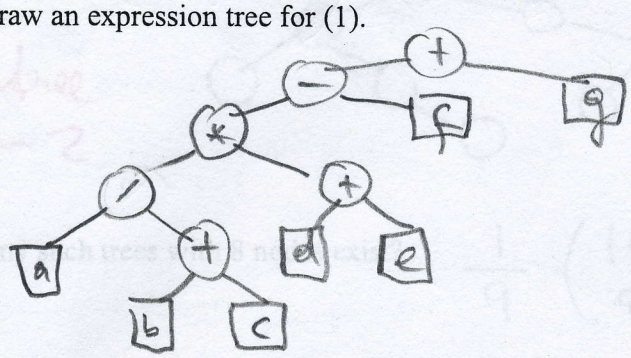
2009

Problem 1 (12 points): Consider the following mathematical expression in **prefix** notation, assuming that each of the operators $+, -, *, /$ has two operands.

$$+ - * / a + bc + defg \tag{1}$$

(a, 4pts) Please draw an expression tree for (1).



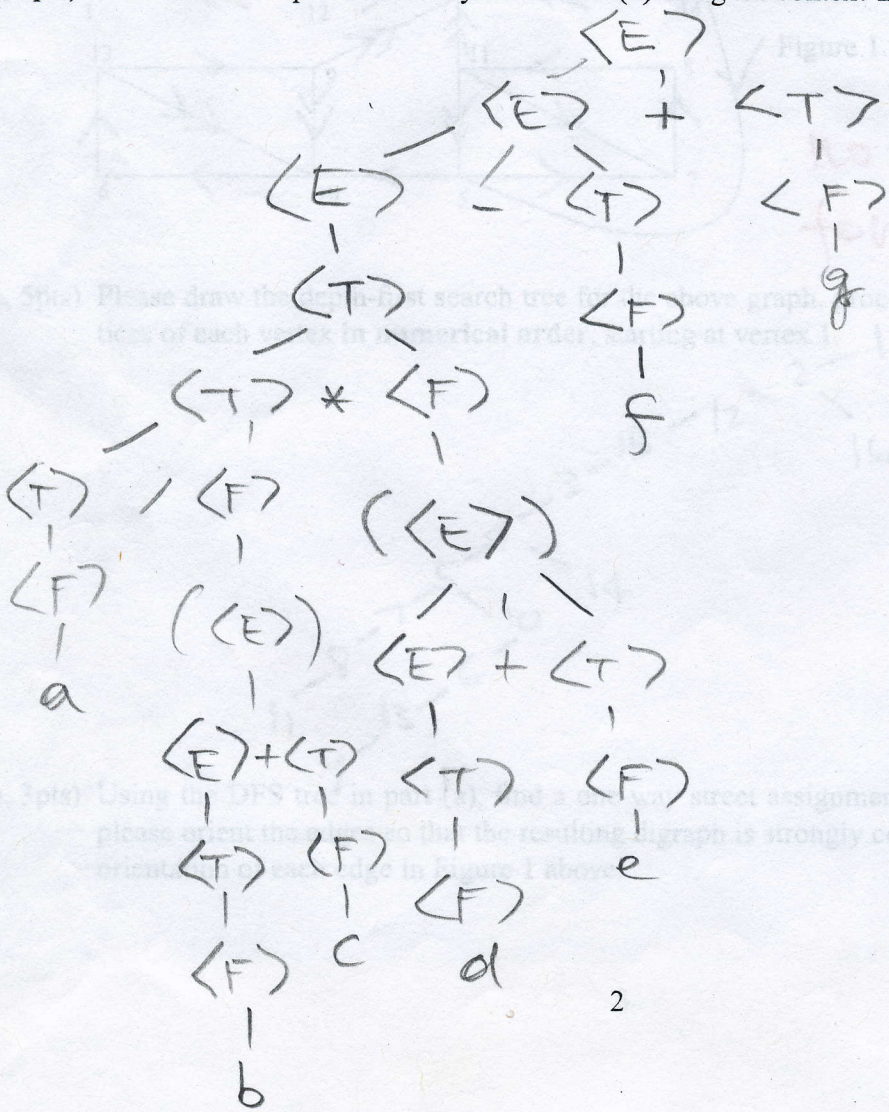
no penalty if OK f. (a) not minimal

(b, 4pts) Please give a **minimally parenthesized infix** expression for (1).

