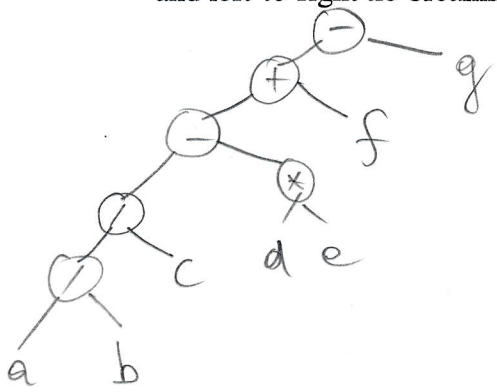
06

Problem 1 (14 points) Consider the following mathematical formula:

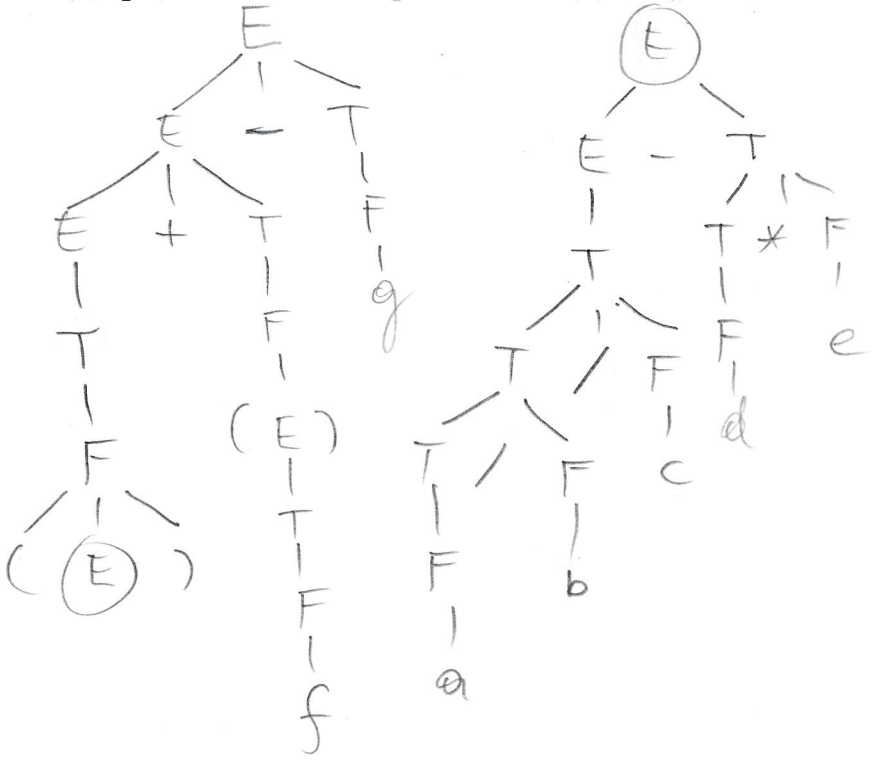
$$(a/b/c-d*e)+(f)-g \tag{1}$$

(a, 5pts) Please draw an expression tree for (1) that complies with the usual operator precedence rules and left-to-right tie-breaking for operators of equal precedence.



right-to-left
-2

(b, 5pts) Please draw the parse tree for (1) using the context-free grammar given in class.



( ) missing for 1st term
-1

( ) missing for (f) -1

(c, 4pts) Please give **both** a **fully parenthesized** infix string of variables, operators and parentheses **and** a **prefix** string of only variables and operators that represent the tree given under part (a).

((((((a/b)/c)-(d*e))+f)-g)

- + - / / a b c * d e f g

no penalty if correct for (a)

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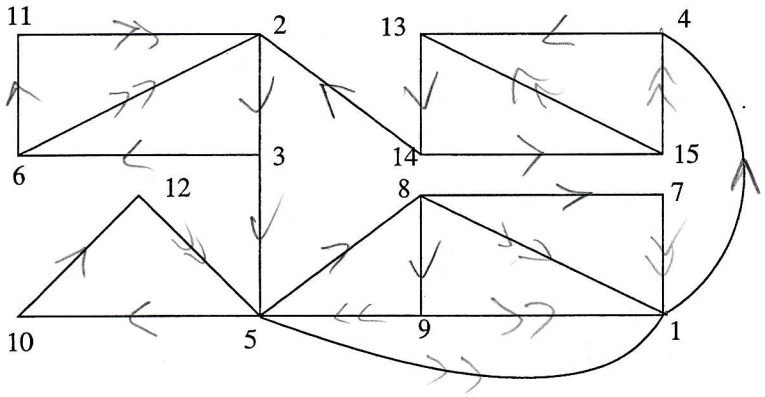
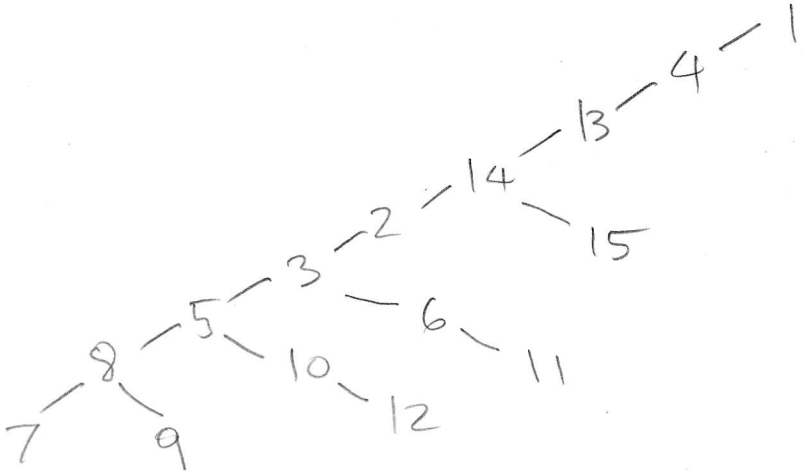
Problem 2 (13 points): Consider the following graph:



(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.



