assume on the contrary the firm possesses a profitable deviation by setting a different first-period price, which would lead to a different pooling equilibrium and give it higher profit. If the deviation maintains $\alpha = 0$, it would have been possible in the full-recognition benchmark — a contradiction. Thus, $\alpha > 0$ must occur in this deviation. Since $\alpha > 0$, we have $p_2^1 - p_2^0 = \bar{c}$ to satisfy indifference between anonymizing and not. However, then the same deviation is possible in the full-recognition benchmark, only without consumers' ability to anonymize, and thus a higher profit, contradicting the firm setting prices optimally.

An analogous argument can be made for any $c > \bar{c}$. It follows that for any $c \geq \bar{c}$, the pooling equilibrium outcome coincides with the full-recognition benchmark outcome. ■

Lemma 6

Proof: From Lemma 5, it immediately follows that there exists $\hat{c} \in (0, \bar{c}]$ such that $\alpha(\hat{c}) = 0$. Since $\hat{c}$ is the smallest cost at which no consumer anonymizes, the first-order condition (henceforth FOC) of the firm's first-period problem (4) is satisfied at $\hat{c}$. We can then apply Lemma 4 to obtain $p_2^1 - p_2^0 = \tilde{v} - p_2^0 = \hat{c}$.

We recall from (5) that

$$\alpha(c) = \frac{F(\tilde{v}(c) - c) + (\tilde{v}(c) - c)f(\tilde{v}(c) - c) - F(\tilde{v}(c))}{1 - F(\tilde{v}(c))} \tag{23}$$

From $\alpha(\hat{c}) = 0$, we have (where we henceforth use $\tilde{v}$ in leu of $\tilde{v}(c)$ to simplify expressions)

$$F(\tilde{v} - \hat{c}) + (\tilde{v} - \hat{c})f(\tilde{v} - \hat{c}) = F(\tilde{v}) \tag{24}$$

Deriving (23) with respect to $\tilde{v}$ and simplifying using (24), we have:

$$\alpha'(\hat{c}) = \left.\frac{\partial \alpha}{\partial \tilde{v}}\right|_{c=\hat{c}} = \frac{2f(\tilde{v} - \hat{c}) + (\tilde{v} - \hat{c})f'(\tilde{v} - \hat{c}) - f(\tilde{v})}{1 - F(\tilde{v})} \tag{25}$$

We recall from (21) the (expanded) FOC of the firm's first-period problem:

$$\delta c + \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \delta \frac{1 - F(\tilde{v} - c) - (\tilde{v} - c)f(\tilde{v} - c) - 2cf(\tilde{v} - c) - c(\tilde{v} - c)f'(\tilde{v} - c)}{f(\tilde{v})} - \tilde{v} = 0$$

$p_1 \in (0, 1)$, strictly increasing its first-period profit, while it is still able to obtain the same second-period profit by setting $p_2^0 = p_2^1 = p^\star$.

Using (24) and simplifying, the above evaluated at $\hat{c}$ reduces to:

$$\delta\hat{c} + \frac{1-F(\tilde{v})}{f(\tilde{v})}(1+\delta) - \hat{c}\delta\frac{2f(\tilde{v}-\hat{c}) + (\tilde{v}-\hat{c})f'(\tilde{v}-\hat{c}) - f(\tilde{v}) + f(\tilde{v})}{f(\tilde{v})} - \tilde{v} = 0$$

where $-f(\tilde{v}) + f(\tilde{v})$ was added as a wash. We can now simplify further using (25) to obtain

$$\tilde{v} = \frac{1-F(\tilde{v})}{f(\tilde{v})}(1+\delta(1-\alpha'(\hat{c})\hat{c})) \tag{26}$$

We now compare (26) with the corresponding FOC in the full-recognition benchmark to illustrate the relationship between $\hat{c}$ and $\bar{c}$. The firm's FOC in the full-recognition benchmark gives

$$\tilde{v}^{FR} = \frac{1-F(\tilde{v}^{FR})}{f(\tilde{v}^{FR})}(1+\delta p_2^{0',FR}) \tag{27}$$

where

$$p_2^{0',FR} = \frac{f(\tilde{v}^{FR})}{2f(\tilde{v}^{FR}-\bar{c}) + (\tilde{v}^{FR}-\bar{c})f'(\tilde{v}^{FR}-\bar{c})} \tag{28}$$

Since the price $p_2^{0,FR}$ only targets consumers in $[0, \tilde{v}(\bar{c})]$ and is set to maximize $p(F(\tilde{v}(\bar{c})) - F(p))$, it follows that $p_2^{0,FR}$ satisfies $F(\tilde{v}(\bar{c})) = F(p_2^{0,FR}) + p_2^{0,FR}f(p_2^{0,FR})$. From concavity of $p(1-F(p))$, we then have $p_2^{0',FR} < 1$.

