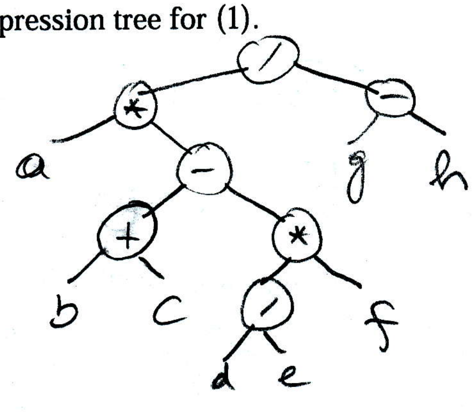
2012

Problem 1 (12 points): Consider the following mathematical expression in parenthesized **infix** notation.

$$a*(b+c-d/e*f)/(g-h) \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).



(b, 4pts) Please give both the **prefix** and the **postfix** representations for the expression (1), both of which only have variables and operators.

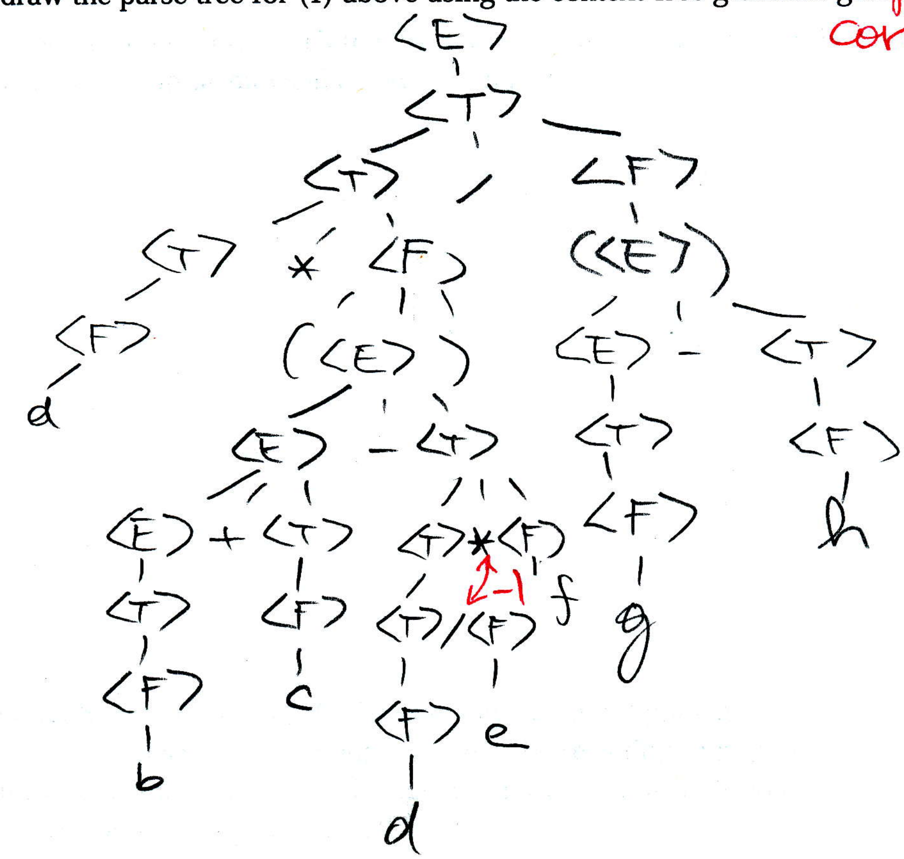
PREFIX: / * a − + b c * / d e f − g h

POSTFIX: a b c + d e / f * − * g h − /

X, • for *

no penalty if correct for (a)

(c, 4pts) Please draw the parse tree for (1) above using the context-free grammar given in class.



2012

explanation 2 pts
example 3 pts

voting theory OK

Problem 2 (5 points): Please explain what is the *Alabama Paradox*, using an example.