(b, 5pts) Using the tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected.

one arc wrong: no penalty
2 arcs -"- : -1 pt

(c, 3pts) What is the size of the maximum clique in the above graph? Please explain.

K_5 ruled out by planarity +1 bonus

4
K_5 is impossible: $V = \{1, 2, 5, 8, 9\}$ the graph is planar
3 clique
+1
4 clique not shown -2
K_5 not ruled out.
no penalty

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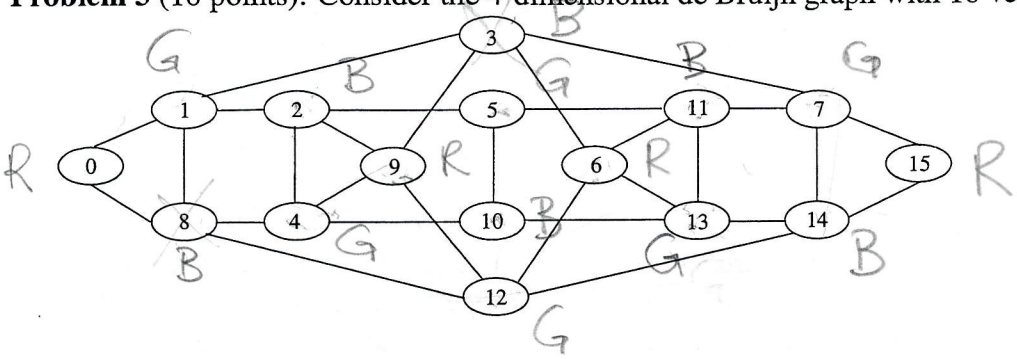
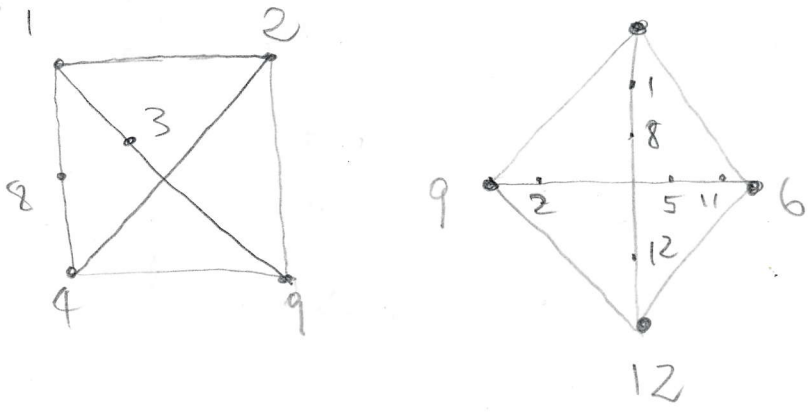
Problem 3 (10 points): Consider the 4-dimensional de Bruijn graph with 16 vertices:



(a, 5pts) Please draw a subgraph that is homeomorphic to K_4 (the complete graph with **four** vertices).



(b, 5pts) What is the chromatic number of the above de Bruijn graph? Please justify your answer.

$\chi = 3$ as shown. $\chi \geq 3$ because the graph has triangles K_3

$\chi = 3$ 2

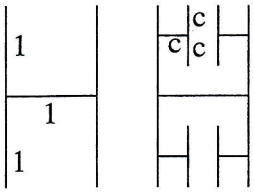
coloring 2 1

4 coloring +2

$\chi = 4$ bec. K_4 no credit.

06

Problem 4 (8 points): Please consider the H-tree.



Here one starts with an H-figure, whose side lengths are 1. The figure thus has 5 lines of total length 5. Then one adds on the 4 vertical lines 4 H-figures of side length $c < 1$ as shown. At this stage, a total of $12c$ in line length is added. One continues to add 16 H-figures of side length c^2, 64 ~~32~~ H-figures of side length c^3, etc.
If the process of adding smaller and smaller H-figures is continued to infinity, what is the total length of lines drawn in dependence on c? Please show your computation.

$5 + 12c + 4 \cdot 4 \cdot 3c^2 + 4 \cdot 4 \cdot 4 \cdot 3c^3 + \dots + 4^i 3 c^i + \dots$

$2 + 3 \cdot \sum_{i=0}^{\infty} (4c)^i \begin{cases} = \infty & c \geq \frac{1}{4} \\ = 2 + 3 \frac{1}{1-4c} \end{cases}$

4th term correct +2

errors in denom -2

series correct $+6$

$= \frac{5 - 18c}{1-4c}$

Problem 5 (4 points): Consider the following Lindenmeyer system: $A \to BC$, $B \to AC$, $C \to D\alpha$, $D \to AB$, $\alpha \to \alpha$. Please write down the first 4 new generations of strings starting with A.

$A \to BC \to ACD\alpha \to BCD\alpha AB\alpha$

$\to ACD\alpha AB\alpha BCAC\alpha$