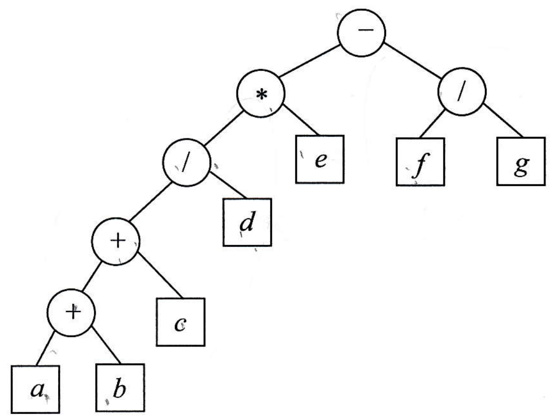
D8

Problem 1 (16 points): Consider the following mathematical expression in **postfix** notation, assuming that each of the operators $+, -, *, /$ has two operands.

$$ab + c + d/e * fg/- \tag{1}$$

(a, 4pts) Please draw an expression tree for (1).



(b, 4pts) Ignoring the operators and variables in the tree vertices, please give the serialization of your tree in (a) using an expression consisting of 13 ('s and 13)'s that are balanced (our second [Riley's] method).

first method −2

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(- (* (/ (+ (+ (a) +) (b) +) (c) /) (d) *) (e)  -)  (/ (f) /)(g)
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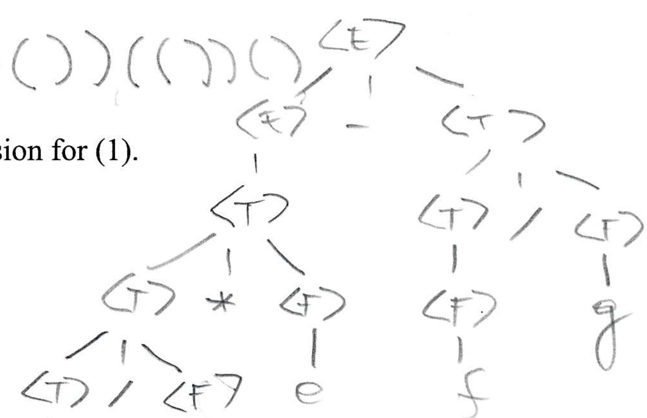
(((((( ))())())())())(())()



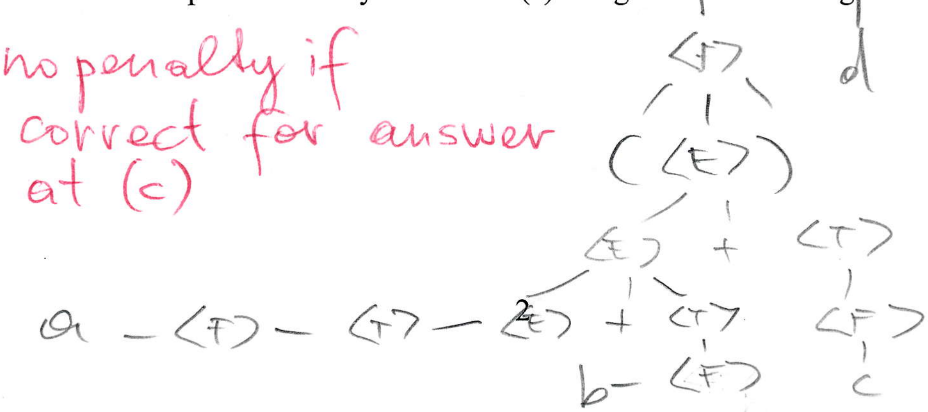
(c, 4pts) Please give a **minimally parenthesized infix** expression for (1).

$(a+b+c)/d*e-f/g$

not minimal −1

(d, 4pts) Please draw the parse tree for your answer (c) using the context-free grammar given in class.

no penalty if correct for answer at (c)



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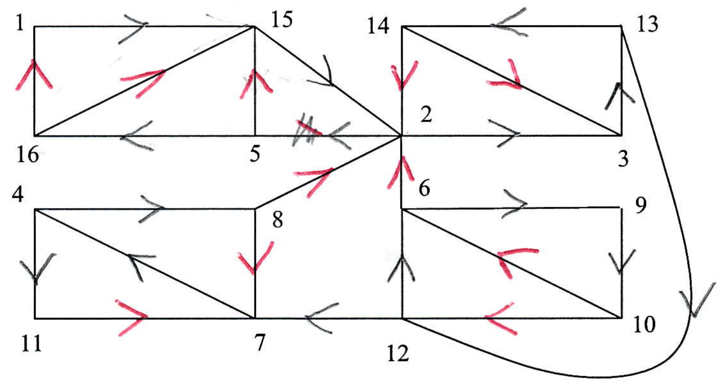
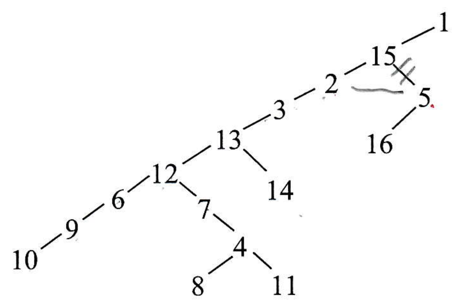
Problem 2 (13 points): Consider the following graph:



