

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Nov 9, 2006
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www.math.ncsu.edu/~kaltofen/courses/DiscreteModels/Fall06/index.html (URL)

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Your Name: _____

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 5 problems, which are subdivided into 10 questions, where each question counts for the explicitly given number of points, adding to a total of **49 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work**, if necessary. You are allowed to consult **two** 8.5in \times 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

5 _____

Total _____

Problem 1 (14 points) Consider the following mathematical formula:

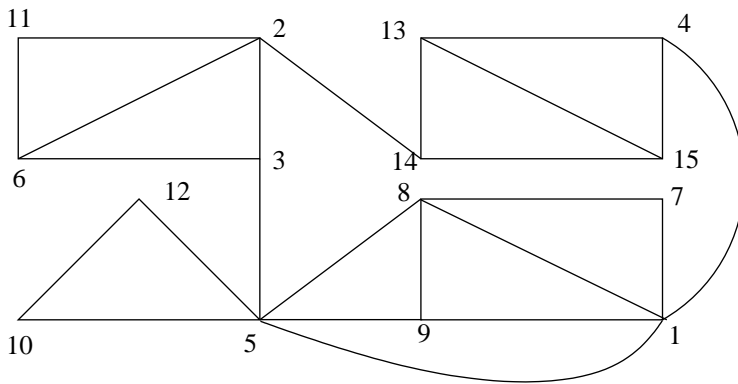
$$(a/b/c - d * e) + (f) - g \quad (1)$$

(a, 5pts) Please draw an expression tree for (1) that complies with the usual operator precedence rules and left-to-right tie-breaking for operators of equal precedence.

(b, 5pts) Please draw the parse tree for (1) using the context-free grammar given in class.

(c, 4pts) Please give **both** a **fully parenthesized** infix string of variables, operators and parentheses **and** a **prefix** string of only variables and operators that represent the tree given under part (a).

Problem 2 (13 points): Consider the following graph:

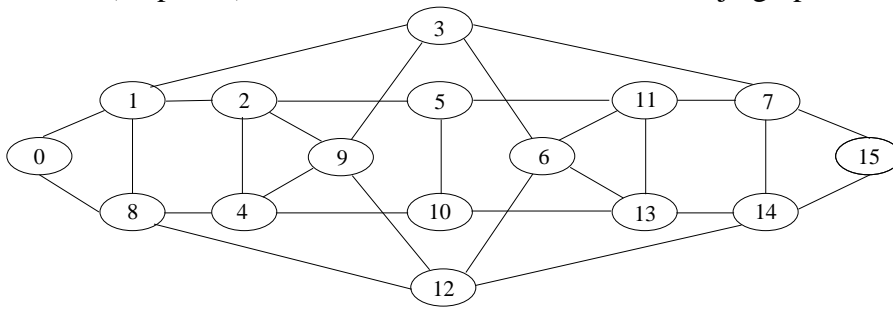


(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

(b, 5pts) Using the tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected.

(c, 3pts) What is the size of the maximum clique in the above graph? Please explain.

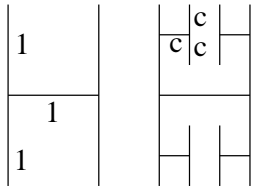
Problem 3 (10 points): Consider the 4-dimensional de Bruijn graph with 16 vertices:



(a, 5pts) Please draw a subgraph that is homeomorphic to K_4 (the complete graph with **four** vertices).

(b, 5pts) What is the chromatic number of the above de Bruijn graph? Please justify your answer.

Problem 4 (8 points): Please consider the H-tree.



Here one starts with an H-figure, whose side lengths are 1. The figure thus has 5 lines of total length 5. Then one adds on the 4 vertical lines 4 H-figures of side length $c < 1$ as shown. At this stage, a total of $12c$ in line length is added. One continues to add 16 H-figures of side length c^2 , 64 H-figures of side length c^3 , etc.

If the process of adding smaller and smaller H-figures is continued to infinity, what is the total length of lines drawn in dependence on c ? Please show your computation.

Problem 5 (4 points): Consider the following Lindenmayer system: $A \rightarrow BC$, $B \rightarrow AC$, $C \rightarrow D\alpha$, $D \rightarrow AB$, $\alpha \rightarrow \alpha$. Please write down the first 4 new generations of strings starting with A .