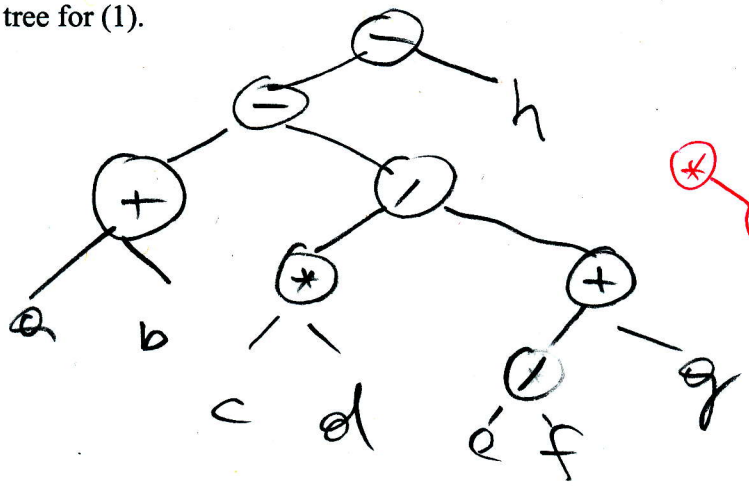
2010

Problem 1 (10 points): Consider the following mathematical expression in parenthesized **in**fix notation.

$$a+b-c*d/(e/f+g)-(h) \tag{1}$$

(a, 4pts) Using precedence rules and left-to-right tie breaking for operator priority, please draw the expression tree for (1).



