

Loss Functions for Classification

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Loss Functions for Classification

Classification error:

$$\text{Fraction of times } \text{sign}(f(x_i)) \text{ is not } y_i = \frac{1}{n} \sum_{i=1}^n [y_i \neq \text{sign}(f(x_i))]$$

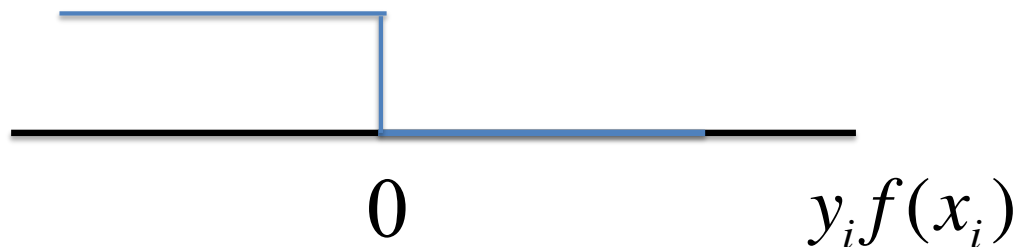
Minimizing this directly is usually computationally hard.

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Classification error:

$$\text{Fraction of times } \text{sign}(f(x_i)) \text{ is not } y_i = \frac{1}{n} \sum_{i=1}^n [y_i \neq \text{sign}(f(x_i))]$$

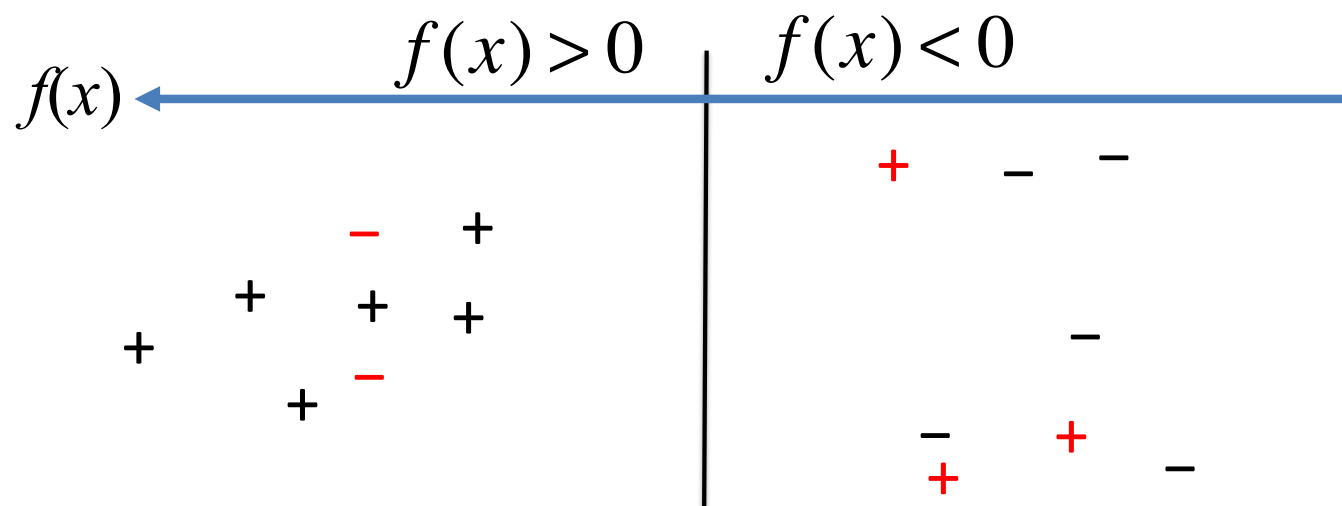
$$[y_i f(x_i) \leq 0] = \frac{1}{n} \sum_{i=1}^n 1_{[y_i f(x_i) \leq 0]}$$



Loss Functions for Classification

Classification error:

Fraction of times $\text{sign}(f(x_i))$ is not $y_i = \frac{1}{n} \sum_{i=1}^n [y_i \neq \text{sign}(f(x_i))]$



$$yf(x) < 0$$

$-$
 $-$
 $+$
 $+$

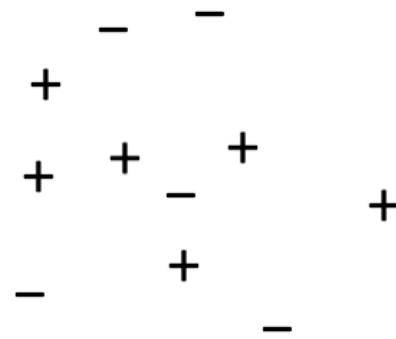
$$yf(x) > 0$$

$-$ $-$
 $+$
 $+$ $+$ $+$
 $-$ $+$ $+$
 $-$ $-$

$$yf(x) < 0$$

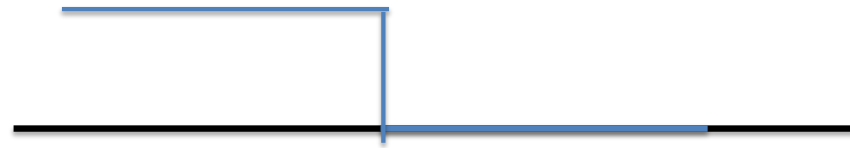


$$yf(x) > 0$$



1

0



$yf(x)$

“margin”

