North Carolina State University is a land-
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of The University of North Carolina

Department of Mathematics

NC STATE UNIVERSITY

Your Name: _____

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

   This examination consists of 4 problems, which are subdivided into 10 questions, where each question counts for the explicitly given number of points, adding to a total of **46 points**. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult **two** 8.5in × 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

   You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

Total _____

**Problem 1** (14 points) Consider the following mathematical formula:

$$(a/(b - c * d + e) * f) \tag{1}$$

(a, 5pts) Please draw an expression tree for (1) that complies with the usual operator precendence rules and left-to-right tie-breaking for operators of equal precendence.

(b, 5pts) Please draw the parse tree for (1) using the context-free grammar given in class.

(c, 4pts) Please give a **post**fix string of operators and variables, but with no parentheses, that represents the tree given under part (a).

Problem 2 (10 points): Consider the following graph:
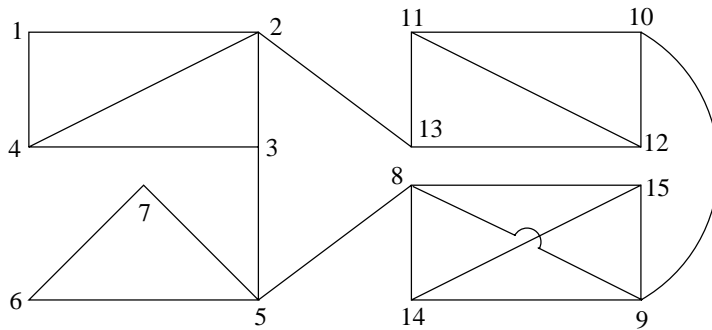
(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex 1.

(b, 5pts) Using the tree in part (a), find a one-way street assignment for the above graph, i.e., orient the edges so that the resulting digraph is strongly connected.

Problem 3 (12 points):
Consider the following graph
(here the vertices are pairs of
integers):

0,0   0,1   0,2   0,3

1,0   1,1   1,2   1,3

2,0   2,1   2,2   2,3
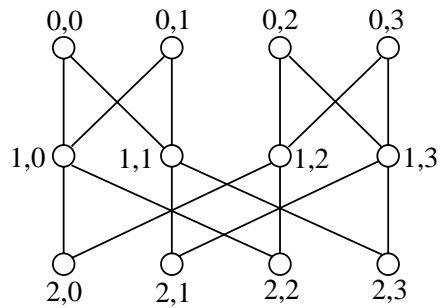
(a, 5pts) Is the above "butterfly" graph planar? Please explain.

(b, 2pts) What is the chromatic number of the above "butterfly" graph? Please justify your answer.

(c, 5pts) True or false: if a graph has n vertices and chromatic number $\chi = n$ it must be the complete graph K_n. Please justify your answer.

Problem 4 (10 points):
Consider the following fractal
polygonal line.
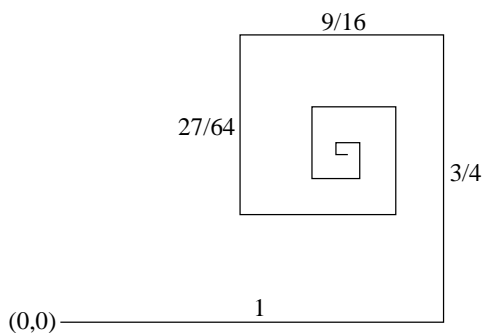
Here you start out at the origin of the plane and move in the x-axis direction by $+1$ unit. Then you move in the y-axis direction by $+3/4$ units, then in the x-axis direction by $-(3/4)\cdot(3/4)$ units, then in the y-axis direction by $-(3/4)^3$ units, then in the x-axis direction by $+(3/4)^4$ units, and so on.

(a, 5pts) Please determine the x- and y-coordinates of the point on the plane to which this polygonal line is converging.

(b, 5pts) Please give a Lindenmeyer system that would draw the above polygonal line. In addition to the productions, please also give the semantics, i.e., the interpretation, of each variable.