* / -1pt

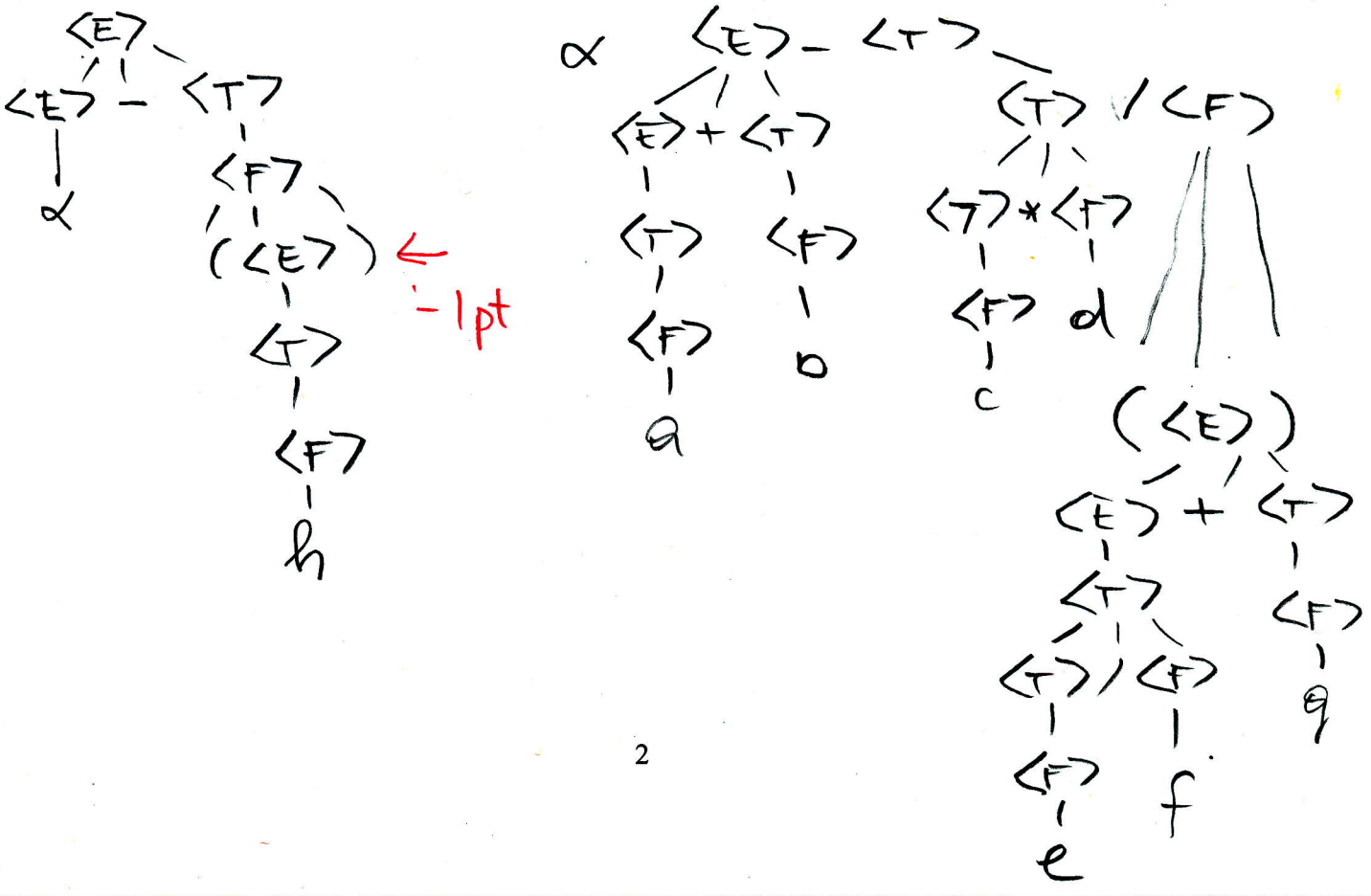
backslash

(b, 2pts) Please give both the **pre**fix and **post**fix representation, which only has variables and operators, for the expression (1)

- - + a b / * c d + / e f g h

a b + c d * e f / g + / - h -

(c, 4pts) Please draw the parse tree for (1) above using the context-free grammar given in class.



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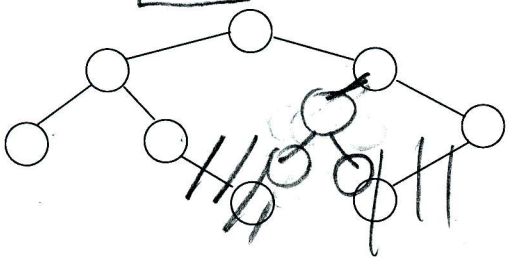
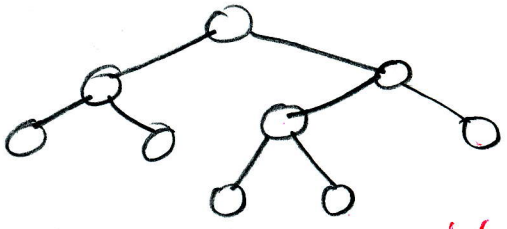
Problem 2 (6 points): Consider binary trees in which each node has either 0 children, or one left or one right child, or both.

(a, 4pts) Such a tree with 9 nodes has been linearized by our method from Homework 3 to ((())())((())()). Please draw the tree.





right subtree wrong −2

(b, 2pts) How many strings with 9 **balanced** pairs of parentheses like the one above exist?

$\frac{1}{10}\binom{18}{9} = \frac{18\cdot 17\cdot 16\cdot 15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9\cdot 10}$

$= 11\cdot 13\cdot 17\cdot 2$

$= 4862$

Problem 3 (6 points): Consider the following graph:

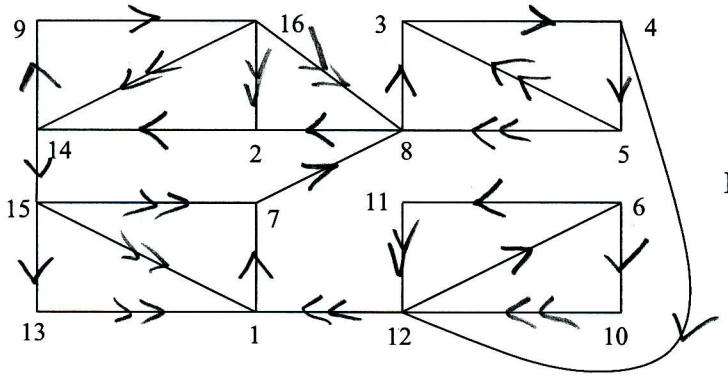
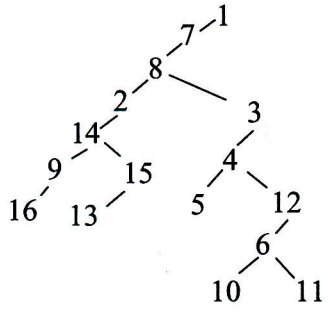


Figure 1.