$$yf(x) < 0$$

$-$
 $-$
 $+$
 $+$

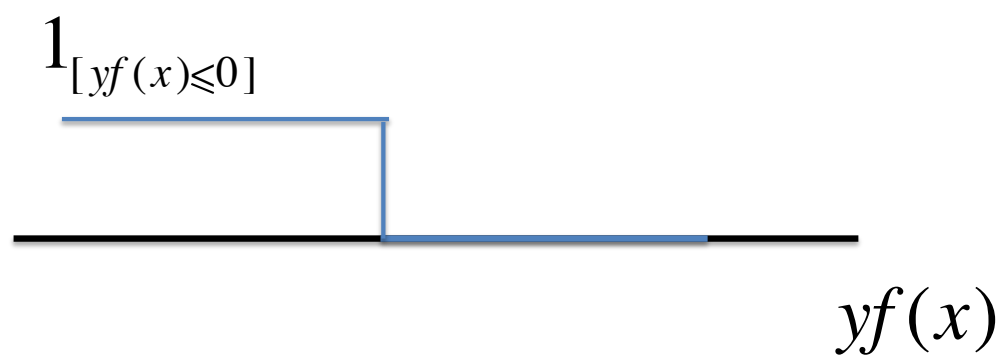
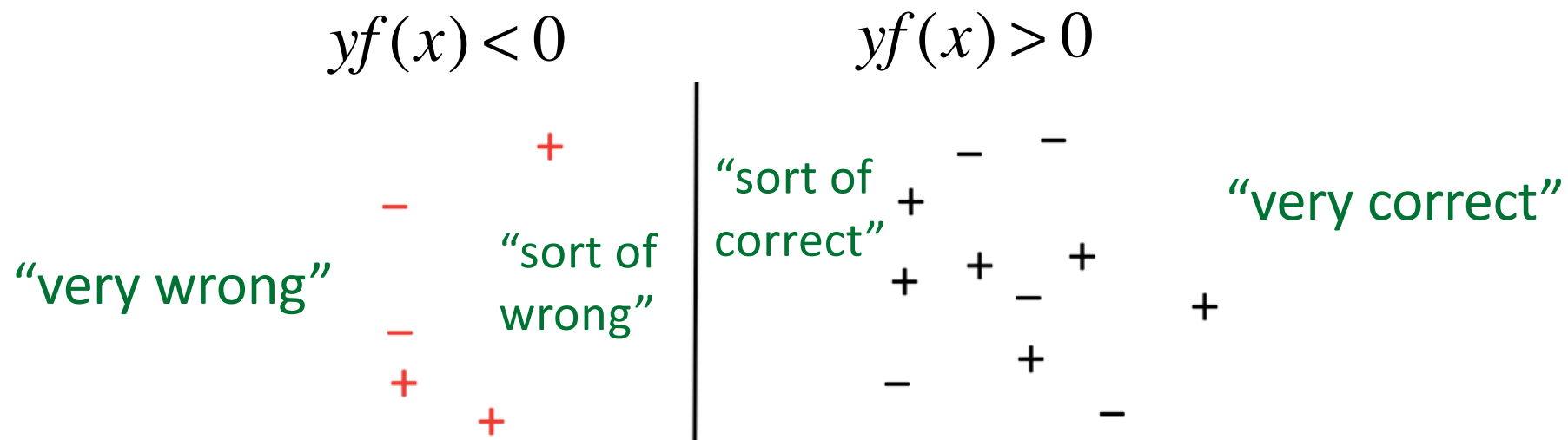
$$yf(x) > 0$$

