

**NC STATE UNIVERSITY**

MA 351 Intro Discrete Math Models, second mid-semester examination, Nov 4, 2010  
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[www.math.ncsu.edu/~kaltofen/courses/DiscreteModels/Fall10/index.html](http://www.math.ncsu.edu/~kaltofen/courses/DiscreteModels/Fall10/index.html) (URL)

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Your Name: \_\_\_\_\_

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 7 problems, which are subdivided into 13 questions, where each question counts for the explicitly given number of points, adding to a total of **50 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work**, if necessary. You are allowed to consult **two** 8.5in  $\times$  11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 \_\_\_\_\_

2 \_\_\_\_\_

3 \_\_\_\_\_

4 \_\_\_\_\_

5 \_\_\_\_\_

6 \_\_\_\_\_

7 \_\_\_\_\_

Total \_\_\_\_\_

**Problem 1** (10 points): Consider the following mathematical expression in parenthesized **infix** notation.

$$a + b - c * d / (e / f + g) - (h) \quad (1)$$

(a, 4pts) Using precedence rules and left-to-right tie breaking for operator priority, please draw the expression tree for (1).

(b, 2pts) Please give both the **prefix** and **postfix** representation, which only has variables and operators, for the expression (1)

(c, 4pts) Please draw the parse tree for (1) above using the context-free grammar given in class.

**Problem 2** (6 points): Consider binary trees in which each node has either 0 children, or one left or one right child, or both.

(a, 4pts) Such a tree with 9 nodes has been linearized by our method from Homework 3 to  $((())())((())())()$ . Please draw the tree.

(b, 2pts) How many strings with 9 **balanced** pairs of parentheses like the one above exist?

**Problem 3** (6 points): Consider the following graph:

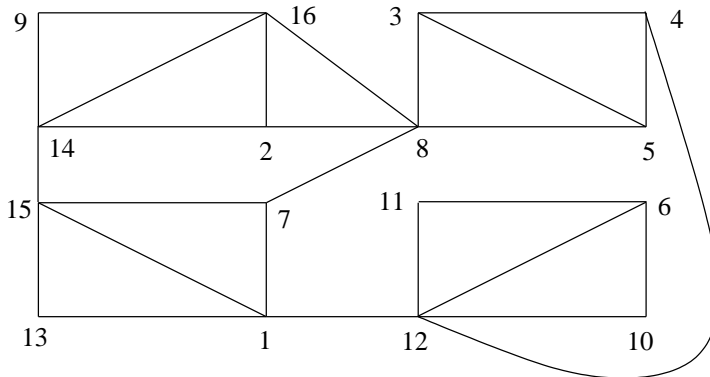


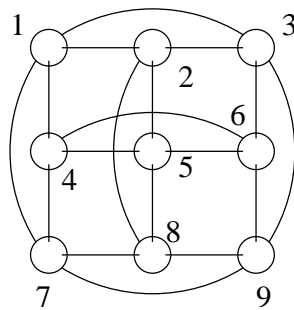
Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1 above, using a different arrow head for those arcs that correspond to edges in the DFS tree.

**Problem 4** (6 points): What is the chromatic number of the  $n$ -dimensional hypercube? Please explain.

**Problem 5** (8 points): Please consider the  $3 \times 3$  toric mesh with 9 vertices.



Please draw a subgraph that is homeomorphic to  $K_5$  (the complete graph with 5 vertices).

**Problem 6** (8 points): Please consider the modified square snowflake fractal:



Here one starts with a line segment, whose length is 1 (left figure above). Each line is exuded in the first iteration by a square of side length  $1/3$  in the middle of the segment, creating 5 line segments of length  $1/3$  (middle figure above). The process continues on each of the 3 **horizontal** of those 5 line segments, creating 9 horizontal line segments of length  $1/9$  for the next step. Please give (a) the total length  $L_i$  of all horizontal and vertical line segments after  $i$  iterations, where  $L_0 = 1$  and  $L_1 = 5/3$ , and (b) the area of the squares  $\lim_{i \rightarrow \infty} A_i$ , where  $A_1 = 1/9$  and  $A_2 = 4/27$ .

**Problem 7** (6 points): Consider the following variant of Fibonacci's rabbits problem: *Each pair takes 2 months to mature, and then after every additional month gives birth to 2 pairs of rabbits. Of those, only one pair is fertile and long-lived, while both rabbits of the other pair die after 2 months.* Please (a) model the variant by a Lindenmayer system, annotating each variable by what type of pair it represents, and (b) give the first 5 new generations of the system, starting at generation 0 with a single pair of newly born fertile and long-lived rabbits.