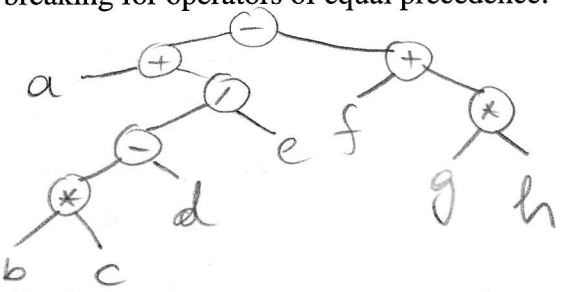
07

Problem 1 (14 points) Consider the following mathematical formula:

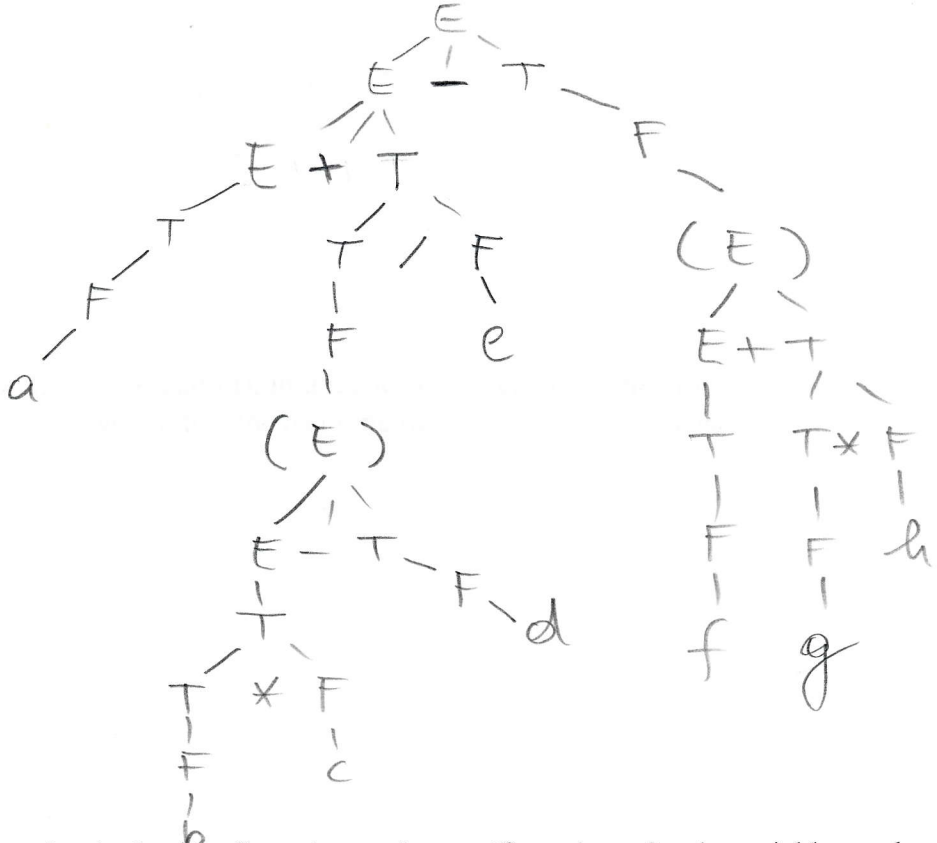
$$a+(b*c-d)/e-(f+g*h) \tag{1}$$

(a, 5pts) Please draw an expression tree for (1) that complies with the usual operator precedence rules and left-to-right tie-breaking for operators of equal precedence.



+ −
−2

(b, 5pts) Please draw the parse tree for (1) using the context-free grammar given in class.



(c, 4pts) Please give both a **prefix** string and a **postfix** string of only variables and operators that represent the tree given under part (a).

− + a / − * b c d e + f * g h

a b c * d − e / + f g h * + −

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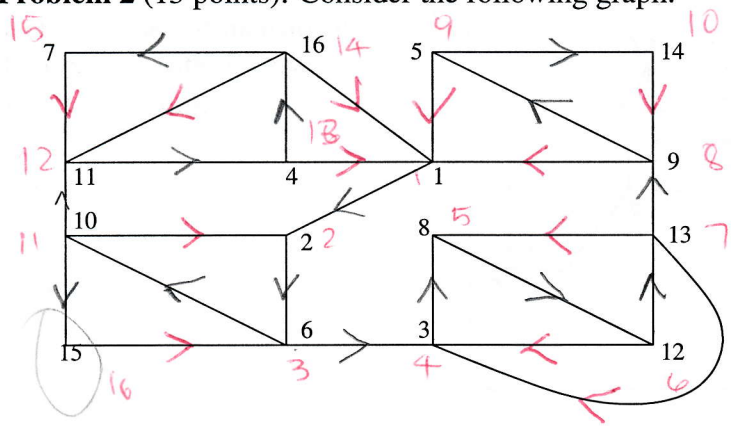
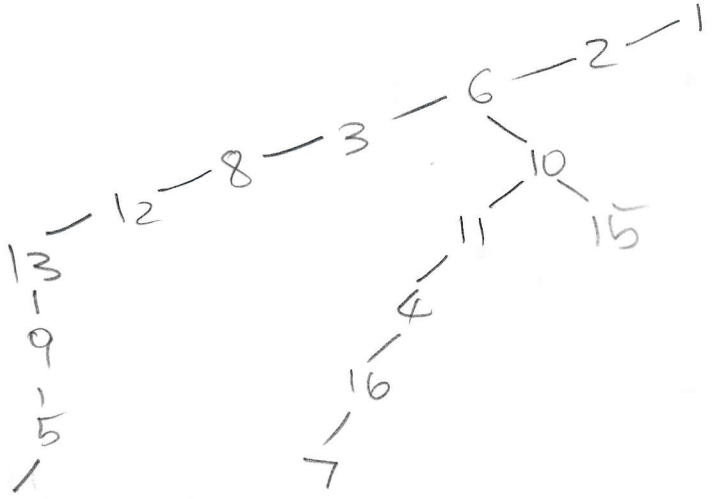
Problem 2 (13 points): Consider the following graph:



