

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Nov 8, 2001
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Your Name: SOLUTION

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 4 problems, which are subdivided into 10 questions, where each question counts for the explicitly given number of points, adding to a total of **46 points**. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult **two** 8.5in \times 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

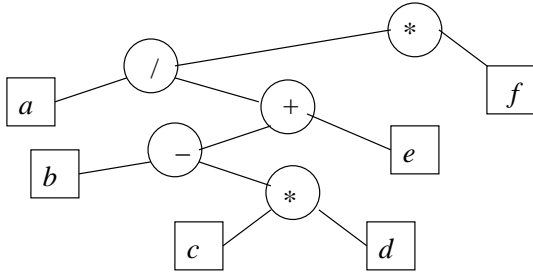
4 _____

Total _____

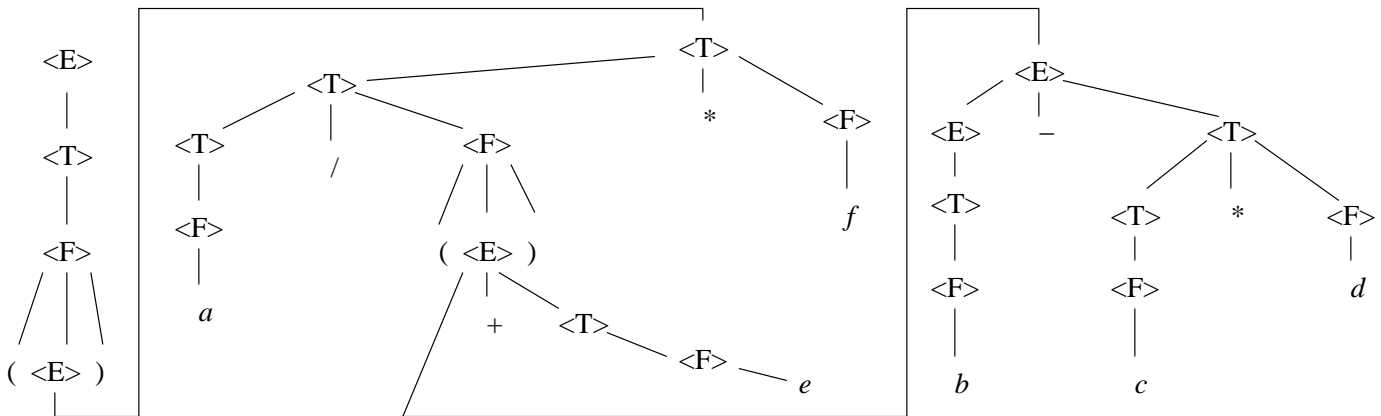
Problem 1 (14 points) Consider the following mathematical formula:

$$(a/(b - c * d + e) * f) \tag{1}$$

(a, 5pts) Please draw an expression tree for (1) that complies with the usual operator precedence rules and left-to-right tie-breaking for operators of equal precedence.



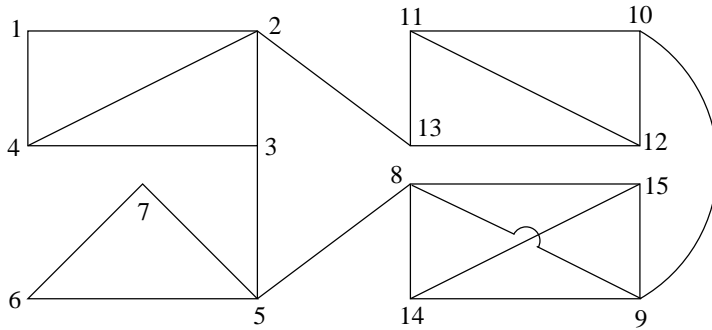
(b, 5pts) Please draw the parse tree for (1) using the context-free grammar given in class.



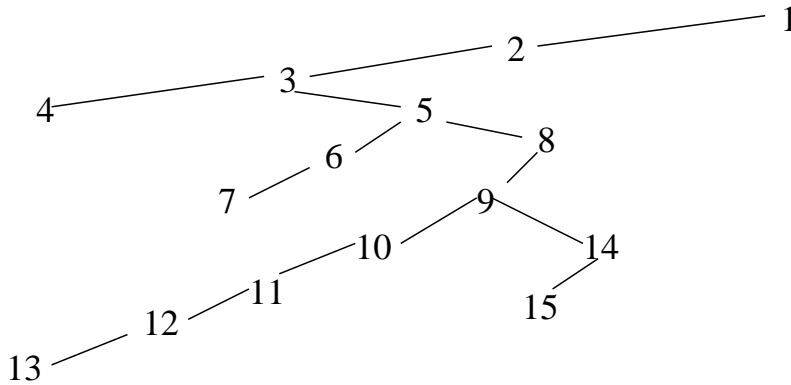
(c, 4pts) Please give a **postfix** string of operators and variables, but with no parentheses, that represents the tree given under part (a).

