

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Thu, Nov 3, 2011
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www.math.ncsu.edu/~kaltofen/courses/DiscreteModels/Fall11/index.html (URL)

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Your Name: _____

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 7 problems, which are subdivided into 10 questions, where each question counts for the explicitly given number of points, adding to a total of **50 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work**, if necessary. You are allowed to consult **two** 8.5in \times 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

5 _____

6 _____

7 _____

Total _____

Problem 1 (8 points): Consider the following mathematical expression in **postfix** notation. assuming that each of the operators $+$, $-$, $*$, $/$, \uparrow has two operands (\uparrow is exponentiation).

$$abc \uparrow \uparrow de / * fgh - / + \quad (1)$$

(a, 4pts) Please draw the expression tree for (1).

(b, 4pts) Please give both the **minimally parenthesized infix** and the **prefix** representations for the expression (1), the latter of which only has variables and operators.

Problem 2 (7 points): Please parse the string

$$(\perp)(\perp)((\perp)\perp)\perp$$

with the context-free grammar of three meta-symbols $\langle T \rangle$, $\langle L \rangle$, $\langle R \rangle$, three terminal symbols $(,), \perp$, rules

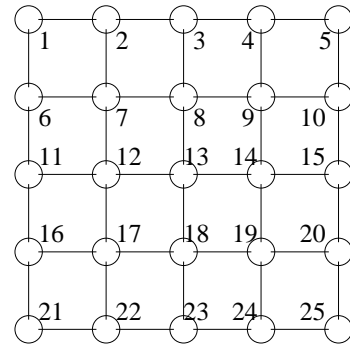
$$\langle T \rangle \rightarrow (\langle L \rangle)\langle R \rangle, \quad \langle L \rangle \rightarrow \langle T \rangle, \quad \langle L \rangle \rightarrow \perp, \quad \langle R \rangle \rightarrow \langle T \rangle, \quad \langle R \rangle \rightarrow \perp,$$

and start symbol $\langle T \rangle$.

Problem 3 (7 points):

Please consider the 5×5 grid graph
(with the given vertex labeling):

How many of the shortest paths from
vertex 1 to vertex 25 do not cross
the diagonal, that is, do not contain any
of the vertices 2, 3, 4, 5, 8, 9, 10, 14, 15, 20? Please explain.



Problem 4 (6 points): Consider the following graph:

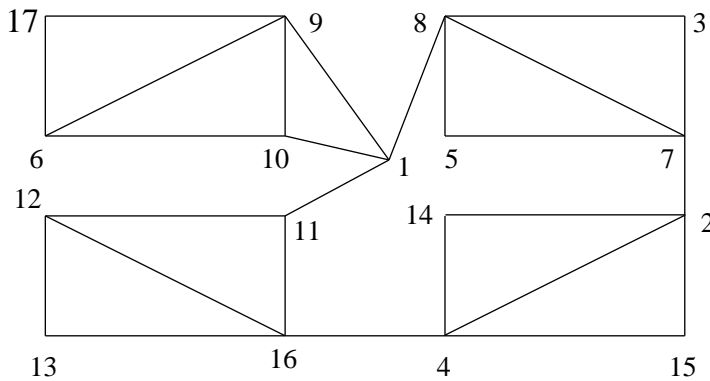
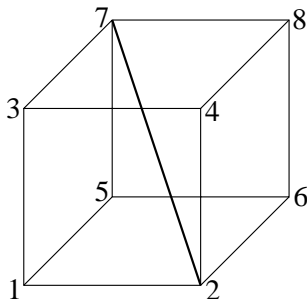


Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

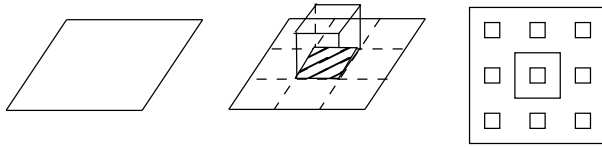
Problem 5 (8 points): Please consider the 3-D cube graph with an additional interior diagonal edge $\{2, 7\}$.



Please draw a subgraph that is homeomorphic to $K_{3,3}$, which denotes the complete bipartite graph from 3 to 3 vertices.

Problem 6 (4 points): Consider the following Lindenmayer system: $X \rightarrow YaZ$, $a \rightarrow a$, $Y \rightarrow Xb$, $b \rightarrow b$, $Z \rightarrow dX$, $d \rightarrow d$. Please write down the first 4 new generations of strings starting with X .

Problem 7 (10 points): Please consider the following cubic fractal:



Here one starts with a square, whose length is 1 (left figure above). The middle square of side length $1/3$ is exuded by a cube of side length $1/3$ (middle figure above).

In the second iteration, the middle squares (of side length $1/9$) of each of the 9 horizontal squares of side lengths $1/3$, that is, the 8 exposed bottom horizontal squares + the top square face of the cube, are exuded by cubes of side length $1/9$ (right figure, bird's eye view).

The process continues with 81 horizontal squares of side length $1/9$, who have their middle squares of side length $1/27$ exuded by cubes of side length $1/27$.

(a, 5 pts) Please give the total area A_i of all horizontal and vertical square faces after i iterations, where $A_0 = 1$ and $A_1 = 13/9$ (note that the bottom hashed face of the cube is not added).

(b, 5 pts) Please give the total volume of all the cubes $\lim_{i \rightarrow \infty} V_i$, where $V_1 = 1/27$ and $V_2 = 4/81$.