(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.



(b, 5pts) Using the tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected.

2 arcs wrong −1 pt

(c, 3pts) What is the maximum degree in the above graph? Please explain.

5

vertex 1 has 5 neighbours

digraph degree − 1

4 1pt

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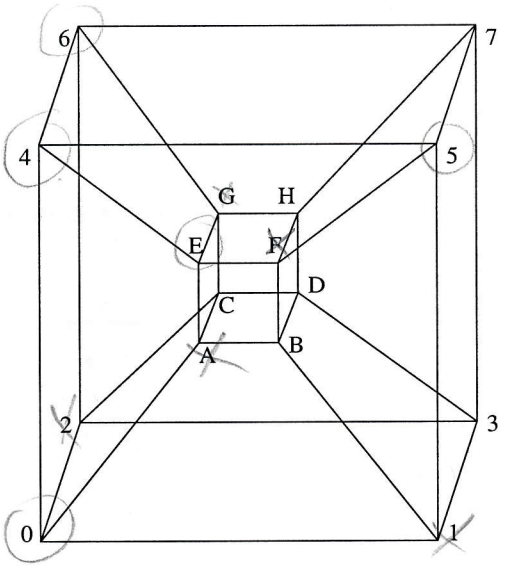
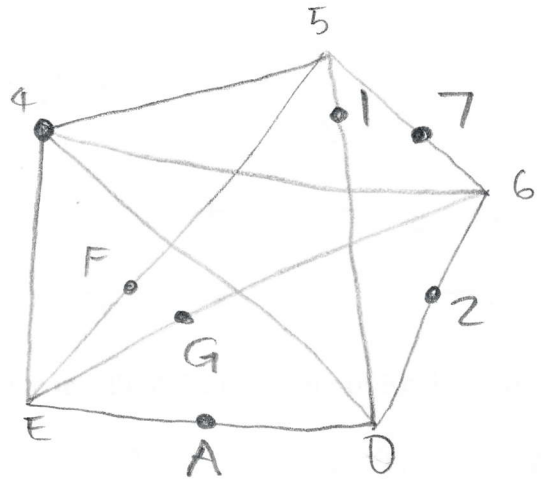
Problem 3 (10 points): Consider the 4-dimensional hypercube (with the given vertex labeling):

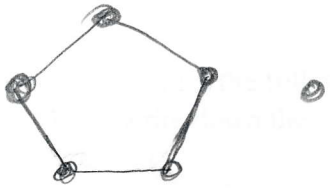


(a, 5pts) Please draw a subgraph that is homeomorphic to K_5 (the complete graph with **five** vertices).



Problem 4

(b, 5pts) Please draw a graph with 6 vertices, clique number 2 and chromatic number 3.



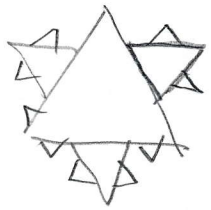
$\omega = 3$ no credit

$\chi = 2$ ~~no credit~~ +0 1pt

07

Problem 4 (8 points):

(a, 3pts) Please describe the construction of Koch's snowflake.



(b, 3pts) Please derive the area of Koch's snowflake, assuming that the initial triangle has area 1.

$$1 + 3\cdot\left(\frac{1}{3}\right)^2 + 3\cdot 4\cdot\left(\frac{1}{9}\right)^2 + 3\cdot 4\cdot 4\left(\frac{1}{27}\right)^2 + \cdots + 3\cdot 4^{i-1}\frac{1}{9^i} + \cdots$$

$$1 + \frac{1}{3}\sum_{i=1}^{\infty}\frac{4^{i-1}}{9^{i-1}} = 1 + \frac{1}{3}\,\frac{1}{1-\frac{4}{9}} = 1 + \frac{1}{3}\,\frac{9}{5} = \frac{8}{5}$$

(c, 2pts) Please derive the length of the boundary of Koch's snowflake.

L grows by $\frac{4}{3}$ each step $\Rightarrow L = \infty$.

Problem 5 (4 points): Consider the following Lindenmeyer system: $A \to BA$, $B \to CB$, $C \to DC$, $D \to A\alpha$, $\alpha \to \alpha$. Please write down the first 4 new generations of strings starting with A.

$A \to BA \to CBBA \to DCCBCBBA$

$\to A\alpha DCDCCBDCCBCBBA$

3 generations −1 pt