For the sake of contradiction, assume that $\hat{c} = \bar{c}$. Then $\tilde{v}(\hat{c}) = \tilde{v}^{FR}$ follows from Lemma 5. For (26) and (27) to yield the same solution, it is required that $p_2^{0',FR} = 1 - \alpha'(\hat{c})\hat{c} = 1 - \alpha'(\bar{c})\bar{c}$. This holds if

$$\frac{f(\tilde{v})}{2f(\tilde{v}-\bar{c}) + (\tilde{v}-\bar{c})f'(\tilde{v}-\bar{c})} = 1 - \bar{c}\frac{2f(\tilde{v}-\bar{c}) + (\tilde{v}-\bar{c})f'(\tilde{v}-\bar{c}) - f(\tilde{v})}{1-F(\tilde{v})} \tag{29}$$

Let $h'(v-c) = 2f(\tilde{v}-\bar{c}) + (\tilde{v}-\bar{c})f'(\tilde{v}-\bar{c})$. Then the above expression reduces to:

$$\frac{f(\tilde{v})}{h'(\tilde{v}-\bar{c})} = 1 - \bar{c}\frac{h'(\tilde{v}-\bar{c}) - f(\tilde{v})}{1-F(\tilde{v})}$$

which further reduces to:

$$f(\tilde{v}) = h'(\tilde{v}-\bar{c})$$

In other words, $\hat{c} = \bar{c}$ requires $f(\tilde{v}) = 2f(\tilde{v}-\bar{c}) + (\tilde{v}-\bar{c})f'(\tilde{v}-\bar{c})$. If we substitute this back in (28) we obtain $p_2^{0',FR} = 1$, a contradiction. ■

Lemma 7

Proof: From (26), the firm's FOC at $\hat{c}$ gives

$$\tilde{v} = \frac{1-F(\tilde{v})}{f(\tilde{v})}(1+\delta(1-\alpha'(\hat{c})\hat{c}))$$

where $\alpha' = \partial\alpha/\partial\tilde{v}$. From (25), we also have

$$\alpha'(\hat{c}) = \frac{2f(\tilde{v}-\hat{c}) + f'(\tilde{v}-\hat{c})(\tilde{v}-\hat{c})}{1-F(\tilde{v})} - \frac{f(\tilde{v})}{1-F(\tilde{v})}$$

Substituting α' into the above FOC and simplifying, we have

$$\frac{1-F(\tilde{v})}{f(\tilde{v})}(1+\delta(1-\frac{2f(\tilde{v}-\hat{c})+f'(\tilde{v}-\hat{c})(\tilde{v}-\hat{c})}{1-F(\tilde{v})}\hat{c}))-(\tilde{v}-\delta\hat{c})=0 \tag{30}$$

For the sake of contradiction, assume that there exists some $k' \in (\hat{c}, \bar{c})$ such that $\alpha(k') > 0$. Since $\alpha(\bar{c}) = 0$, there must exist $k > k'$ such that $\alpha(k) = 0$ *and* the first-order condition (30) is satisfied at $c = k$ (i.e., α turns 0 at k and Lemma 4 can be applied). From Lemma 4, we also have $p_2^0(k) = \tilde{v}(k) - k$. Substituting this into the firm's second-period problem, we obtain

$$(\tilde{v}(k)-k)f(\tilde{v}(k)-k)+F(\tilde{v}(k)-k)=F(\tilde{v}(k)) \tag{31}$$

Due to concavity of $p(1-F(p))$, the latter expression together with (31) can both hold only if $\tilde{v}(k) - \tilde{v}(\hat{c}) > k - \hat{c}$. We thus have $\tilde{v}(k) - \tilde{v}(\hat{c}) > k - \hat{c} \geq \delta(k - \hat{c})$, so that $\tilde{v}(k) - \delta k > \tilde{v}(\hat{c}) - \delta\hat{c}$. Since $\alpha(k) = 0$, the first-order condition at k similarly gives:

$$\underbrace{\frac{1-F(\tilde{v}(k))}{f(\tilde{v}(k))}(1+\delta(\overbrace{\frac{1-F(\tilde{v}(k))-k\big(2f(\tilde{v}(k)-k)+f'(\tilde{v}(k)-k)(\tilde{v}(k)-k)\big)}{1-F(\tilde{v}(k))}}^{(\star\star)}))}_{(\star\star\star)}\underbrace{-(\tilde{v}(k)-\delta k)}_{(\star)} \tag{32}$$

