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**Department of Mathematics**

**NC STATE** UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Nov 7, 2002

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For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 5 problems, which are subdivided into 10 questions, where each question counts for the explicitly given number of points, adding to a total of **47 points**. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult **two** 8.5in $\times$ 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

5 _____

Total _____

**Problem 1** (14 points) Consider the following mathematical formula:

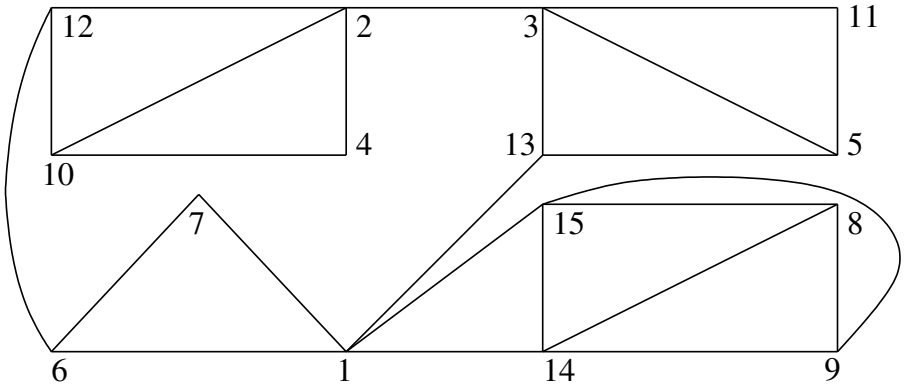
$$a + b/c - d * e/(f * g) \tag{1}$$

(a, 5pts) Please draw an expression tree for (1) that complies with the usual operator precedence rules and left-to-right tie-breaking for operators of equal precedence.

(b, 5pts) Please draw the parse tree for (1) using the context-free grammar given in class.

(c, 4pts) Please give a **pre**fix string of operators and variables, but with no parentheses, that represents the tree given under part (a).

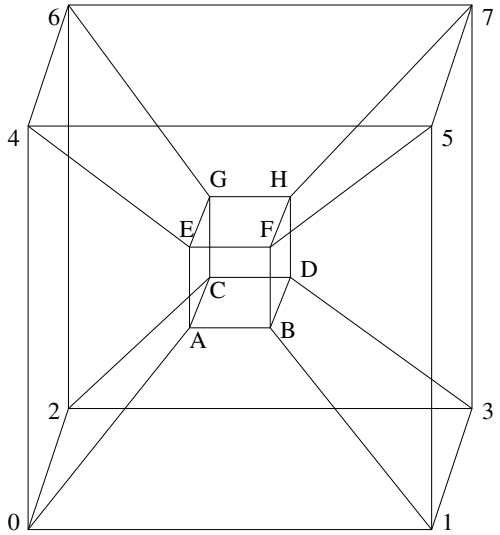
**Problem 2** (10 points): Consider the following graph:




(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex 1.

(b, 5pts) Using the tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected.

**Problem 3** (13 points): Consider the 4-dimensional hypercube (with the given vertex labeling):

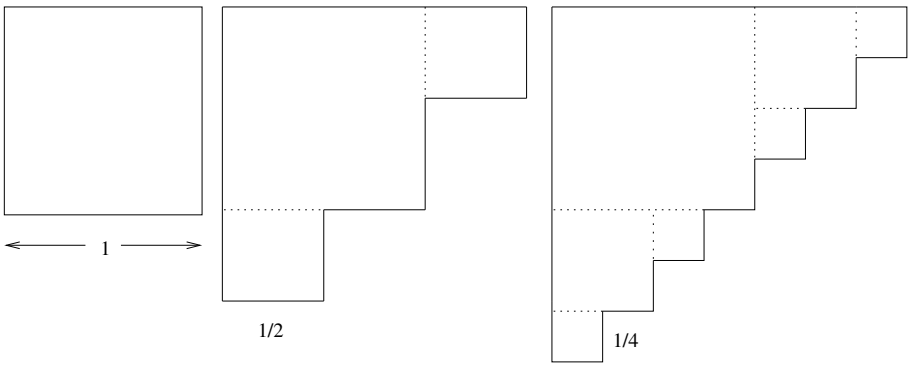



(a, 6pts) Please draw a subgraph that is homeomorphic to $K_{3,3}$. [Hint: choose as the first subset $\{0,3,5\}$ and as the second another 3 vertices on the outer cube.]

(b, 5pts) What is the chromatic number of the above 4-D hypercube? Please justify your answer.

(c, 2pts) Give an example of a graph such that the clique number is smaller than the chromatic number.

**Problem 4** (6 points): Please consider the following fractal staircase.




Here you start with a square of sidelength = 1 unit. You add a square of half the sidelength on the right-top and left-bottom. For each square added, you continue this process, to infinity. Please determine the area and length of the boundary for the fractal.

**Problem 5** (4 points): Consider the following Lindenmeyer system: $A \rightarrow AB$, $B \rightarrow AC$, $C \rightarrow BD$, $D \rightarrow B$. Please write down the first 4 new generations of strings starting with $A$.