More on ROC Curves for Individual Models

Cynthia Rudin

Machine Learning Course, Duke
ROC Curves

• Adjust the decision boundary

\[ f(x) \]
ROC Curves

- Adjust the decision boundary
ROC Curves

- Adjust the decision boundary

\[ f(x) = 7 \]

- TPR = 1/11
- FPR = 0/12
ROC Curves

- Adjust the decision boundary

\[ f(x) = 3 \]

\[ \begin{array}{cccccccccc}
+ & - & + & - & + & - & - \\
- & + & + & - & - & + & - \\
+ & + & + & + & - & - & - \\
+ & - & + & - & - & - & - \\
\end{array} \]

- TPR = 3/11
- FPR = 2/12
ROC Curves

- Adjust the decision boundary

\[ f(x) = 0 \]

- TPR = 7/11
- FPR = 3/12
ROC Curves

- Adjust the decision boundary

\[ f(x) = -4 \]

- TPR = 10/11
- FPR = 7/12
ROC Curves

• For a particular False Positive Rate (FPR), what is the True Positive Rate (TPR)?
ROC Curves

• Let’s do it without scaling
ROC Curves

- Let’s do it without scaling
ROC Curves

- To do this, you need only values of $f(x)$ and $y$.
  - e.g., $f(x) = 15, 12, 10, 8, 6, 2, -1, -3, -14, ...$
  - e.g., $y(x) = -1, 1, 1, -1, 1, -1, -1, -1, 1, -1, 1, -1, ...$
ROC Curves

- If \( f(x) \) is really good:
  - e.g., \( f(x) = 15, 12, 10, 8, 6, 2, -1, -3, -14, \ldots \)
  - e.g., \( y(x) = 1, 1, 1, 1, -1, -1, -1, -1, -1, -1, \ldots \)
ROC Curves

- If $f(x)$ is really bad:
  - e.g., $f(x) = 15, 12, 10, 8, 6, 2, -1, -3, -14, \ldots$
  - e.g., $y(x) = -1, 1, -1, 1, -1, 1, -1, 1, -1, \ldots$
ROC Curves
Imbalanced Data

Cynthia Rudin
Machine Learning Course, Duke
Evaluation (from earlier lecture)

Many ways to evaluate a classifier:
• Confusion matrix (TP, TN, FP, FN)
• Accuracy / misclassification error
• Precision, Recall, F1-score
• ROC curves, AUC/AUROC

When dataset is balanced, accuracy might be ok... but what about when an FP is different than an FN?
Imbalanced Data

• One of the most annoying and difficult problems in ML.
Imbalanced Data

- One of the most annoying and difficult problems in ML.
Imbalanced Data

- One of the most annoying and difficult problems in ML.

99% accurate but totally meaningless
Imbalanced Data

- One of the most annoying and difficult problems in ML.
Imbalanced Data

- One of the most annoying and difficult problems in ML.
Starting with the standard approach:

\[
\frac{1}{n} \sum_{i=1}^{n} \ell(y_i, f(x_i)) + \text{Regularization}(f)
\]

A misclassified positive is worth the same as a misclassified negative.
\[ \frac{1}{n} \sum_{i=1}^{n} \ell(y_i, f(x_i)) + \text{Regularization}(f) \]
Let’s weigh losses for positives and negatives differently.
Each positive is worth $C$ times a negative

Let’s weigh losses for positives and negatives differently.
• Perhaps the algorithm will choose this model now.
Each positive is worth $C$ times a negative

\[
\frac{1}{n} \left( C \sum_{i \text{ where } y_i = 1}^{n} \ell(y_i f(x_i)) + \sum_{k \text{ where } y_k = -1}^{n} \ell(y_k f(x_k)) \right) + \text{Regularization}(f)
\]
Imbalanced Data

• Don’t report plain accuracy.

• Adjust imbalance parameter C to obtain your ideal balance between TP/FP.

• Look at the confusion matrix to assess FP’s and FN’s separately.
ROC Curves for Algorithms

Cynthia Rudin
Machine Learning Course, Duke
• ROC Curves can be produced in 2 ways:
  – Using a single real-valued classifier. In that case the ROC curve evaluates the classifier. (In earlier lectures)
  – Using a single algorithm and sweeping the imbalance parameter across the full range. In that case, the ROC curve evaluates the algorithm. (This lecture)
ROC Curves for algorithms

- Run the algorithm, sweeping across C values.

\[ C = .0001 \]

\[ f_{.0001}(x) = 0 \]

\[ f_{.0001}(x) > 0 \]
ROC Curves for algorithms

- Run the algorithm, sweeping across C values.

\[ C = 0.2 \]

\[ f_2(x) = 0 \]

\[ f_2(x) > 0 \]
ROC Curves for algorithms

- Run the algorithm, sweeping across C values.

\[ C = 1 \]

\[ f_1(x) = 0 \]

\[ f_1(x) > 0 \]
ROC Curves for algorithms

- Run the algorithm, sweeping across $C$ values.

\[
C = 2
\]

\[
f_2(x) = 0
\]

\[
f_2(x) > 0
\]
ROC Curves for algorithms

- Run the algorithm, sweeping across C values.

\[ C = 1000 \]
\[ f_{1000}(x) = 0 \]
ROC Curves for algorithms

• For a particular False Positive Rate (FPR), what is the True Positive Rate (TPR)?
• **ROC Curves can be produced in 2 ways:**
  – From a single real-valued classifier. In that case the ROC curve evaluates the classifier.
  – From an algorithm, sweeping the imbalance parameter across the full range. In that case, the ROC curve evaluates the algorithm.
ROC Curves

ROC curve from the whole algorithm

ROC curve for single classifier