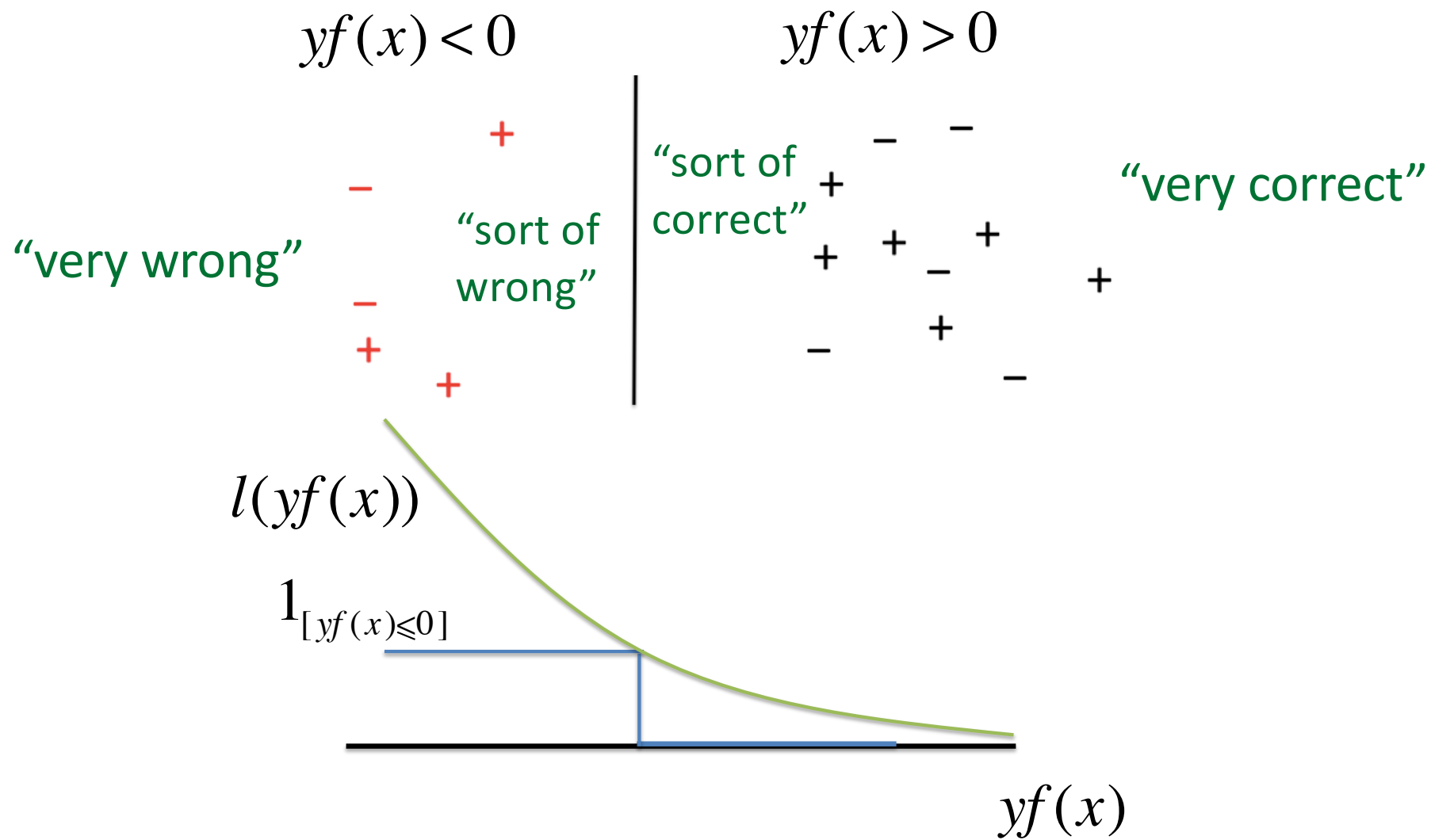
$+$
 $+$
 $-$
 $+$
 $-$

$$1_{[yf(x) \leq 0]}$$