To see that this expression is negative, we first note that $(\star)$ is lower (more negative) than its corresponding term in (30). In addition, the hazard rate $\frac{1-F(v)}{v}$ is decreasing, and since $\tilde{v}(k) > \tilde{v}(\hat{c})$, $1 - F(\tilde{v}(k)) < 1 - F(\tilde{v}(\hat{c}))$.

It remains to show that $(\star\star)$ is smaller, but this follows directly from concavity of $p(1 - F(p))$ and its derivative. Hence, the FOC is violated at $c = k$, a contradiction. It follows that for all $c \in [\hat{c}, \bar{c}]$, $\alpha(c) = 0$. ■

Proposition 3

Proof: Lemmas 5-7 prove parts (1) and (5). Proposition 2 proves part (2) and Lemma 4 proves part (3). It remains to prove part (4).

By Lemma 7, for $c \in [\hat{c}, \bar{c})$, the equilibrium first-period price p_1 and corresponding cutoff type $\tilde{v}$ are low enough to satisfy $\alpha(c) = 0$. From the firm's second-period problem, when $\alpha = 0$, p_2^0 is derived from

$$F(p_2^0(\tilde{v}))+f(p_2^0(\tilde{v}))p_2^0(\tilde{v})=F(\tilde{v}) \tag{33}$$

Let $\bar{v}(c)$ satisfy

$$F(\bar{v}-c)+f(\bar{v}-c)(\bar{v}-c)=F(\bar{v}) \tag{34}$$

From (5), we have

$$\alpha(c)=\frac{F(\tilde{v}(c)-c)+(\tilde{v}(c)-c)f(\tilde{v}(c)-c)-F(\tilde{v}(c))}{1-F(\tilde{v}(c))}$$

Since $\alpha(c) = 0$ holds on $[\hat{c}, \bar{c}]$, $\bar{v}(c)$ denotes the highest cutoff type such that $\alpha = 0$ in this region. The firm's first-period problem is given by

$$\max_{\tilde{v}(c)}\tilde{v}(c)(1-F(\tilde{v}(c)))(1+\delta)+\delta p_2^0(\tilde{v}(c))(F(\tilde{v}(c))-F(p_2^0(\tilde{v}(c)))) \tag{35}$$

subject to $\tilde{v}(c) \leq \bar{v}(c)$, i.e., subject to $\alpha(c) = 0$. Since $\partial \bar{v}(c)/\partial c > 0$, the firm is less constrained as c increases, thus profit is non-decreasing on $[\hat{c}, \bar{c}]$.

Assume on the contrary that $\tilde{v}'(c) < 0$ holds at some $c_1 \in (\hat{c}, \bar{c})$ so that $\bar{v}(c_1) - \tilde{v}(c_1) = \epsilon$. Let $k < c_1$ satisfy $\bar{v}(c_1) - \bar{v}(k) \leq \epsilon$. Then the same strategy is feasible for the firm for all $c \in [k, c_1]$. Since the firm is less constrained as c increases in this region, and an optimal strategy and corresponding outcome in a less constrained problem is feasible in a more constrained problem, $\tilde{v}(c_1)$ would indeed be optimal for $c \in [k, c_1]$. However, one can make a similar argument for any $c \in (c_1, \bar{c})$. In particular, one can make a similar argument for c slightly above c_1. But then $\tilde{v}'(c_1) \geq 0$, a contradiction. Thus, $\tilde{v}'(c) \geq 0$.

From the fact that both $\hat{c}$ and $\bar{c}$ satisfy $F(\tilde{v} - c) + f(\tilde{v} - c)(\tilde{v} - c) = F(\tilde{v})$ and $\hat{c} < \bar{c}$, we have $\tilde{v}(\hat{c}) < \tilde{v}(\bar{c})$. Thus, $\tilde{v}'(c) > 0$ holds for some $c \in (\hat{c}, \bar{c})$. ■

Lemma 8

Proof: Following the proof of part (4) of Proposition 3, the firm's constraint in the region of cost $[\hat{c}, \bar{c}]$ is $\tilde{v} \leq \bar{v}$. Because $\partial \bar{v}(c)/\partial c > 0$, the firm is less constrained as c increases (with the firm's objective and other constraints remaining the same). Thus, profit is non-decreasing. ■