$$a/(b+c)*(d+e)-f+g$$

−1

(c, 4pts) Please draw the parse tree for your answer (b) using the context-free grammar given in class.



no penalty if correct for b

different expr.

−1

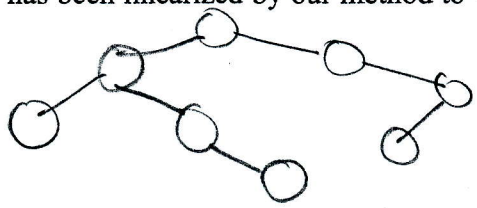
incorrect rule

−1

2009

Problem 2 (8 points): Consider binary trees in which each node has either 0 children, or one left or one right child, or both.

(a, 4pts) Such a tree with 8 nodes has been linearized by our method to `(((())()())()(()))`. Please draw the tree.



left subtree wrong −2

(b, 4pts) How many such trees with 8 nodes exist?

$$\frac{1}{9}\binom{16}{8} = \frac{\cancel{16}\cdot\cancel{15}\cdot\cancel{14}\cdot 13\cdot\cancel{12}\cdot 11\cdot\cancel{10}\cdot\cancel{9}}{1\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{4}\cdot\cancel{5}\cdot\cancel{6}\cdot\cancel{7}\cdot\cancel{8}\cdot\cancel{9}}$$

$\cancel{2}\ 5\ 2$

$= 10\cdot 13\cdot 11$

$= 130\cdot 11 = 1430$

Problem 3 (8 points): Consider the following graph:

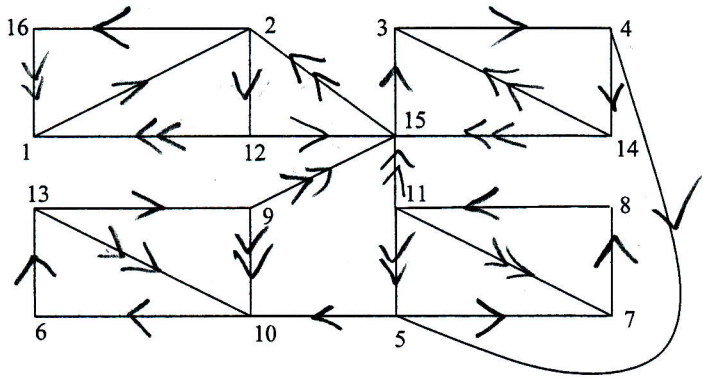
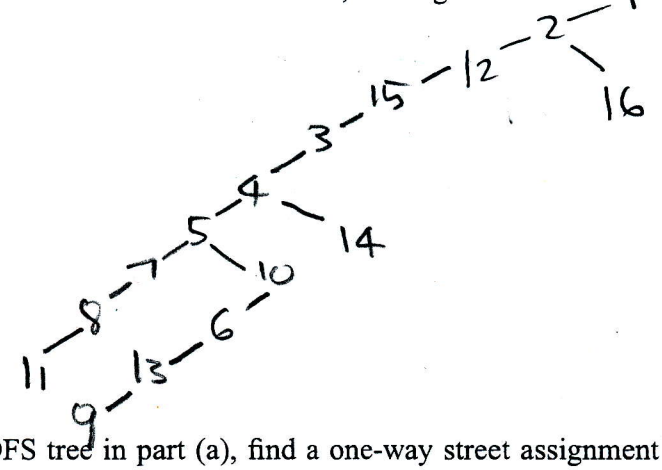


Figure 1.

no penalty if OK for tree in a)

(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.



