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Department of Mathematics

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Nov 4, 2004

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Your Name: SOLUTION

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 5 problems, which are subdivided into 10 questions, where each question counts for the explicitly given number of points, adding to a total of **50 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **two** 8.5in $\times$ 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

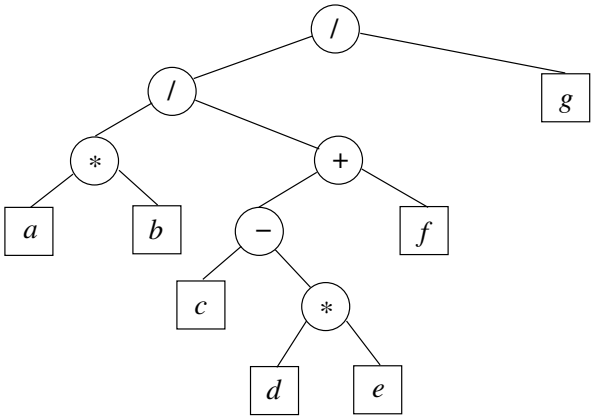
5 _____

Total _____

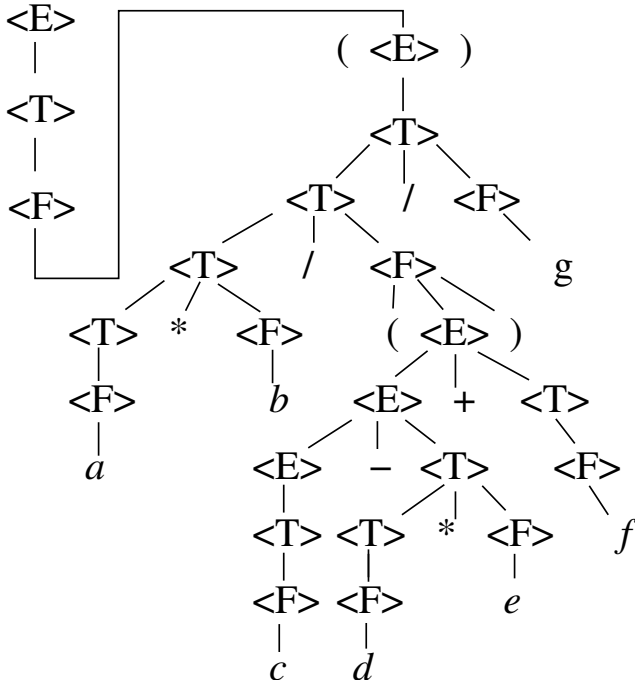
Problem 1 (14 points) Consider the following mathematical formula:

$$(a*b/(c-d*e+f)/g) \tag{1}$$

(a, 5pts) Please draw an expression tree for (1) that complies with the usual operator precedence rules and left-to-right tie-breaking for operators of equal precedence.



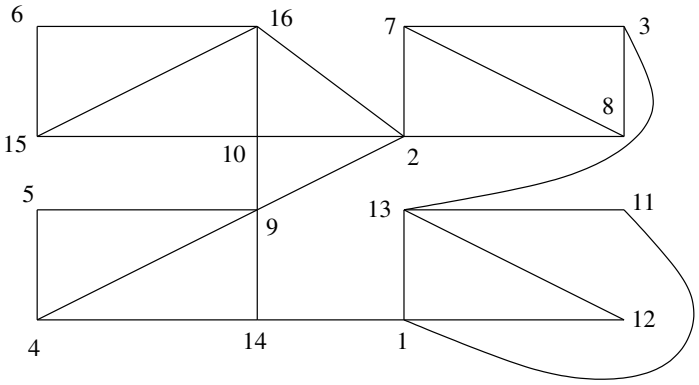
(b, 5pts) Please draw the parse tree for (1) using the context-free grammar given in class.



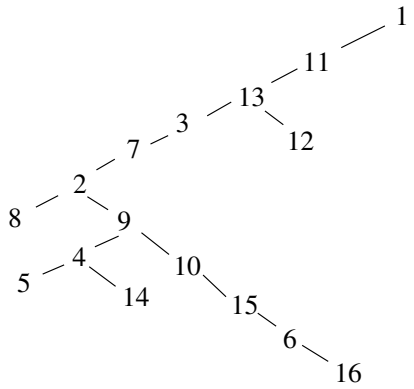
(c, 4pts) Please give **both** a **fully parenthesized** infix string of variables, operators and parentheses **and** a postfix string of only variables and operators that represent the tree given under part (a).

Infix: $(((([a]*[b])/(([c]-([d]*[e]))+[f]))/[g])$
Postfix: $ab*cde*-f+/g/$

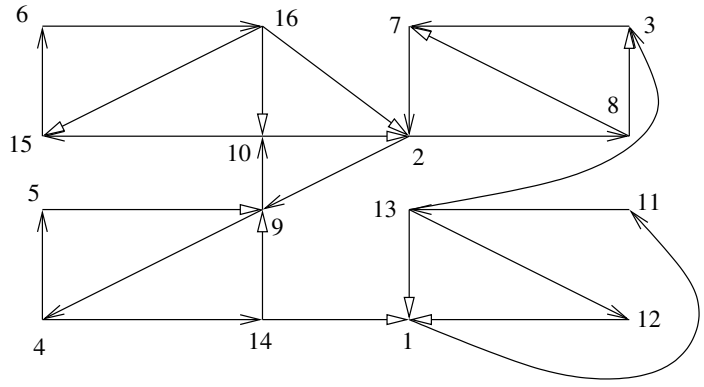
Problem 2 (10 points): Consider the following graph:



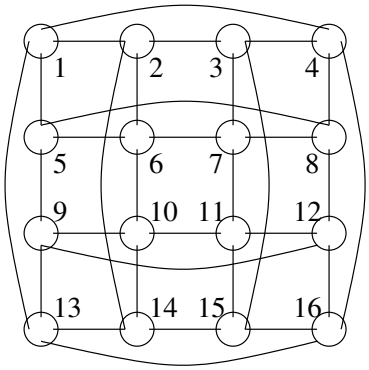
(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.