13 · 11
13
143 17
1001
2431

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

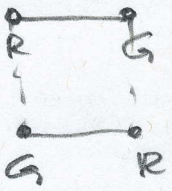


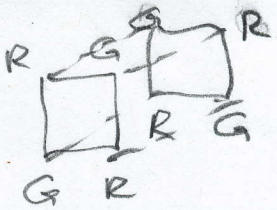
DFS tree but random order +1

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1 above, using a different arrow head for those arcs that correspond to edges in the DFS tree.

≥ 3 arrow wrong no penalty

−1





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etc.

$\chi(H_0) = 1$

Problem 4 (6 points): What is the chromatic number of the n-dimensional hypercube? Please explain.

Stephen Ramsey

$\chi(H_n) = 2$ Color each vertex with an even number of 1's R, the others G. Since an edge connects 2 vertices with Hamming distance 1, the number of 1's changes by 1, so do the colors.

≥ 2

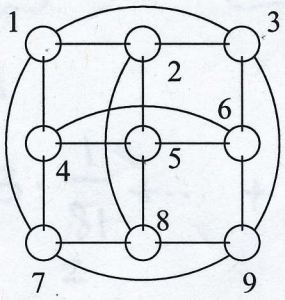
$+1$

2 3pts

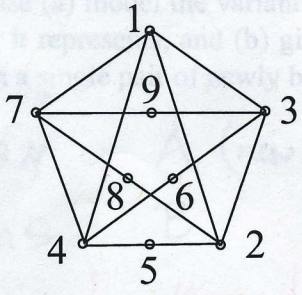
explan. 3pts

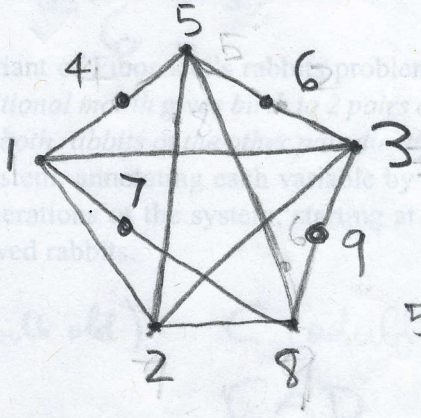
$\chi(G) \leq \omega(G)$ no credit

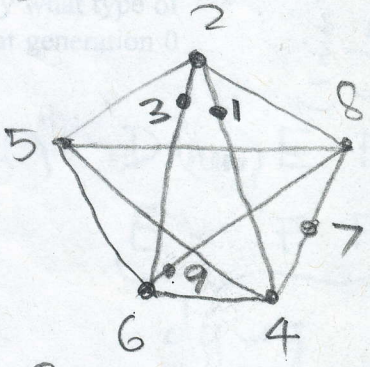
Problem 5 (8 points): Please consider the 3×3 toric mesh with 9 vertices.