Proposition 5

Proof: From part (4) of Proposition 3, $\tilde{v}(c)$ is non-decreasing on $[\hat{c}, \bar{c}]$ and is strictly increasing for some c in this region. Combining this result with $p_2^1 = \tilde{v}$ and the fact that p_2^0 satisfies $F(p_2^0) + p_2^0 f(p_2^0) = F(\tilde{v})$, where $p(1 - F(p))$ is concave, it follows that in this region, p_2^0 and p_2^1 are non-decreasing in c and strictly increasing for some c. From the indifference condition of type $\tilde{v}$, we also have $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$, or $p_1 = \tilde{v}(1 - \delta) + \delta p_2^0$; hence, p_1 exhibits the same behavior as p_2^0 and p_2^1. Therefore, CS is non-increasing in c over this region, with $CS(c) = CS(\bar{c}) < CS(\hat{c})\ \forall c \geq \bar{c}$. ■

Proposition 7

Proof: For a given cost of anonymizing, c, the firm's profit is bounded above by its profit when it collects c. The expression for the latter on $c \in [0, \hat{c}]$ and $c \geq \bar{c}$ readily follows from (9) and is given by $\tilde{v}(c)(1 - F(\tilde{v}(c))) + \delta p_2^0(1 - F(p_2^0))$.[36] This profit is clearly maximized at $\tilde{v} = p_2^0 = p^\star$, which, by Proposition 2, can only be obtained with $c = 0$, thus proving part (i) of the proposition.

For part (ii), if the firm collects c, it would set $p_2^1 - p_2^0 \leq c$. This follows from observing that if $p_2^1 - p_2^0 > c$, an identified consumer would anonymize, and the firm profits $p_2^0 + c$. However, if the firm sets this fee at c' such that $c' \in (c, p_2^1 - p_2^0)$, an identified consumer would still anonymize (or be indifferent about doing so, in which case the firm's profit is the same whether or not an identified consumer anonymizes), but the firm's profit is strictly higher. The same applies for any $c < p_2^1 - p_2^0$. Hence, $p_2^1 - p_2^0 \leq c$ holds. Anticipating that by anonymizing after the first period, they cannot increase their utility, consumers behave in the first period as they do in the full-recognition outcome. Thus, the firm sets second-period prices as in the full-recognition outcome, giving $c \geq \bar{c}$. Finally, the same outcome results when the firm does not collect c, since its profit is bounded above by its profit when it collects c, and the full-recognition outcome is feasible. ■

Proposition 8

Proof: For part (i), $c^\star = \arg\max_c \delta\alpha(c)(1 - F(\tilde{v}(c)))c$ maximizes the gatekeeper's profit. It is straightforward to see that $c^\star \in (0, \hat{c})$ holds, as when $c = 0$ or $c \geq \hat{c}$, no consumer anonymizes in equilibrium, resulting in 0 profit for the gatekeeper. For part (ii), from Proposition 4, the firm's

[36]For $c \in (\hat{c}, \bar{c})$, since no consumer anonymizes, the expression for profit is bounded above by $\tilde{v}(\bar{c})(1 - F(\tilde{v}(\bar{c}))) + \delta p_2^0(1 - F(p_2^0))$.

profit, $\Pi(c)$, is uniquely maximized at $c = 0$. Hence, for a given $c > 0$, the agreement surplus from switching to a different level of c is no greater than $\Pi(0) - \Pi(c)$. The expression for $\Pi(c)$ in (9) is bounded above by $\Pi(c) + \delta\alpha(c)(1 - F(\tilde{v}(c)))c = \tilde{v}(c)(1 - F(\tilde{v}(c))) + \delta p_2^0(c)(1 - F(p_2^0(c)))$, which is also uniquely maximized at $c = 0$ (for $\tilde{v} = p_2^0 = p^\star$). Thus, for any $c > 0$,

$$\Pi(0) - \left[\tilde{v}(c)(1 - F(\tilde{v}(c))) + \delta p_2^0(c)(1 - F(p_2^0(c)))\right] > 0$$

Rearranging, we obtain

$$\Pi(0) - \Pi(c) > \delta\alpha(c)(1 - F(\tilde{v}(c)))c \tag{36}$$

The RHS in (36) gives the gatekeeper's profit from consumers who anonymize for a given $c > 0$. Therefore, the surprlus from reaching an agreement with the firm for setting $c = 0$ always exceeds the gatekeeper's profit from anonymizing consumers given any other level of c. ■

