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**Department of Mathematics**

**NC STATE** UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Thur, Oct 31, 2013

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www.math.ncsu.edu/~kaltofen/courses/DiscreteModels/Fall13/index.html (URL)

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*Your Name:* _____

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 7 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of **48 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **two** 8.5in $\times$ 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

5 _____

6 _____

7 _____

Total _____

If you are taking the exam later, please sign the following statement:

*I,* _____*, affirm that I have no knowledge of the contents of this exam.*

_____

Signature

**Problem 1** (13 points): Consider the following mathematical expression in **in**fix notation, assuming that each of the binary operators $+, -, *, /, \uparrow$ has two operands, where $\uparrow$ is exponentiation with highest precedence, which is evaluated right-to-left: $a \uparrow b \uparrow c = a \uparrow (b \uparrow c)$:

$$a - b + c \uparrow (d * e / f \uparrow g \uparrow h). \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).

(b, 4pts) Please give both the **pre**fix and the **post**fix representations for the expression (1), both of which only have variables and operators.

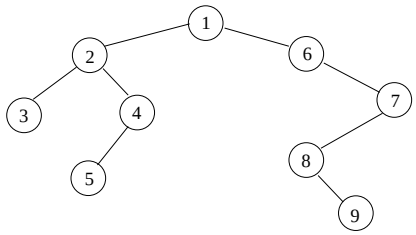
PREFIX:

POSTFIX:

(c, 5pts) Please draw the parse tree for (1) above using the following context-free grammar $G = (N, T, P, s)$, which is the one in class extended by exponentiation: $N = \{\langle E\rangle, \langle T\rangle, \langle F\rangle, \langle B\rangle\}$; note that $\langle E\rangle$ is an expression, $\langle T\rangle$ is a term, $\langle F\rangle$ is a factor and $\langle B\rangle$ is the base for a power. $T = \{a, b, \ldots, z, (, ), +, -, *, /, \uparrow\}$. The start symbol $s = \langle E\rangle$.

$$P = \{\langle E\rangle \to \langle E\rangle + \langle T\rangle, \quad \langle T\rangle \to \langle T\rangle * \langle F\rangle, \quad \langle F\rangle \to \langle B\rangle \uparrow \langle F\rangle, \quad \langle B\rangle \to (\langle E\rangle),$$
$$|\langle E\rangle - \langle T\rangle, \quad |\langle T\rangle / \langle F\rangle, \quad |\langle B\rangle, \quad |a|b|\ldots|z\}.$$
$$|\langle T\rangle, \quad |\langle F\rangle,$$

**Problem 2** (4 points):
Please consider the binary tree
(with left and right children identified):
Please give the parentheses-only string from class for the tree, labelling each pair of parentheses with the corresponding vertex




**Problem 3** (6 points): Consider the following graph:

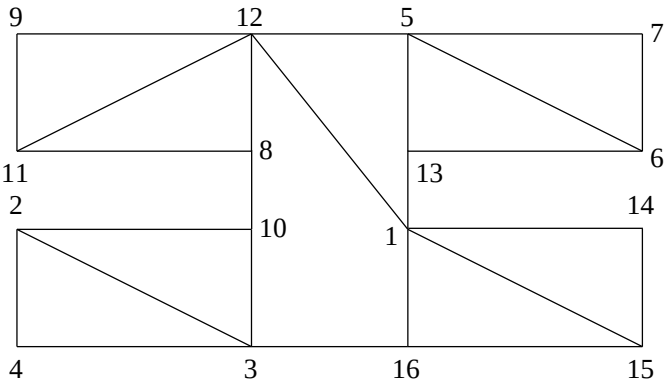



Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

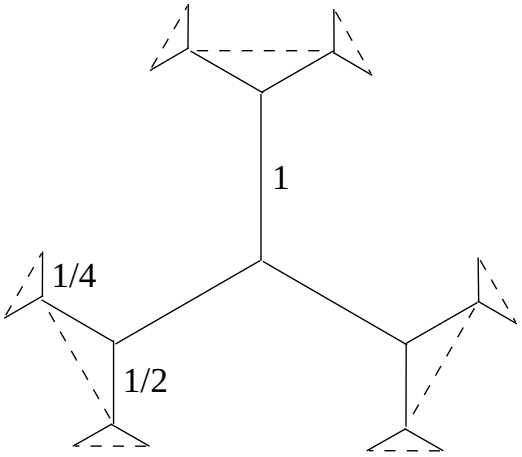
(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

**Problem 4** (6 points): Consider the following variant of Fibonacci's rabbits problem: *Each pair takes 1 or 2 months to mature, and then after every additional month gives birth to 2 pairs of rabbits. Of those, one pair takes 1 month to mature while the other pair takes 2 months to mature.* Please (a) model the variant by a Lindenmayer system, annotating each variable by what type of pair it represents, and (b) give the first 5 new generations of the system, starting at generation 0 with a single pair of newly born rabbits that takes 1 month to mature.

**Problem 5** (5 points): Please define the Julia set $J_c$ for $c = -1$, that is, $J_{-1}$. Please show that $-1 \in J_{-1}$ and $2 \notin J_{-1}$.

**Problem 6** (4 points): The "butterfly effect" is used as a ficticious state in a system that is chaotic. Please describe how the butterfly effect metaphor explains chaos.

**Problem 7** (10 points): Please consider the following (Steiner) tree fractal.




Here one starts at iteration 1 with three line segements of length 1 arranged at a root point with angle $2\pi/3$ (120 degrees). At each tip of the 3 segments, away from the root, one adds at iteration 2 two line segments of length $1/2$, again at an angle of $2\pi/3$ to the longer already drawn first segment. At iteration $i = 3$, one adds at each of the 6 tips a total of 12 segments of lenght $1/4$, which is the iteration shown above.

(a, 5 pts) Please give the total length $L_i$ of all line segments, drawn above as solid lines, in the tree after $i$ iterations, where $L_1 = 3$.

(b, 5 pts) Please give the total area of all obtuse isosceles triangles with dashed base lines and obtuse angle $2\pi/3$ that are added at iteration $i$: note $A_1 = 0$ and $A_2 = 3 \times \sqrt{3}/16$. Finally, please compute $\sum_{i=1}^{\infty} A_i$.