(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

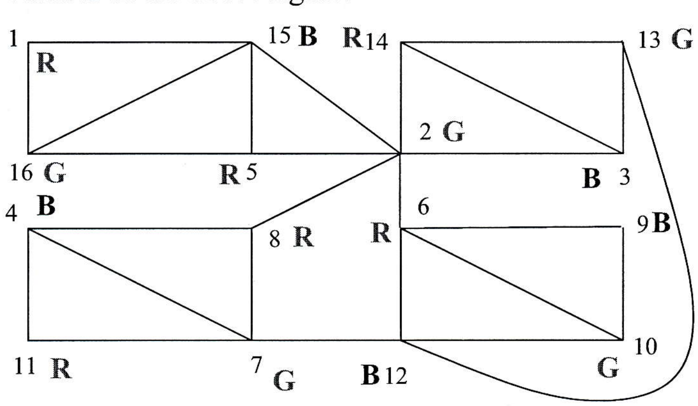


(b, 4pts) Using the tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected.

no penalty if OK for tree in (a)

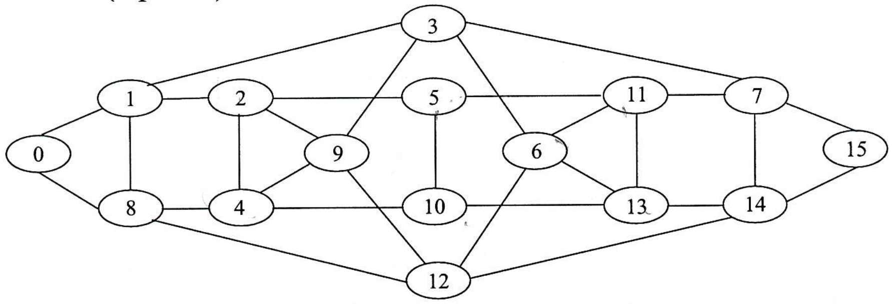
2 arcs missing −1

(c, 4pts) Please find a 3-coloring of the above graph. You may place your RGB colors next to the vertices in the above figure.

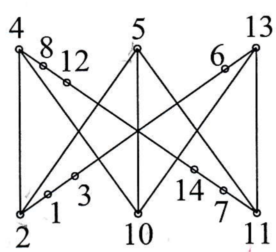


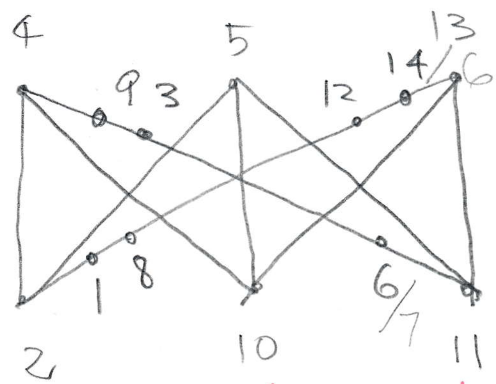
08

Problem 3 (8 points): Consider the 4-dimensional de Bruijn graph with 16 vertices:



Please draw a subgraph that is homeomorphic to $K_{3,3}$ (the complete bi-partite graph from 3 to 3 vertices). Hint: Choose 4, 5, and 13 as one set of vertices.





Shared vertex −2

intermediates missing −3

not a subgraph ≤3

Problem 4 (5 points): Please define Benoit Mandelbrot's set $M_0 \subset \mathbb{C}$ for start value $a = 0$ and discuss what is chaotic about it.

$$M_0 = \{ c \in \mathbb{C} \mid \exists B \in \mathbb{R}_{>0} : \forall i \geq 0 : z_{i+1} = z_i^2 + c, z_0 = 0 \Rightarrow |z_{i+1}| \leq B \}$$

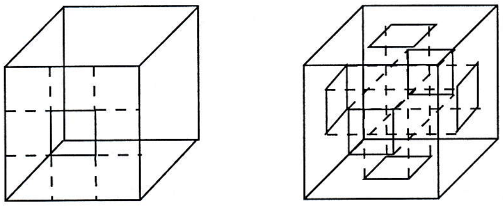
Def 3pts

Chaos 2pts

Finite area, infinite boundary +1

08

Problem 5 (8 points): Please consider the Sierpinski sponge fractal:



Here one starts with a cube, whose side length is 1. The cube consists of 27 cubes of side length 1/3. The front face of the first cube shows how each face is divided by those smaller cubes. By drilling out the middle cubes one removes $6+1=7$ smaller cubes, one adjacent to each face and one in the middle of the cube (see second cube above) . There remain $27-7=20$ smaller cubes. In the first iteration, one has thus removed $7\cdot(1/3)^3 = 7/27$ of the volume. In the second iteration, the process is continued for the remaining 20 cubes of side length 1/3, removing $20\cdot 7$ cubes of side length $1/3^2 = 1/9$.

(a, 4 pts) At iteration i, please state how much volume V_i is removed. Note that $V_1 = 7/27$.

$$V_i = 7\cdot 20^{i-1}\cdot\left(\frac{1}{3^i}\right)^3 = \frac{7}{27}\cdot\left(\frac{20}{27}\right)^{i-1}$$

$$7\cdot\left(\frac{20}{27}\right)^{i-1} \quad -1\text{pt}$$

(b, 4 pts) Please compute $\sum_{i=1}^{\infty} V_i$. Please show your work.

$$\frac{7}{27}\quad\frac{1}{1-\frac{20}{27}} = \frac{7}{27}\quad\frac{27}{7} = 1$$

1 not without explanation

+2