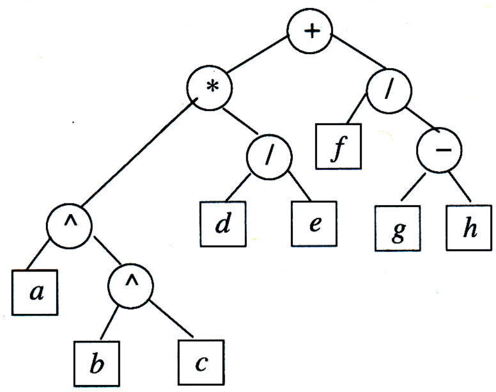
2011.

Problem 1 (8 points): Consider the following mathematical expression in **post**fix notation. assuming that each of the operators $+, -, *, /, \uparrow$ has two operands ($\uparrow$ is exponentiation).

$$abc \uparrow\uparrow de/ * fgh - /+ \qquad (1)$$

(a, 4pts) Please draw the expression tree for (1).



(b, 4pts) Please give both the **minimally parenthesized in**fix and the **pre**fix representations for the expression (1), the latter of which only has variables and operators.

2pts $a \uparrow b \uparrow c * (d/e) + f/(g-h)$ not min no penalty if OK for (a) −1pt. no ( ) around d/e no extra penalty

2pts $+ * \uparrow a \uparrow bc/de/f - gh$

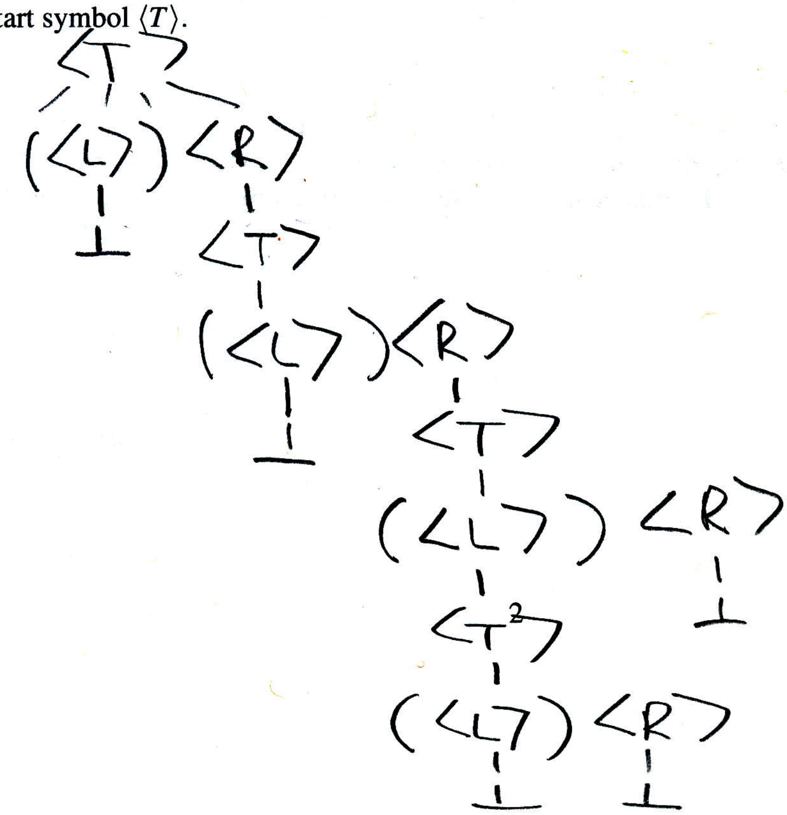
Problem 2 (7 points): Please parse the string

$$(\perp)(\perp)((\perp)\perp)\perp$$

with the context-free grammar of three meta-symbols $\langle T\rangle, \langle L\rangle, \langle R\rangle$, three terminal symbols $(,), \perp$, rules

