Problem 1 (14 points) Consider the following mathematical formula:

\[ a + \frac{(b \times c - d)}{e} - (f + g \times h) \]  

(a, 5pts) Please draw an expression tree for (1) that complies with the usual operator precedence rules and left-to-right tie-breaking for operators of equal precedence.

(b, 5pts) Please draw the parse tree for (1) using the context-free grammar given in class.

(c, 4pts) Please give both a prefix string and a postfix string of only variables and operators that represent the tree given under part (a).

\[- + a / - \times b c d e + f x g h\]
\[a b c \times d - e / + f g h \times + -\]
Problem 2 (13 points): Consider the following graph:

(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex in numerical order, starting at vertex 1.

(b, 5pts) Using the tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected.

(c, 3pts) What is the maximum degree in the above graph? Please explain.
Problem 3 (10 points):
Consider the 4-dimensional hypercube (with the given vertex labeling):

(a, 5pts) Please draw a subgraph that is homeomorphic to $K_5$ (the complete graph with five vertices).

(b, 5pts) Please draw a graph with 6 vertices, clique number 2 and chromatic number 3.
Problem 4 (8 points):

(a, 3pts) Please describe the construction of Koch’s snowflake.

(b, 3pts) Please derive the area of Koch’s snowflake, assuming that the initial triangle has area 1.

\[
1 + 3 \cdot \frac{1}{3} + 3 \cdot 4 \cdot \left(\frac{1}{9}\right) + 3 \cdot 4 \cdot 4 \left(\frac{1}{27}\right) + \ldots + 3 \cdot 4 \cdot \frac{1}{9^i} + \ldots
\]

\[
1 + \frac{1}{3} \sum_{i=1}^{\infty} \frac{4}{9^i} = 1 + \frac{1}{3} \cdot \frac{1}{1 - \frac{4}{9}} = 1 + \frac{1}{3} \cdot \frac{9}{5} = \frac{8}{5}
\]

(c, 2pts) Please derive the length of the boundary of Koch’s snowflake.

\[
L \text{ grows by } \frac{4}{3} \text{ each step } \rightarrow L = \infty.
\]

Problem 5 (4 points): Consider the following Lindenmeyer system: \(A \rightarrow BA, B \rightarrow CB, C \rightarrow DC,\)
\(D \rightarrow A\alpha, \alpha \rightarrow \alpha.\) Please write down the first 4 new generations of strings starting with \(A.\)

\[
\begin{align*}
A & \rightarrow BA \\
& \rightarrow CBBA \\
& \rightarrow DCCBCBBB \\
& \rightarrow A\alpha DCDCCBDCBCB \\
& \rightarrow \alpha DCDCCBDCBCB \\
& \rightarrow \alpha DCDCCBDCBCB
\end{align*}
\]

3 generations \(-1\) pt