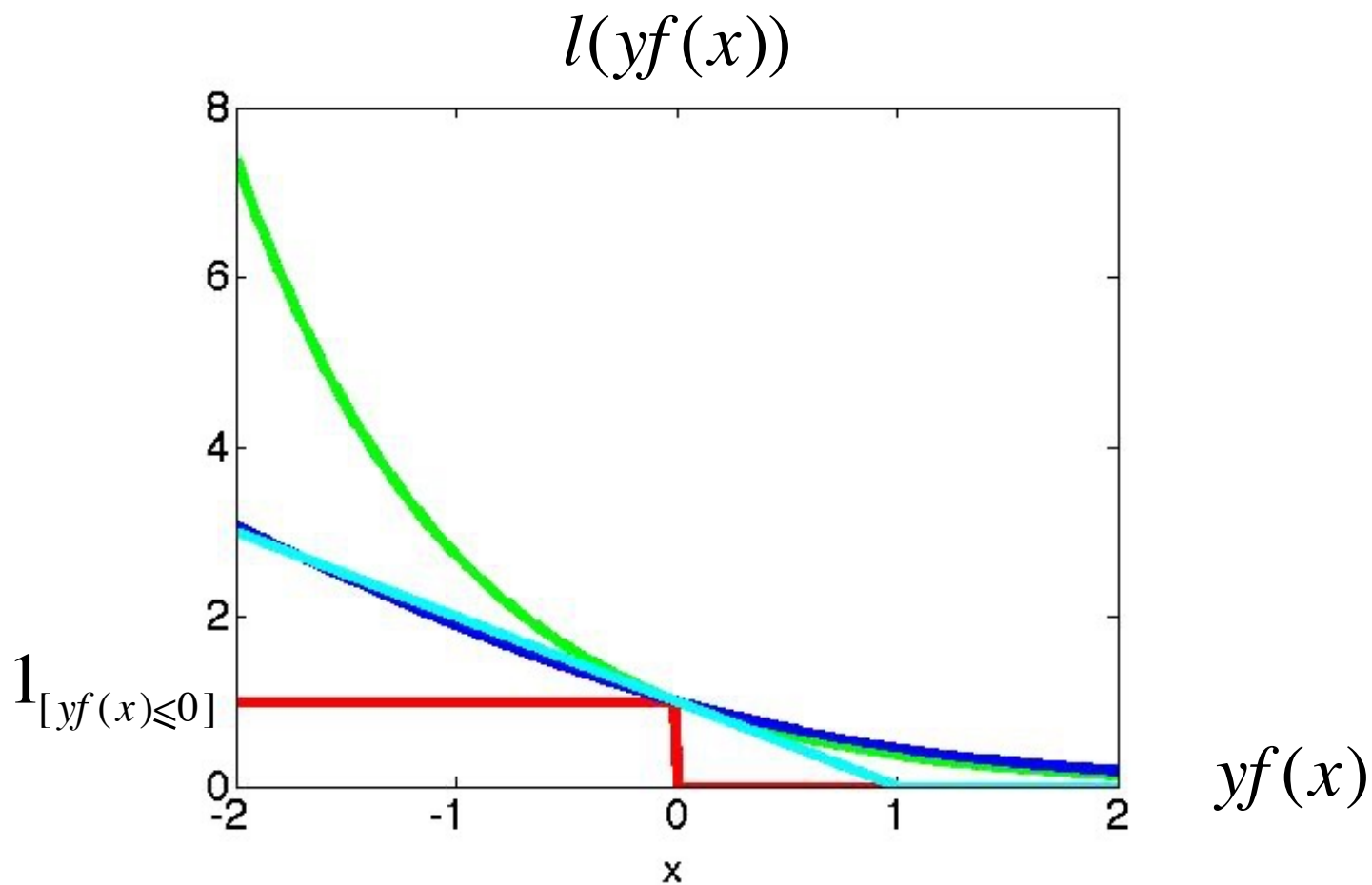


$$yf(x)$$



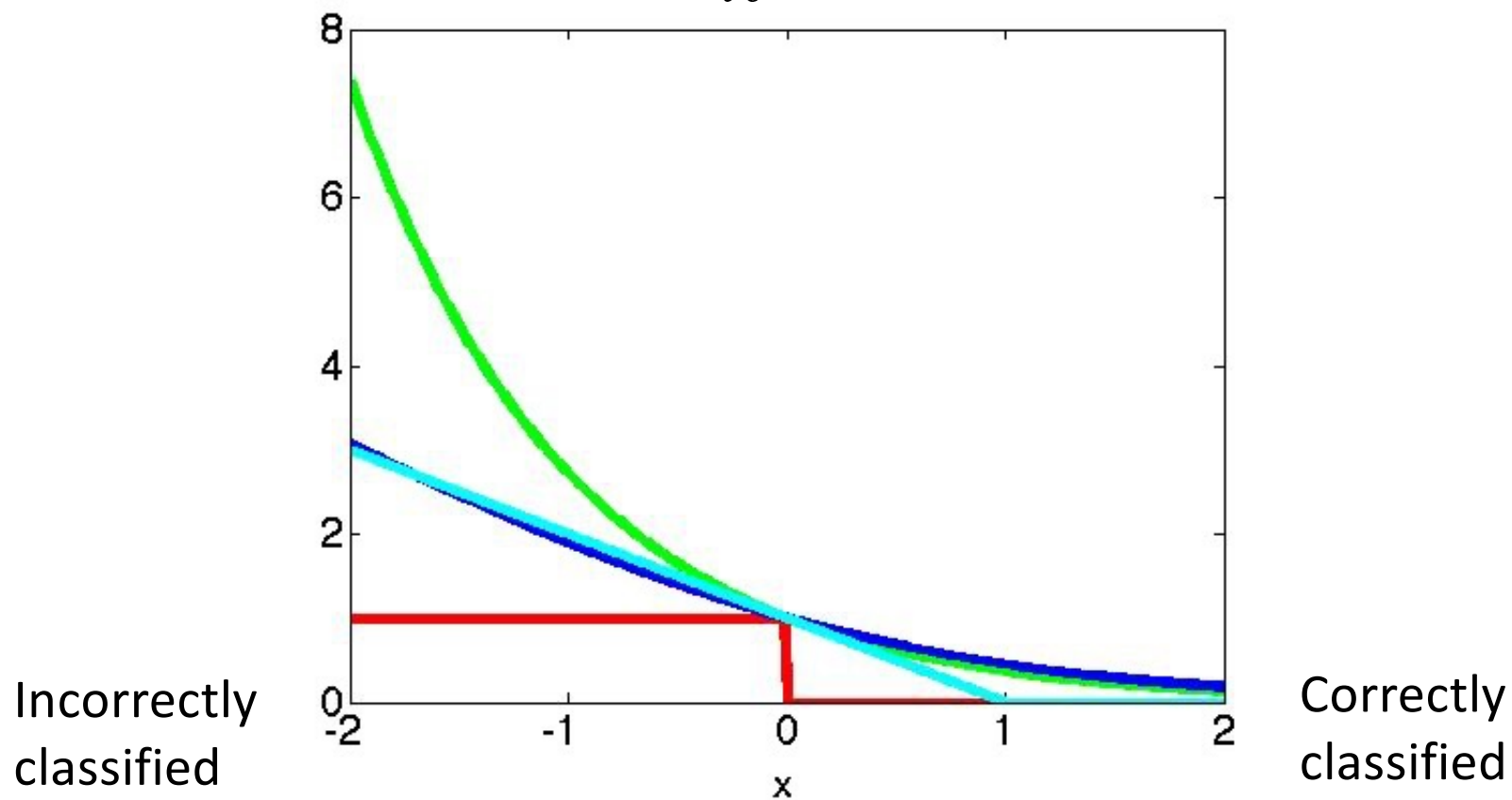