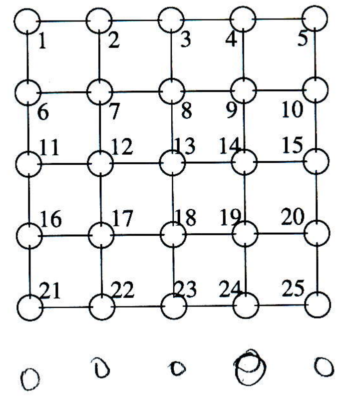
$$\langle T\rangle \to (\langle L\rangle)\langle R\rangle, \quad \langle L\rangle \to \langle T\rangle, \quad \langle L\rangle \to \perp, \quad \langle R\rangle \to \langle T\rangle, \quad \langle R\rangle \to \perp,$$

and start symbol $\langle T\rangle$.



2011

Problem 3 (7 points):
Please consider the 5×5 grid graph (with the given vertex labeling):
How many of the shortest paths from vertex 1 to vertex 25 do not cross the diagonal, that is, do not contain any of the vertices 2, 3, 4, 5, 8, 9, 10, 14, 15, 20? Please explain.



$$\binom{8}{4} - \binom{8}{3} = \frac{1}{5}\binom{8}{4}$$

$$= \frac{1}{5}\,\frac{8\cdot 7\cdot 6\cdot 5}{1\cdot 2\cdot 3\cdot 4} = 14$$

explicit count, but short 5pts

$$\binom{10}{5} - \binom{10}{4} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{1\cdot 2\cdot 3\cdot 4\cdot 5} - \frac{10\cdot 9\cdot 8\cdot 7}{1\cdot 2\cdot 3\cdot 4} = 3\cdot 7(2\cdot 6-10) = 42$$ 5pts

$\binom{8}{4} = \frac{8\cdot 7\cdot 6\cdot 5}{1\cdot 2\cdot 3\cdot 4} = 70$ 3pts $\quad \binom{2n}{n} - \binom{2n}{n-1}$ 4pts

Problem 4 (6 points): Consider the following graph:

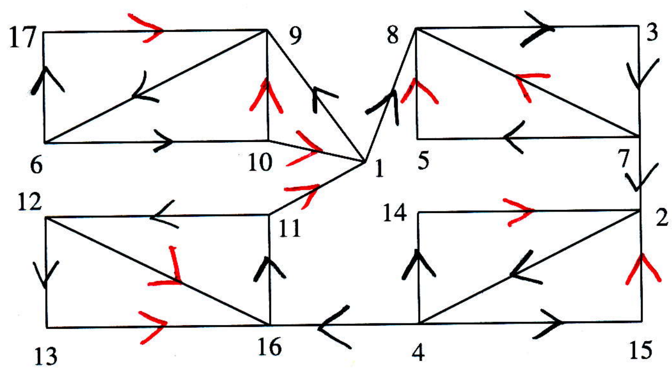
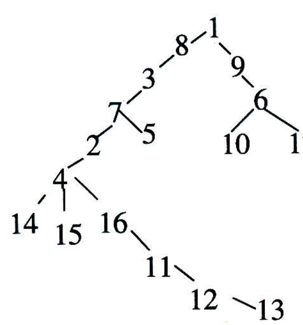


Figure 1.

≤ 2 back arcs missing no penalty

no penalty if OK for (a)

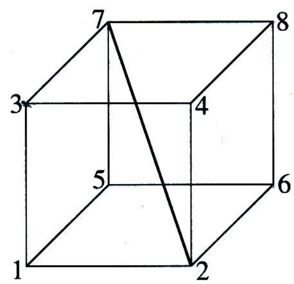
(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.