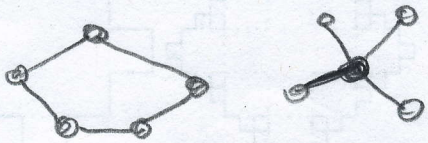
(b, 3pts) Using the DFS tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1 above.

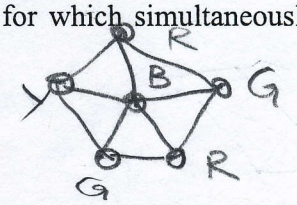
2009

Problem 4 (4 points): Suppose $\omega(G)$ is the clique number, $\chi(G)$ the chromatic number and $\Delta(G)$ the maximum vertex degree of a graph G. Please draw a graph for which simultaneously the inequalities $\omega(G) < \chi(G)$ and $\chi(G) < \Delta(G)$ hold.

one inequ.

+2





$\omega = 3$
$\chi = 4$
$\Delta = 5$

$\omega(G) = 2$, $\chi(G) = 3$, $\Delta(G) = 4$

Problem 5 (8 points): Please consider the 4-dimensional "de Bruijn plus graph" with 16 vertices, which has the additional edge $\{6, 9\}$.

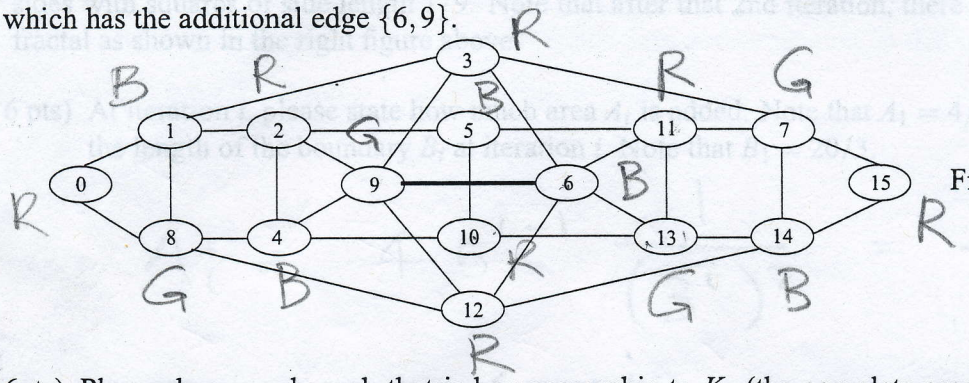
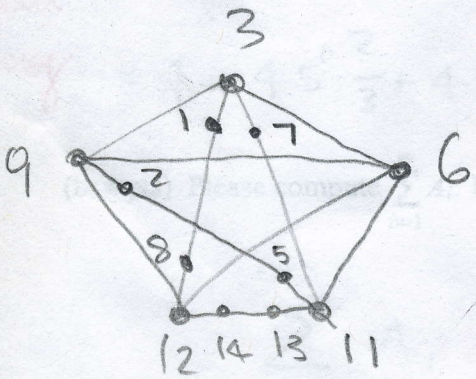
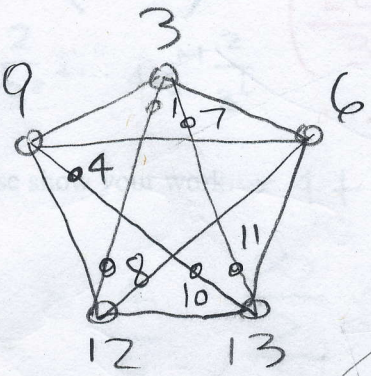
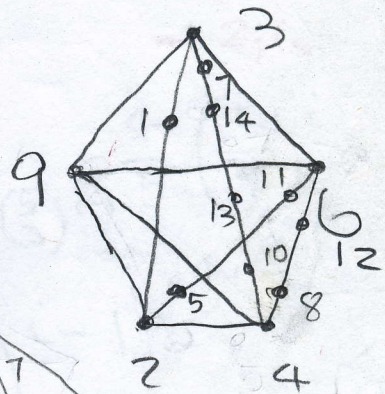