Popu.		Reps = 3		Reps = 4	
A	3	$\frac{3}{7} \cdot 3 = 1\frac{2}{7}$	1	$\frac{3}{7} \cdot 4 = 1\frac{5}{7}$	2
B	3	$\frac{3}{7} \cdot 3 = 1\frac{2}{7}$	1	$\frac{3}{7} \cdot 4 = 1\frac{5}{7}$	2
C	1	$\frac{1}{7} \cdot 3 = \frac{3}{7}$	1	$\frac{1}{7} \cdot 4 = \frac{4}{7}$	0

Condorcet's paradox: no credit

population increase: ~~no credit~~ +1

Problem 3 (8 points): Consider the following graph:

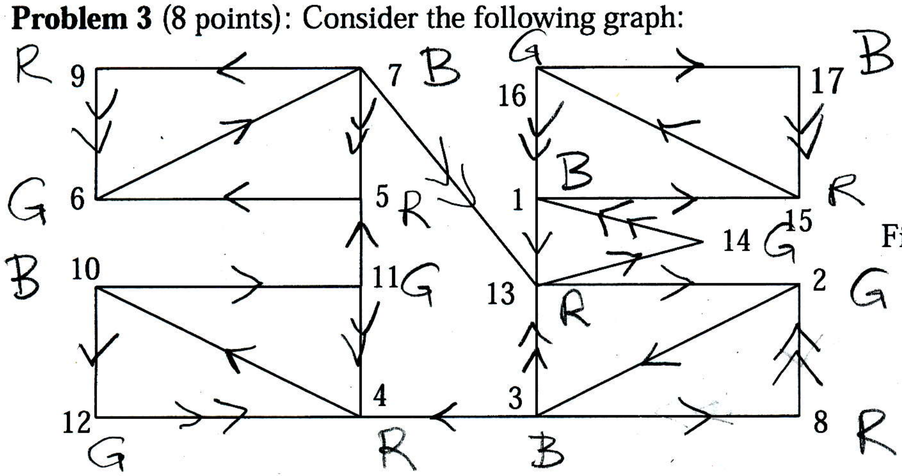
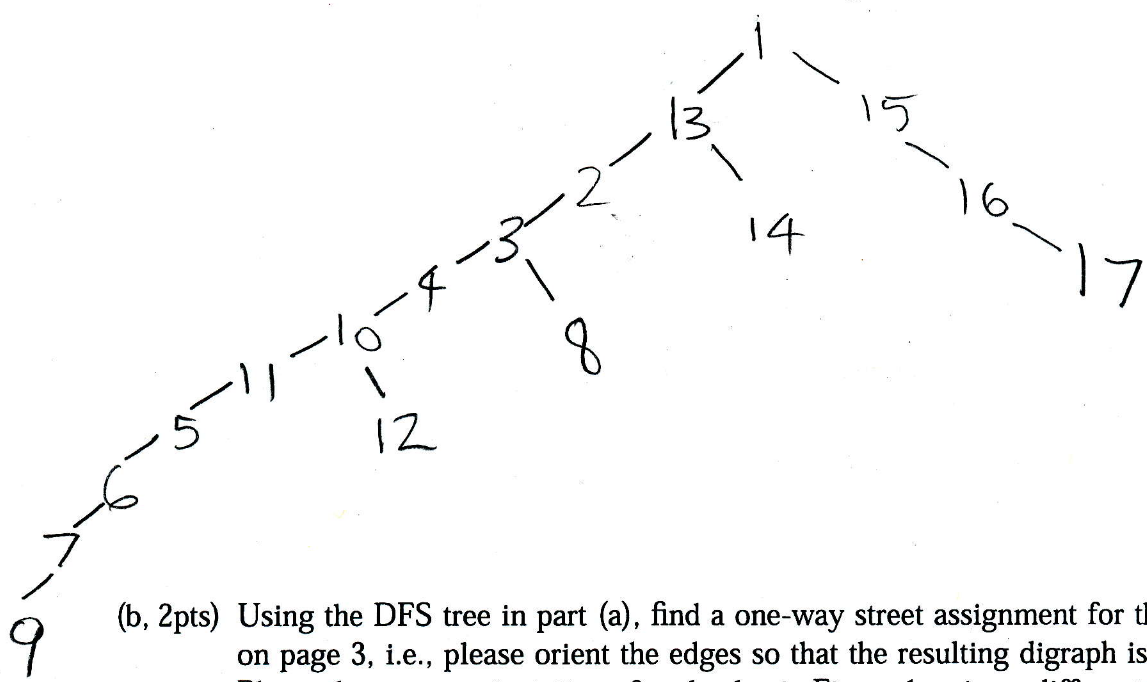


Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.



