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Department of Mathematics

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Nov 5, 2009

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www.math.ncsu.edu/~kaltofen/courses/DiscreteModels/Fall09/index.html (URL)

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Your Name: SOLUTION

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 6 problems, which are subdivided into 12 questions, where each question counts for the explicitly given number of points, adding to a total of **50 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **two** 8.5in $\times$ 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

5 _____

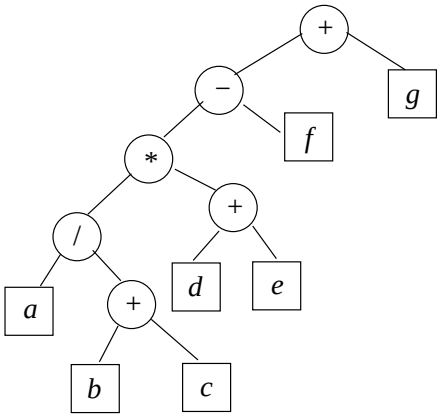
6 _____

Total _____

Problem 1 (12 points): Consider the following mathematical expression in **prefix** notation, assuming that each of the operators $+, -, *, /$ has two operands.

$$+ - */a + bc + defg \tag{1}$$

(a, 4pts) Please draw an expression tree for (1).



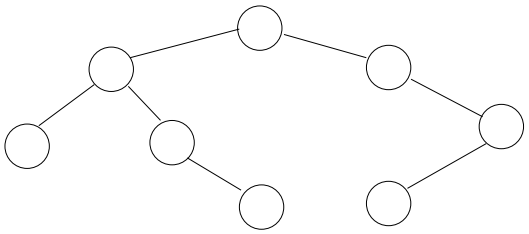
(b, 4pts) Please give a **minimally parenthesized infix** expression for (1).

$a/(b+c)*(d+e)-f+g$

(c, 4pts) Please draw the parse tree for your answer (b) using the context-free grammar given in class.

Problem 2 (8 points): Consider binary trees in which each node has either 0 children, or one left or one right child, or both.

(a, 4pts) Such a tree with 8 nodes has been linearized by our method to `(((())()())()(()))`. Please draw the tree.



(b, 4pts) How many such trees with 8 nodes exist?

Problem 3 (8 points): Consider the following graph:

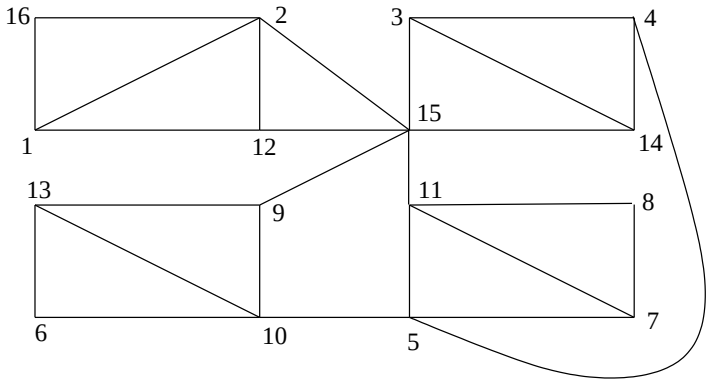
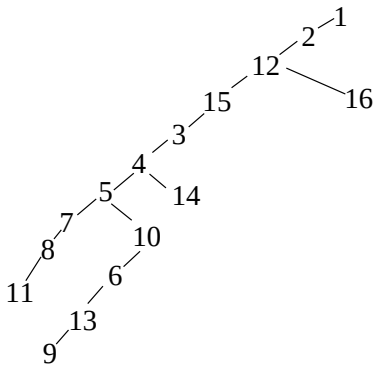


Figure 1.

(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.



(b, 3pts) Using the DFS tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1 above.

Problem 4 (4 points): Suppose $\omega(G)$ is the clique number, $\chi(G)$ the chromatic number and $\Delta(G)$ the maximum vertex degree of a graph G. Please draw a graph for which simultaneously the inequalities $\omega(G) < \chi(G)$ and $\chi(G) < \Delta(G)$ hold.

Problem 5 (8 points): Please consider the 4-dimensional "de Bruijn plus graph " with 16 vertices, which has the additional edge $\{6, 9\}$.

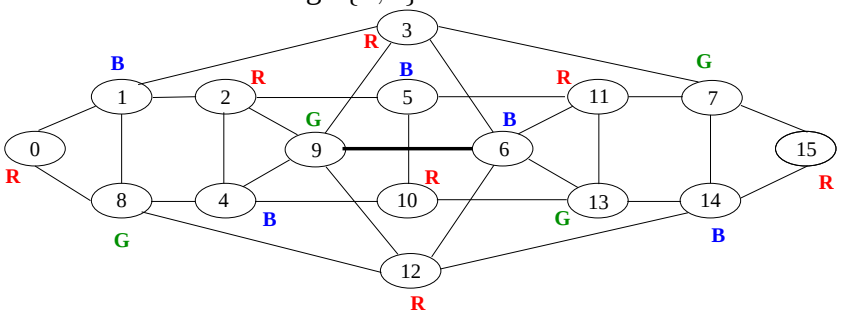
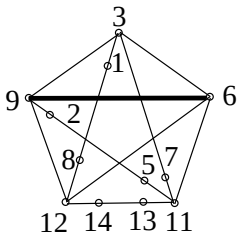


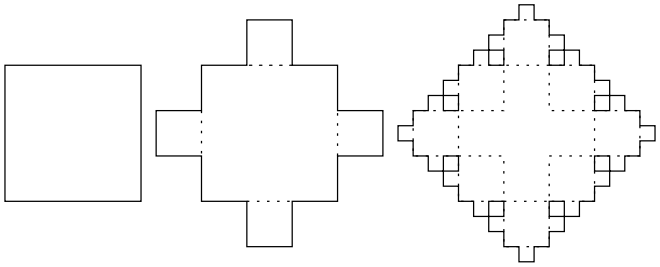
Figure 2.

(a, 6pts) Please draw a subgraph that is homeomorphic to K_5 (the complete graph with 5 vertices). Hint: Choose 3, 6, 9 and two more vertices as the corner vertices of the K_5-like subgraph.



(b, 2pts) Please 3-color the de Bruijn plus graph by marking the vertices in Figure 2 above with the colors R,G,B.

Problem 6 (10 points): Please consider the square snowflake fractal:



Here one starts with a square, whose side length is 1 (left figure above). Each side is exuded in the first iteration by a square of side length $1/3$ in the middle of the side, creating $4 \cdot 5 = 20$ sides of length $1/3$ on the boundary (middle figure above). The process continues on each of those 20 sides with squares of side length $1/9$. Note that after that 2nd iteration, there are 8 "holes" in the fractal as shown in the right figure above.

(a, 6 pts) At iteration i, please state how much area A_i is added. Note that $A_1 = 4/9$. Please also state the length of the boundary B_i at iteration i. Note that $B_1 = 20/3$.

(b, 4 pts) Please compute $\sum_{i=1}^{\infty} A_i$. Please show your work.