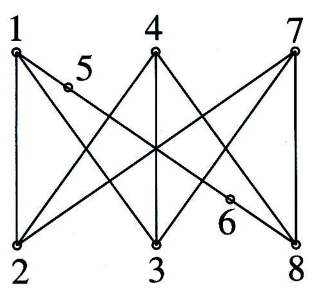
2011

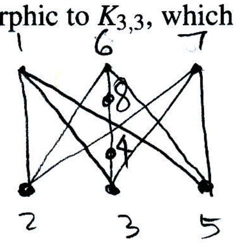
(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

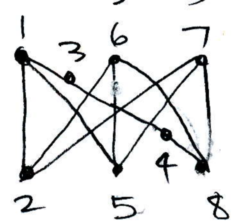
Problem 5 (8 points): Please consider the 3-D cube graph with an additional interior diagonal edge $\{2,7\}$.

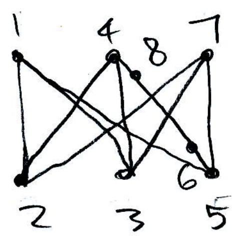


Please draw a subgraph that is homeomorphic to $K_{3,3}$, which denotes the complete bipartite graph from 3 to 3 vertices.

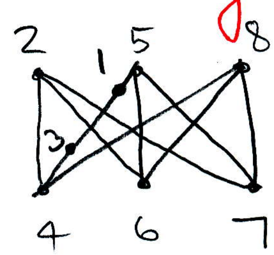


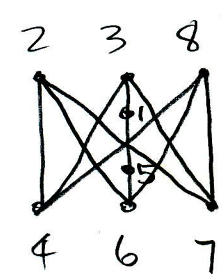






Incomplete leg
1 edge −2
2 edges −5





Problem 6 (4 points): Consider the following Lindenmayer system: $X \to YaZ$, $a \to a$, $Y \to Xb$, $b \to b$, $Z \to dX$, $d \to d$. Please write down the first 4 new generations of strings starting with X.

$X \to YaZ \to XbadX \to YaZbadYaZ \to XbadXbadXbadX$

1pt 2 3 4

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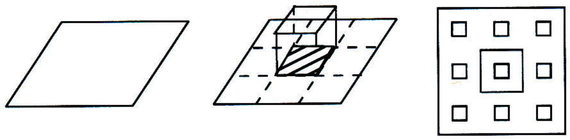
Problem 7 (10 points): Please consider the following cubic fractal:



Here one starts with a square, whose length is 1 (left figure above). The middle square of side length 1/3 is exuded by a cube of side length 1/3 (middle figure above).

In the second iteration, the middle squares (of side length 1/9) of each of the 9 horizontal squares of side lengths 1/3, that is, the 8 exposed bottom horizontal squares + the top square face of the cube, are exuded by cubes of side length 1/9 (right figure, bird's eye view).

The process continues with 81 horizontal squares of side length 1/9, who have their middle squares of side length 1/27 exuded by cubes of side length 1/27.

(a, 5 pts) Please give the total area A_i of all horizontal and vertical square faces after i iterations, where $A_0 = 1$ and $A_1 = 13/9$ (note that the bottom hashed face of the cube is not added).

Sum not simplified: no penalty

$A_i = 1+4\cdot(1/3)^2+4\cdot 3^2\cdot(1/3^2)^2+4\cdot(3^2)^2\cdot(1/3^3)^2+\cdots+4\cdot(3^{i-1})^2\cdot(1/3^i)^2 = 1+i\cdot 4/9.$

$\frac{4}{9}$ $\quad 4\cdot\frac{9}{81}$ +1pt.

goes to ∞ : no credit

(b, 5 pts) Please give the total volume of all the cubes $\lim_{i\to\infty} V_i$, where $V_1 = 1/27$ and $V_2 = 4/81$.

$V_i = 1/27+3^2\cdot(1/3^2)^3+\cdots+(3^{i-1})^2\cdot(1/3^i)^3+\cdots = \sum_{i=1}^{\infty} 1/3^{i+2} = 1/27\cdot 1/(1-1/3) =$ 1/18.

$\frac{1}{81}+\frac{1}{243}$ $\quad 9^{i-1}\left(\frac{1}{3^i}\right)^3$