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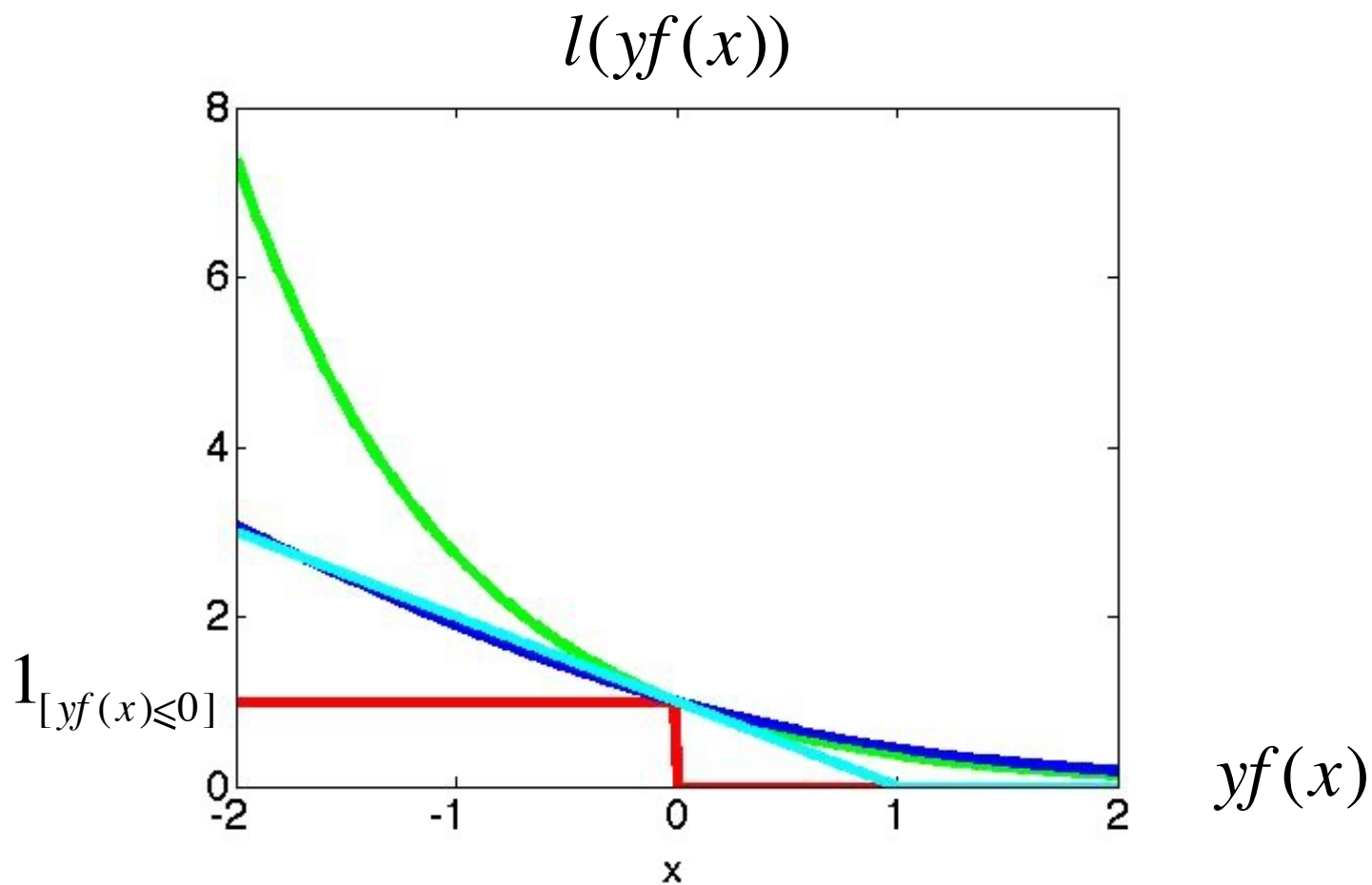


