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Department of Mathematics

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Tue, Oct 30, 2012
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www.math.ncsu.edu/~kaltofen/courses/DiscreteModels/Fall12/index.html (URL)

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Your Name: ____________________

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 6 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of **47 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **two** 8.5in $\times$ 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1	_____
2	_____
3	_____
4	_____
5	_____
6	_____
Total	_____

Problem 1 (12 points): Consider the following mathematical expression in parenthesized **in**fix notation.

$$a * (b + c - d/e * f)/(g - h) \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).

(b, 4pts) Please give both the **pre**fix and the **post**fix representations for the expression (1), both of which only have variables and operators.

PREFIX:

POSTFIX:

(c, 4pts) Please draw the parse tree for (1) above using the context-free grammar given in class.

Problem 2 (5 points): Please explain what is the *Alabama Paradox*, using an example.

Problem 3 (8 points): Consider the following graph:

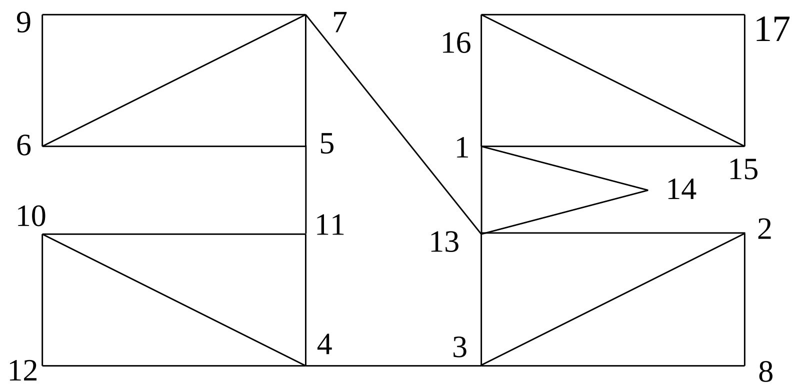


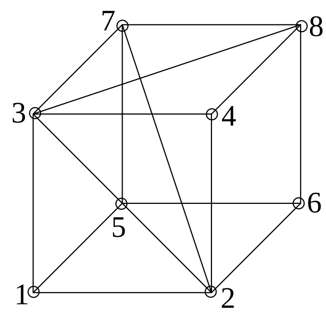
Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

(c, 2pts) Please 3-color the graph of Figure 1 on page 3. Please use the colors Red, Green, Blue, and place color assignments next to vertex numbers in Figure 1.

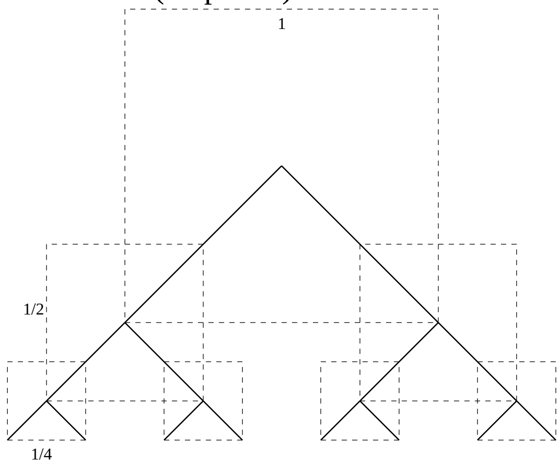
Problem 4 (8 points): Please consider the 3-D cube graph with the 4 additional edges $\{2,5\}$, $\{2,7\}$, $\{3,5\}$, $\{3,8\}$.



Please draw a subgraph that is homeomorphic to K_5, which denotes the complete graph with 5 vertices.

Problem 5 (4 points): Consider the following Lindenmayer system: $C \to cYZ$, $c \to c$, $Y \to hZ$, $h \to h$, $Z \to inW$, $i \to i$, $n \to n$, $W \to aCh$, $a \to a$. Please write down the first 4 new generations of strings starting with C.

Problem 6 (10 points): Please consider the following perfect tree fractal.



Here one starts with a square, whose side length is 1 and draws the edges from the midpoint, which is the root of the tree, to the bottom vertices of the square, which are the root's children.

In the second iteration, one places 2 squares of side length $\frac{1}{2}$ centered at those 2 children and again draws 4 edges to the 4 bottom vertices. The process continues with 4 squares of side length $\frac{1}{4}$, who have their centers at the 4 tree vertices.

(a, 5 pts) Please give the total length L_i of all line segments in the tree after i iterations, where $L_1 = \sqrt{2}$.

(b, 5 pts) Please give the accumulated area of all squares with dashed borders $\lim_{i\to\infty} A_i$, counting overlap areas only once. Note that $A_1 = 1$ and $A_2 = 1 + \frac{3}{8}$.