B An Equilibrium in Pure Strategies

The pooling equilibrium presented in the paper is the only equilibrium that survives the purification refinement mentioned in footnote 22. However, there can exist pure-strategy equilibria of the game. For instance, suppose valuations are uniformly distributed and $\delta = 1$. Consider a PBE in which all types $v \in [\tilde{v}, 1]$ purchase the good in the first period, but the subset of types $v \in [\tilde{v}, \hat{v}]$ choose to be identifiable ($\alpha(v) = 0$) while the complementary subset of types $v \in (\hat{v}, 1]$ choose to be anonymous ($\alpha(v) = 1$). (It can be verified that no equilibrium exists in which the lower interval of types is anonymous and the higher interval is identifiable.) For small values of c, no such equilibrium exists because it would be optimal for the firm to set $p_2^0 = \hat{v}$ and $p_2^1 = \tilde{v}$, which would induce no types to anonymize. When c is sufficiently high, however, the measure of first-period purchasers who choose anonymity is relatively small and the firm essentially 'gives up' on them, setting p_2^0 to attract low valuation consumers who did not purchase in the first period.

Second-period beliefs are then given by

$$F^0(v) = \begin{cases} \frac{F(v)}{F(\tilde{v})+1-F(\hat{v})} & \text{if } v < \tilde{v} \\ \\ \frac{F(\tilde{v})+F(v)-F(\hat{v})}{F(\tilde{v})+1-F(\hat{v})} & \text{if } \hat{v} < v \leq 1 \end{cases}$$

and

$$F^1(v) = \frac{F(v) - F(\tilde{v})}{F(\hat{v}) - F(\tilde{v})} \quad \text{for} \quad \tilde{v} \leq v \leq \hat{v}$$

In the second period, the firm chooses its prices to maximize profits:

$$\max_{p_2^r} \, (1 - F^r(p_2^r))p_2^r \quad \text{for} \quad r = 0, 1$$

for $r \in \{0, 1\}$. The (relevant) solutions are

$$p_2^0 = \frac{1}{2}\left(\tilde{v} + (1 - \hat{v})\right) \tag{37}$$

and

$$p_2^1 = \tilde{v} \tag{38}$$

To determine the marginal consumer type $\tilde{v}$, equate the expected utility from purchasing the good in the first period to the utility from waiting to purchase until the second period:

$$\tilde{v} - p_1 + \tilde{v} - p_2^1 = \tilde{v} - p_2^0$$

Substituting the second-period prices from (37) and (38) and rearranging yields

$$p_1 = \frac{1}{2}\left(\tilde{v} + (1 - \hat{v})\right) \tag{39}$$

The firm's first-period problem is given by

$$\max_{p_1} (1 - F(\tilde{v}))p_1 + (F(\hat{v}) - F(\tilde{v}))p_2^1 + (1 - F(\hat{v}) + F(\tilde{v}) - F(p_2^0))p_2^0 \tag{40}$$

subject to (37), (38), and (39). Eliminating the prices by substituting the constraints into the objective, differentiating with respect to $\tilde{v}$, and rearranging the resulting first-order condition yields

$$\tilde{v} = \frac{2\hat{v} + 1}{5} \tag{41}$$

Since type $\hat{v}$ must be indifferent about anonymizing, we also have $\hat{v} - p_2^1 = \hat{v} - p_2^0 - c$. Substituting for the prices from (37) and (38) and rearranging yields

$$\hat{v} = 2c + 1 - \tilde{v} \tag{42}$$

The equilibrium outcome is then found by solving (41) and (42). In particular

$$\hat{v} = \frac{10c + 4}{7} \tag{43}$$

and

$$\tilde{v} = \frac{4c + 3}{7} \tag{44}$$

As in the equilibrium with mixed strategies, when $c = \frac{3}{10}$, this equilibrium outcome coincides with the full-recognition outcome in which no consumers anonymize($\hat{v} = 1$). We observe that $c = 0$ does not, however, yield the full-commitment outcome. As mentioned above, if c is too small (less than $\frac{1}{109}(4 + 7\sqrt{7}) \approx \frac{1}{5}$ for this example), then no such equilibrium exists, because the firm would rather set $p_2^0 = \hat{v}$ than $p_2^0 = 0.5(\tilde{v} + 1 - \hat{v})$. However, $p_2^0 > p_2^1$ cannot occur in equilibrium, because then no consumer would anonymize.