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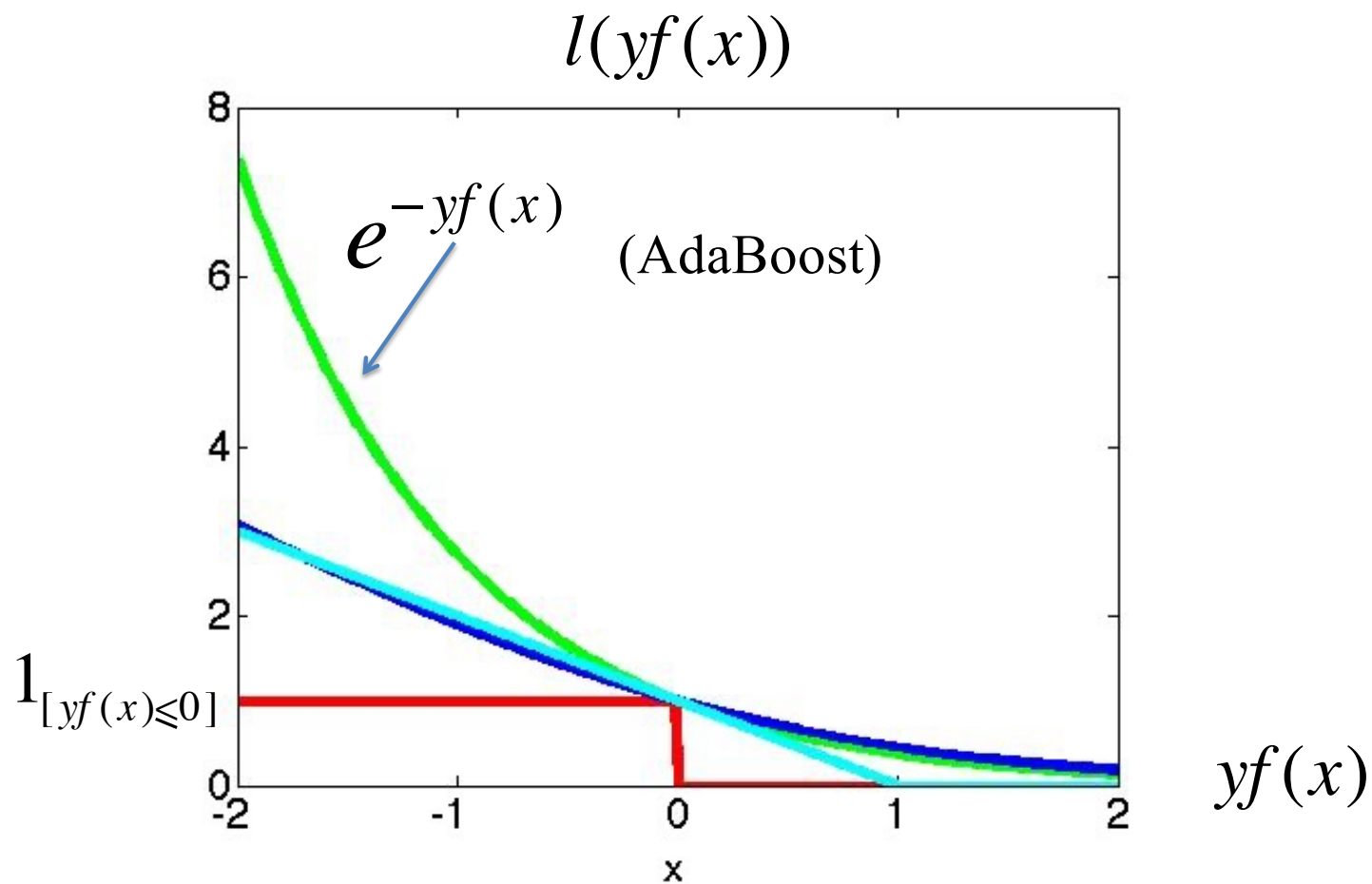
$$l(yf(x))$$