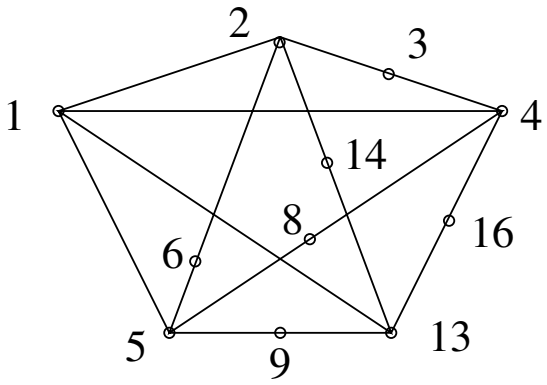
(b, 5pts) Using the tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected.



Problem 3 (12 points):
Consider the 4-D hypercube as a 4×4 toric mesh (with the given vertex labeling):



(a, 6pts) Please draw a subgraph that is homeomorphic to K_5. [Hint: choose as the vertex subset $\{1,2,4,5\}$ and another vertex.]



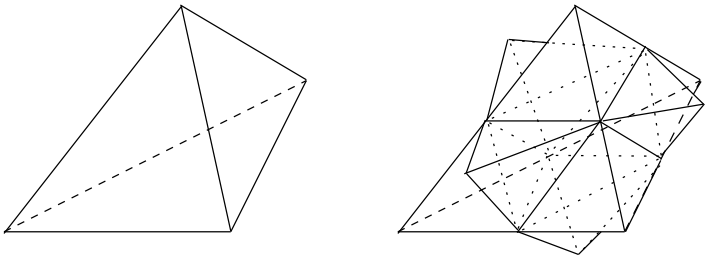
(b, 4pts) What is the chromatic number of the above 4×4 toric mesh? Please justify your answer.

$\chi = 2$*: color the vertices* $1,3,6,8,9,11,14,16$ *red and the others green;* $\chi \geq 2$ *because there is, obviously, a 2-clique.*

(c, 2pts) Please give an example of a graph such that the maximum of the degrees of all the vertices is equal to the chromatic number plus 5.

$G = (\{1,2,3,4,5,6,7,8\},\{\{1,2\},\{1,3\},\{1,4\},\{1,5\},\{1,6\},\{1,7\},\{1,8\}\})$*:* $\chi = 2$*, because one may color vertex* 1 *red and the others green.* $\Delta = 7$ *because vertex* 1 *has* 7 *neighbors.*

Problem 4 (10 points): Please consider the following 3-dimensional snowflake:



Here you start with a regular tetrahedron of edge length 1. On each of the four faces, which are equilateral triangles, you set a regular tetrahedron of edge length $1/2$ as shown: the vertices of the base triangle of each smaller tetrahedron are the midpoints of the edges of the larger tetrahedron. The next iteration sets 24 ($= 6 \cdot 4$) new tetrahedra of edge length $1/4$ on the exposed equilaterally triangular faces. Note that an equilateral triangle of side length a has an area of $A = \frac{\sqrt{3}}{4}a^2$ and that a tetrahedron of edge length a has volume $V = \frac{\sqrt{2}}{12}a^3$.

(a, 5pts) If the process of adding smaller and smaller tetrahedra to the polyhedron is continued to infinity, what is the exposed surface area of the fractal? Please show your computation.

The surface is enlarged by a factor of $6/4$ *at every iteration, hence the fractal has infinite surface area.*

As a series: $4 \cdot A + 4 \cdot 2 \cdot \dfrac{A}{4} + 4 \cdot 6 \cdot 2 \cdot \dfrac{A}{16} + \cdots + 4 \cdot 6^{i-1} \cdot 2 \cdot \dfrac{A}{(2^i)^2} + \cdots \to \infty$.

(b, 5pts) What is the volume of the fractal? Please show your computation.

Let V *be the volume of the first tetrahedron. The fractal has a volume of*

$$V + 4 \cdot \frac{V}{2^3} + 4 \cdot 6 \cdot \frac{V}{(2^2)^3} + 4 \cdot 6 \cdot 6 \cdot \frac{V}{(2^3)^3} + \cdots + 4 \cdot 6^{i-1} \cdot \frac{V}{(2^i)^3} + \cdots$$
$$= V + \frac{4}{8} V \sum_{i=0}^{\infty} \left(\frac{6}{8}\right)^i = V + \frac{1}{2} V \frac{1}{1 - 6/8} = 3V = \sqrt{2}/4.$$

Problem 5 (4 points): Consider the following Lindenmeyer system: $A \to ABC$, $B \to C$, $C \to CDE$, $D \to B$, $E \to E$. Please write down the first 4 new generations of strings starting with A.

$$A \to ABC \to ABCCCDE \to ABCC\underbrace{CDE}_{C}\underbrace{CDE}_{C}\underbrace{CDE}_{C}BE$$
$$\to \underbrace{ABC}_{A}\underbrace{C}_{B}\underbrace{CDE}_{C}\underbrace{CDE}_{C}\underbrace{CDE}_{C}\underbrace{B}_{D}\underbrace{E}_{E}\underbrace{CDE}_{C}BE\underbrace{CDE}_{C}BECE.$$