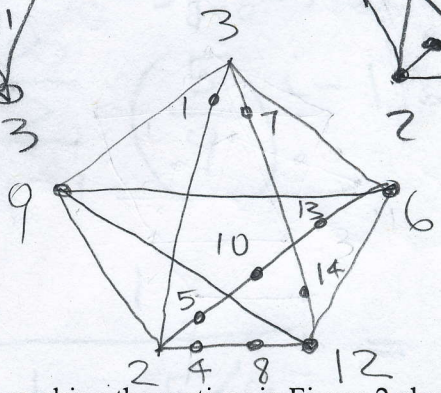
Figure 2.

(a, 6pts) Please draw a subgraph that is homeomorphic to K_5 (the complete graph with 5 vertices). Hint: Choose 3, 6, 9 and two more vertices as the corner vertices of the K_5-like subgraph.









shared vertex

-2

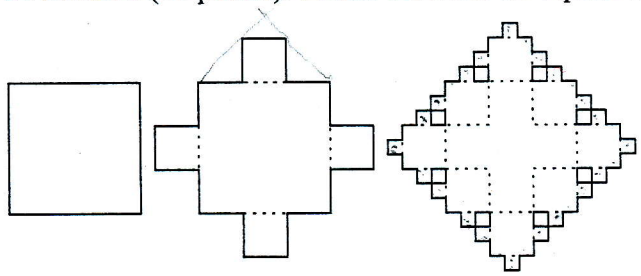
(b, 2pts) Please 3-color the de Bruijn plus graph graph by marking the vertices in Figure 2 above with the colors R,G,B.

color homeomorphic image

no credit

2009

Problem 6 (10 points): Please consider the square snowflake fractal:



Here one starts with a square, whose side length is 1 (left figure above). Each side is exuded in the first iteration by a square of side length $1/3$ in the middle of the side, creating $4 \cdot 5 = 20$ sides of length $1/3$ on the boundary (middle figure above). The process continues on each of those 20 sides with squares of side length $1/9$. Note that after that 2nd iteration, there are 8 "holes" in the fractal as shown in the right figure above.

(a, 6 pts) At iteration i, please state how much area A_i is added. Note that $A_1 = 4/9$. Please also state the length of the boundary B_i at iteration i. Note that $B_1 = 20/3$.

3 $$A_i = 4 \cdot 5^{i-1} \frac{1}{(3^i)^2} = \frac{4}{9}\left(\frac{5}{9}\right)^{i-1}$$

4^i $\quad \frac{5^{i-1}}{9^i}$

3 $$B_i = 4 \cdot \left(\frac{5}{3}\right)^i$$

$\left(\frac{20}{3}\right)^i$ $\quad$ − 2pts

Ryan Story $= 4 + 4 \cdot 5^0 \cdot \frac{2}{3} + 4 \cdot 5^1 \frac{2}{3^2} + \cdots 4 \cdot 5^{i-1} \frac{2}{3^i}$

$+1$ $\quad \frac{4}{5}\left(\frac{5}{9}\right)^i$

(b, 4 pts) Please compute $\sum_{i=1}^{\infty} A_i$. Please show your work. $= 4 + \frac{8}{3} \sum_{j=0}^{i-1} \left(\frac{5}{3}\right)^{i-1} = 4 + \frac{8}{3} \frac{\left(\frac{5}{3}\right)^i - 1}{\frac{5}{3} - 1} = 4 \cdot \left(\frac{5}{3}\right)^i$

$$\sum_{i=1}^{\infty} A_i = \frac{4}{9} \sum_{i=1}^{\infty} \left(\frac{5}{9}\right)^{i-1}$$

2-3 pts if correct for (a)

$$= \frac{4}{9} \frac{1}{1 - 5/9} = 1$$