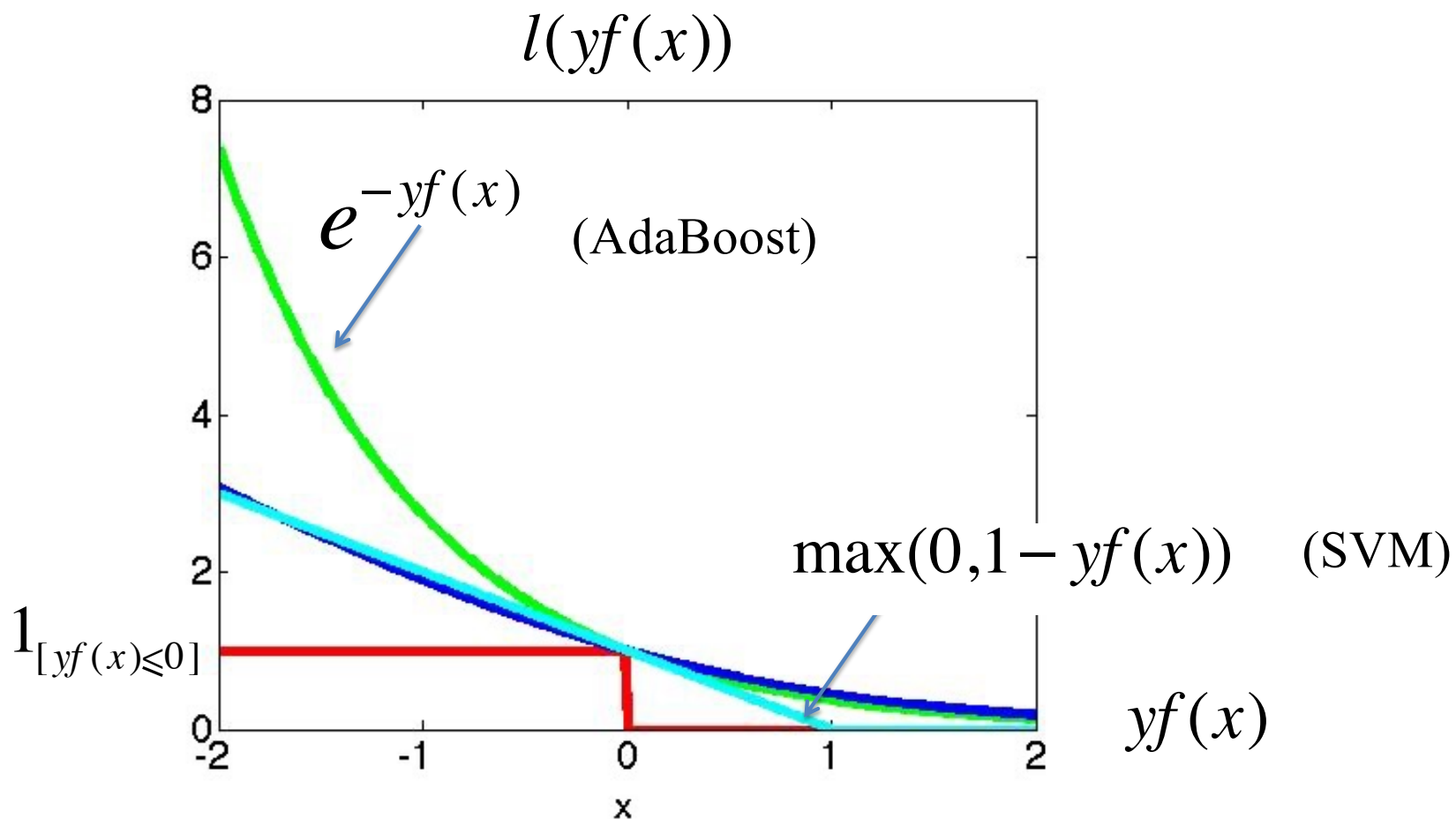
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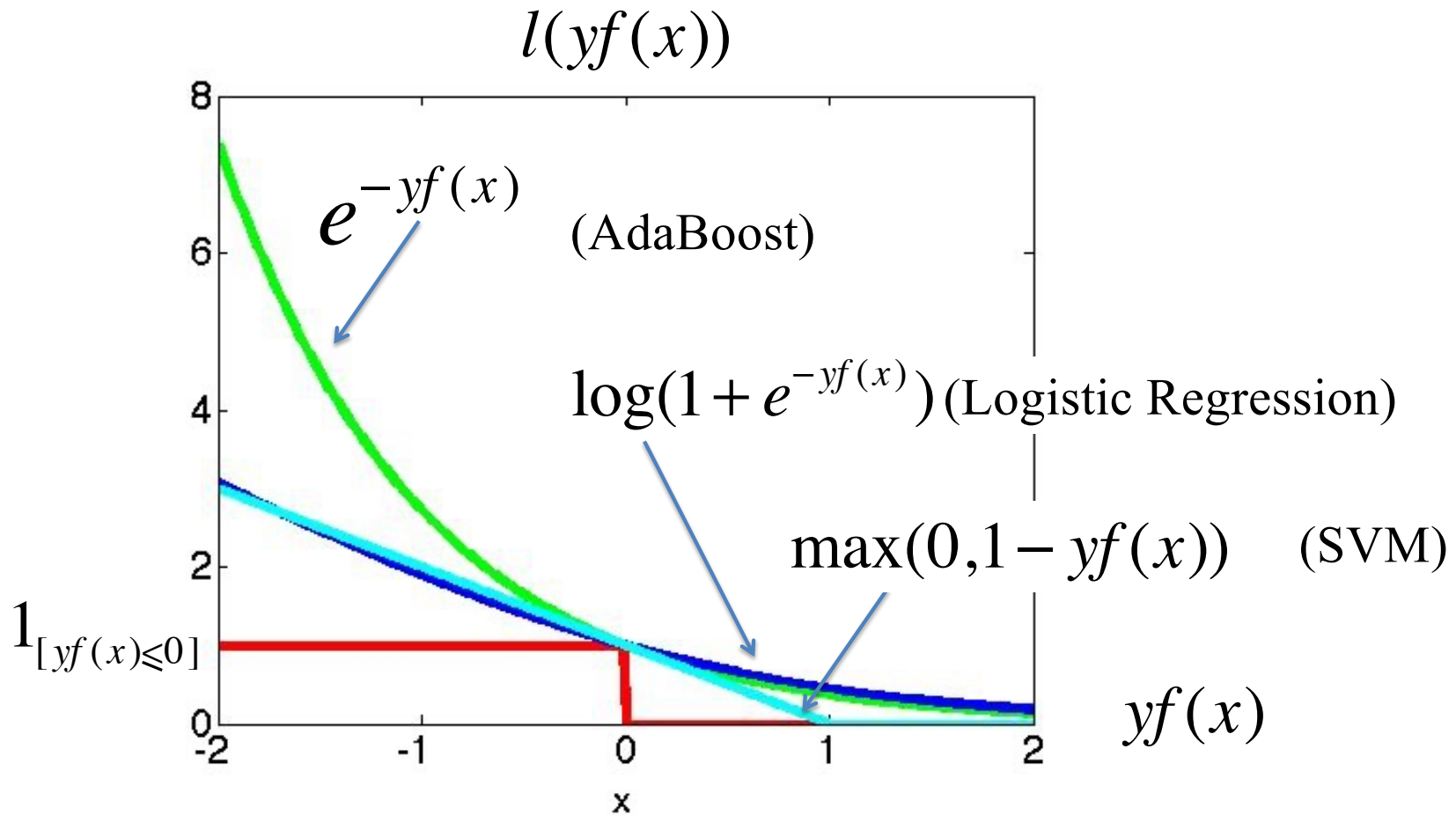
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Loss Functions for Classification