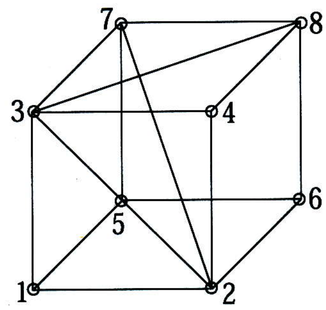
(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

OK if for DFS tree in (a)

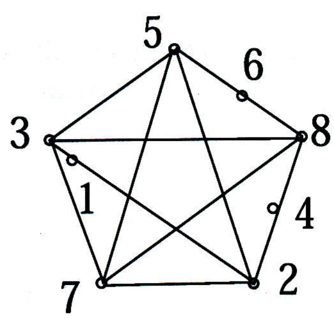
2012

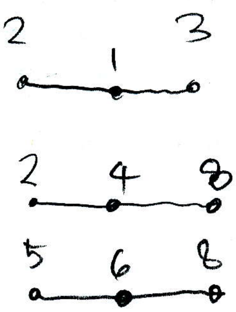
(c, 2pts) Please 3-color the graph of Figure 1 on page 3. Please use the colors Red, Green, Blue, and place color assignments next to vertex numbers in Figure 1.

Problem 4 (8 points): Please consider the 3-D cube graph with the 4 additional edges $\{2,5\}$, $\{2,7\}$, $\{3,5\}$, $\{3,8\}$.



Please draw a subgraph that is homeomorphic to K_5, which denotes the complete graph with 5 vertices.





1 false edge −3

incomplete −3

Problem 5 (4 points): Consider the following Lindenmayer system: $C \to cYZ$, $c \to c$, $Y \to hZ$, $h \to h$, $Z \to inW$, $i \to i$, $n \to n$, $W \to aCh$, $a \to a$. Please write down the first 4 new generations of strings starting with C.

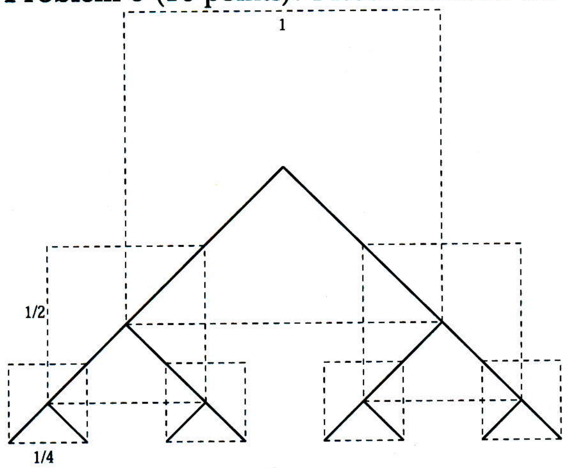
$X \to cYZ \to chZinW \to chinWinaCh \to chinaChinacYZh$

3 generations

−1 pt

2012

Problem 6 (10 points): Please consider the following perfect tree fractal.



Here one starts with a square, whose side length is 1 and draws the edges from the midpoint, which is the root of the tree, to the bottom vertices of the square, which are the root's children.

In the second iteration, one places 2 squares of side length $\frac{1}{2}$ centered at those 2 children and again draws 4 edges to the 4 bottom vertices. The process continues with 4 squares of side length $\frac{1}{4}$, who have their centers at the 4 tree vertices.

(a, 5 pts) Please give the total length L_i of all line segments in the tree after i iterations, where $L_1 = \sqrt{2}$.

$$L_i = \sqrt{2} + 2 \cdot (1/2)\sqrt{2} + 4(1/4)\sqrt{2} + \cdots + 2^{i-1} \cdot (1/2)^{i-1}\sqrt{2} = i \cdot \sqrt{2}.$$

$\sqrt{2} + \frac{1}{2}\sqrt{2} + \frac{1}{4}\sqrt{2} + \cdots$ +1pt

(b, 5 pts) Please give the accumulated area of all squares with dashed borders $\lim_{i\to\infty} A_i$, counting overlap areas only once. Note that $A_1 = 1$ and $A_2 = 1 + \frac{3}{8}$.

$$A_3 = 1 + \frac{3}{8} + \frac{3}{16} \overset{+\frac{3}{32}}{} = 1 + \frac{9}{16}$$ 3pts   +1pt

$$A_i = 1 + 3/4 \cdot 2 \cdot 1/4 + 3/4 \cdot 4 \cdot 1/16 + \cdots + 3/4 \cdot 2^{i-1} 1/4^{i-1} = \underbrace{1/4}_{1} + 3/4 \sum_{i=0}^{\infty} 1/2^i = 1/4 + 3/2 = 7/4.$$

−1pt