$abcd * - e + / f *$

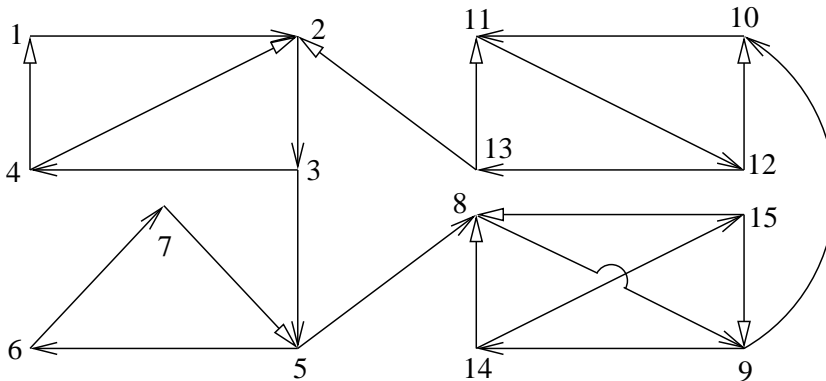
Problem 2 (10 points): Consider the following graph:



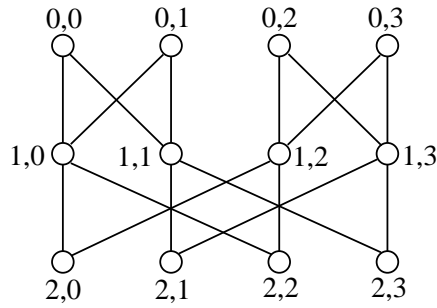
(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex 1.



(b, 5pts) Using the tree in part (a), find a one-way street assignment for the above graph, i.e., orient the edges so that the resulting digraph is strongly connected.

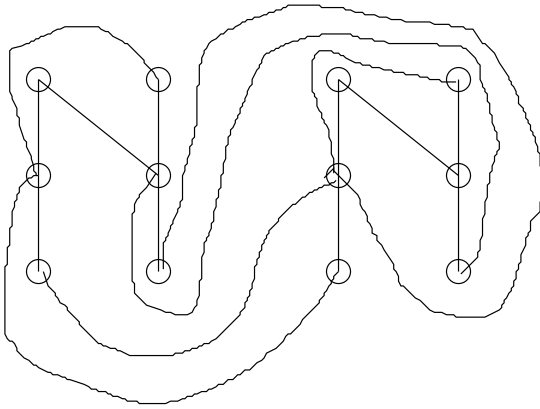


Problem 3 (12 points):
 Consider the following graph
 (here the vertices are pairs of
 integers):



(a, 5pts) Is the above “butterfly” graph planar? Please explain.

Yes.



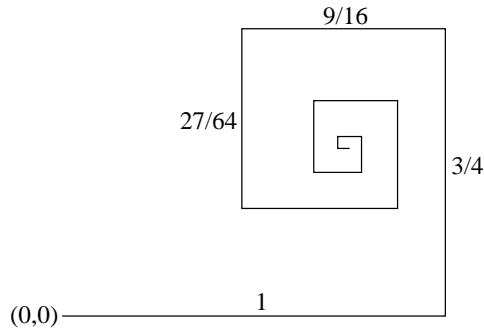
(b, 2pts) What is the chromatic number of the above “butterfly” graph? Please justify your answer.

$\chi = 2$: $(0, i) \rightarrow R, (1, i) \rightarrow G, (2, i) \rightarrow R$.
Since there are edges, 2 is the minimum.

(c, 5pts) True or false: if a graph has n vertices and chromatic number $\chi = n$ it must be the complete graph K_n . Please justify your answer.

TRUE: Suppose G is not complete. Assign a different color to each vertex. Now pick a vertex with fewer than $n - 1$ neighbors. Such a vertex must exist because G is not complete. Replace the color by the one that a vertex has that is not a neighbor. Therefore $\chi(G) \leq n - 1$.

Problem 4 (10 points):
 Consider the following fractal polygonal line.



Here you start out at the origin of the plane and move in the x-axis direction by +1 unit. Then you move in the y-axis direction by +3/4 units, then in the x-axis direction by $-(3/4) \cdot (3/4)$ units, then in the y-axis direction by $-(3/4)^3$ units, then in the x-axis direction by $+(3/4)^4$ units, and so on.

(a, 5pts) Please determine the x- and y-coordinates of the point on the plane to which this polygonal line is converging.

$$\begin{aligned}
 x &= 1 - \frac{9}{16} + \frac{81}{256} - + \dots + \left(-\frac{9}{16}\right)^i + \dots \\
 &= 1 / (1 + 9/16) = 16/25 = 0.64. \\
 y &= \frac{3}{4} - \frac{27}{64} + \frac{243}{1024} - + \dots + \frac{3}{4} \left(-\frac{9}{16}\right)^i + \dots \\
 &= 3/4 \cdot 16/25 = 48/100 = 0.48.
 \end{aligned}$$

(b, 5pts) Please give a Lindenmeyer system that would draw the above polygonal line. In addition to the productions, please also give the semantics, i.e., the interpretation, of each variable.

X_+ : draw a line from current point of length L in positive x-coordinate direction. Then set $L = 3/4 \cdot L$.

X_- : Same but in negative direction. Y_+, Y_- : Same as X_+, X_- , but in y-coordinate direction.

The Lindenmeyer system is with start variable A :

| | | | | | | | | |
|-------------|--------|--------|--------|--------|-------|-------|-------|-------|
| Variables | A | B | C | D | X_+ | Y_+ | X_- | Y_- |
| Productions | X_+B | Y_+C | X_-D | Y_-A | X_+ | Y_+ | X_- | Y_- |