Your Name: ______________________________
For purpose of anonymous grading, please do not write your name on the subsequent pages.

This examination consists of 7 problems, which are subdivided into 13 questions, where each question counts for the explicitly given number of points, adding to a total of 50 points. Please write your answers in the spaces indicated, or below the questions, using the back of the sheets for completing the answers and for all scratch work, if necessary. You are allowed to consult two 8.5in × 11in sheets with notes, but not your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have 75 minutes to do this test.

Problem 1 ___

2 ___

3 ___

4 ___

5 ___

6 ___

7 ___

Total ___
Problem 1 (10 points): Consider the following mathematical expression in parenthesized infix notation.

\[ a + b - c \times d/(e/f + g) - (h) \]  

(a, 4pts) Using precedence rules and left-to-right tie breaking for operator priority, please draw the expression tree for (1).

(b, 2pts) Please give both the prefix and postfix representation, which only has variables and operators, for the expression (1)

(c, 4pts) Please draw the parse tree for (1) above using the context-free grammar given in class.
Problem 2 (6 points): Consider binary trees in which each node has either 0 children, or one left or one right child, or both.

(a, 4pts) Such a tree with 9 nodes has been linearized by our method from Homework 3 to (((())))((())())(). Please draw the tree.

(b, 2pts) How many strings with 9 balanced pairs of parentheses like the one above exist?

Problem 3 (6 points): Consider the following graph:

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex in numerical order, starting at vertex 1.

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1 above, using a different arrow head for those arcs that correspond to edges in the DFS tree.
**Problem 4** (6 points): What is the chromatic number of the \( n \)-dimensional hypercube? Please explain.

**Problem 5** (8 points): Please consider the \( 3 \times 3 \) toric mesh with 9 vertices.

Please draw a subgraph that is homeomorphic to \( K_5 \) (the complete graph with 5 vertices).
Problem 6 (8 points): Please consider the modified square snowflake fractal:

Here one starts with a line segment, whose length is 1 (left figure above). Each line is exuded in the first iteration by a square of side length 1/3 in the middle of the segment, creating 5 line segments of length 1/3 (middle figure above). The process continues on each of the 3 horizontal of those 5 line segments, creating 9 horizontal lines segments of length 1/9 for the next step. Please give (a) the total length $L_i$ of all horizontal and vertical line line segments after $i$ iterations, where $L_0 = 1$ and $L_1 = 5/3$, and (b) the area of the squares $\lim_{i \to \infty} A_i$, where $A_1 = 1/9$ and $A_2 = 4/27$.

Problem 7 (6 points): Consider the following variant of Fibonacci’s rabbits problem: Each pair takes 2 months to mature, and then after every additional month gives birth to 2 pairs of rabbits. Of those, only one pair is fertile and long-lived, while both rabbits of the other pair die after 2 months. Please (a) model the variant by a Lindenmayer system, annotating each variable by what type of pair it represents, and (b) give the first 5 new generations of the system, starting at generation 0 with a single pair of newly born fertile and long-lived rabbits.