North Carolina State University is a land-grant university and a constituent institution of The University of North Carolina

Department of Mathematics

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, first mid-semester examination, Tuesday, September 21, 2010

Prof. Erich Kaltofen <kaltofen@math.ncsu.edu>
www.math.ncsu.edu/~kaltofen/courses/DiscreteModels/Fall10/index.html (URL)
919.515.8785 (phone)
919.515.3798 (fax)

Your Name: _____

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 4 problems, which are subdivided into 15 questions, where each question counts for the explicitly given number of points, adding to a total of **50 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

Total _____

Problem 1 (13 points)

(a, 6pts) Consider the linear recurrence g_n with

$$g_0 = 2, g_1 = 3, \quad \forall n \geq 2 : g_n = 4g_{n-1} - 4g_{n-2}. \tag{1}$$

i. Please list the values of $g_2, g_3, g_4, g_5, g_6, g_7$.

ii. Please compute a closed form solution for g_n in (1) above.

(b, 7pts) i. Please define the 6-dimensional hypercube graph. In particular, define the set of vertices and edges.

ii. Please discuss path-finding (routing) methods on the 6-dimensional hypercube. In particular, describe all shortest paths between a pair of vertices, and discuss the importance of having multiple shortest paths when the distance is larger.

Problem 2 (12 points): Consider the following digraph:

$$D = (\{1,2,3,4,5,6\}, \{(1,5),(2,2),(2,4),(3,4),(3,6),(4,2),(4,4),(5,3),(6,1),(6,2)\})$$

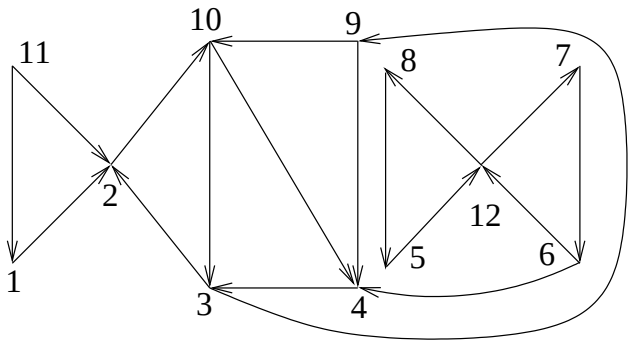
(a, 3pts) Please draw a picture of D.

(b, 3pts) Please write down the adjacency matrix M for D under the vertex order $(1,2,3,4,5,6)$.

(c, 3pts) Please write down M^2.

(d, 3pts) Please write down the reachability matrix R for D under the vertex order $(1,2,3,4,5,6)$.

Problem 3 (12 points):
Consider the following digraph:



(a, 4pts) Please list the strong components of the above digraph.

(b, 4pts) Please draw the digraph that is the condensation of the above digraph.

(c, 4pts) Please give the vertex basis for the condensation digraph (b) and all vertex bases for the above original digraph.

Problem 4 (13 points): Please consider a 5-dimensional de Bruijn digraph $D_5 = (V_5, A_5)$ whose $32 = 2^5$ vertices are bit strings of length 5, $b_1b_2b_3b_4b_5 \in V_5$, where all $b_i \in \{0,1\}$ for all $1 \leq i \leq 5$. As in homework, the arcs are constructed by shifting to the **right**, and shifting in a 0 or 1 for the **left**most bit: $(b_1b_2b_3b_4b_5, 0b_1b_2b_3b_4) \in A_5$ and $(b_1b_2b_3b_4b_5, 1b_1b_2b_3b_4) \in A_5$.

(a, 3pts) How many arcs does D_5 have? Please explain.

(a, 3pts) What is the diameter of D_5? Please explain.

(b, 3pts) Please give a pair of vertices u, v in V_5 such that the distance from u to v is 4.

(c, 4pts) Please give a subgraph that is a simple closed path (cycle) of length 4.