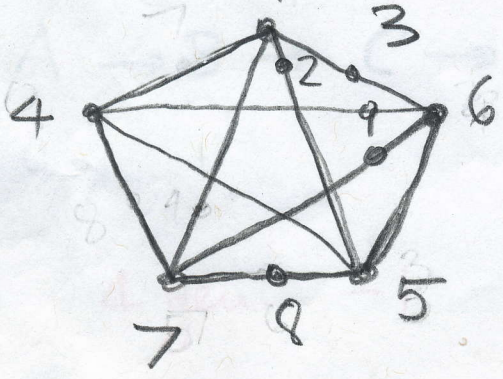


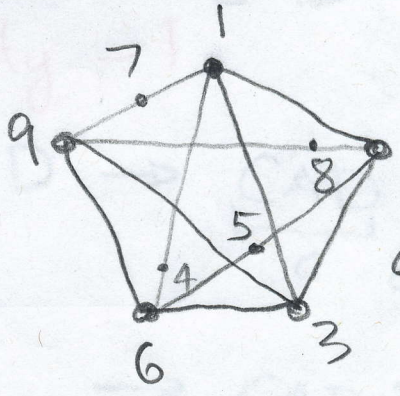
Please draw a subgraph that is homeomorphic to K_5 (the complete graph with 5 vertices).

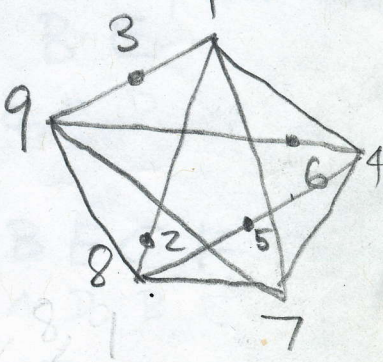












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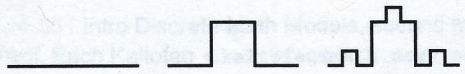
Problem 6 (8 points): Please consider the modified square snowflake fractal:



Here one starts with a line segment, whose length is 1 (left figure above). Each line is exuded in the first iteration by a square of side length $1/3$ in the middle of the segment, creating 5 line segments of length $1/3$ (middle figure above). The process continues on each of the 3 **horizontal** of those 5 line segments, creating 9 horizontal lines segments of length $1/9$ for the next step. Please give (a) the total length L_i of all horizontal and vertical line line segments after i iterations, where $L_0 = 1$ and $L_1 = 5/3$, and (b) the area of the squares $\lim_{i\to\infty} A_i$, where $A_1 = 1/9$ and $A_2 = 4/27$, $A_3 = \frac{13}{81}$

$$L_i = 1 + \frac{2}{3} + 3\cdot\frac{2}{9} + \cdots + 3^{i-1}\frac{2}{3^i} = 1 + \frac{2}{3}i$$

$\left(\frac{5}{3}\right)^i$ ~~no credit~~ 1pt

4pts

$$A_i = 0 + \frac{1}{9} + 3\cdot\frac{1}{81} + \cdots + 3^{i-1}\frac{1}{(3^i)^2}$$

4pts

$\sum_{i=1}^{\infty} x^i = x\cdot\frac{1}{1-x}$

$$\lim_{i\to\infty} A_i = \frac{1}{9}\cdot\frac{1}{1-\frac{1}{3}} = \frac{1}{6}$$

$\frac{1}{3^{i+1}} = \frac{1}{9}\cdot\frac{1}{3^{i-1}}$

Barry Peddycord

Problem 7 (6 points): Consider the following variant of Fibonacci's rabbits problem: *Each pair takes 2 months to mature, and then after every additional month gives birth to 2 pairs of rabbits. Of those, only one pair is fertile and long-lived, while both rabbits of the other pair die after 2 months.* Please (a) model the variant by a Lindenmeyer system, annotating each variable by what type of pair it represents, and (b) give the first 5 new generations of the system, starting at generation 0 with a single pair of newly born fertile and long-lived rabbits.

3pts

var	A (new born)	B (1 month old)	C (adult)	D (new)	E (1 month old)	F (dead)
rhs	B	C	CAD	E	F	F

1 month maturity −1

dying pair

3pts

$$A \to B \to C \to CAD \to \underbrace{CAD}_{C}\ B\ \underbrace{E}_{A}\ \underbrace{D}_{D}$$

0 1 2 3 4

4 gen. −1

$$\to \underbrace{CAD}_{C}\ B\ \underbrace{E}_{A}\ \underbrace{C}_{D}\ \underbrace{F}_{B}\ \underbrace{E}_{E}$$

plain Fibo: no credit

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