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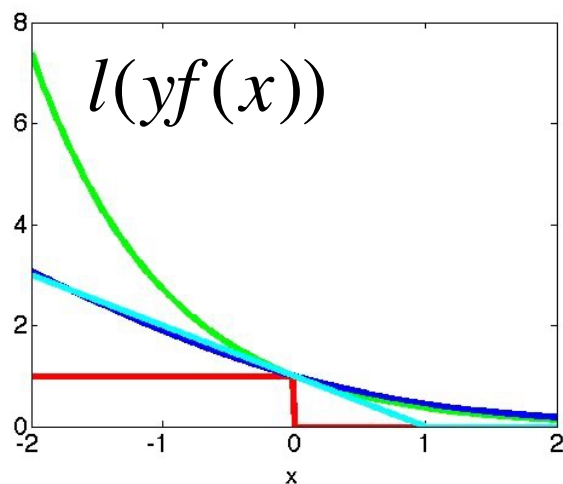


Loss Functions for Classification

Fraction of times $\text{sign}(f(x_i))$ is not y_i $= \frac{1}{n} \sum_{i=1}^n 1_{[y_i \neq \text{sign}(f(x_i))]}$

$$= \frac{1}{n} \sum_{i=1}^n 1_{[y_i f(x_i) \leq 0]}$$

$$\leq \frac{1}{n} \sum_{i=1}^n \ell(y_i f(x_i))$$



To summarize

- Margin: $y_i f(x_i)$ measures how well point i is classified.
- It's computationally hard to minimize the 0-1 loss directly
- Several ML methods minimize upper bounds for the 0-1 loss that involve the margin.