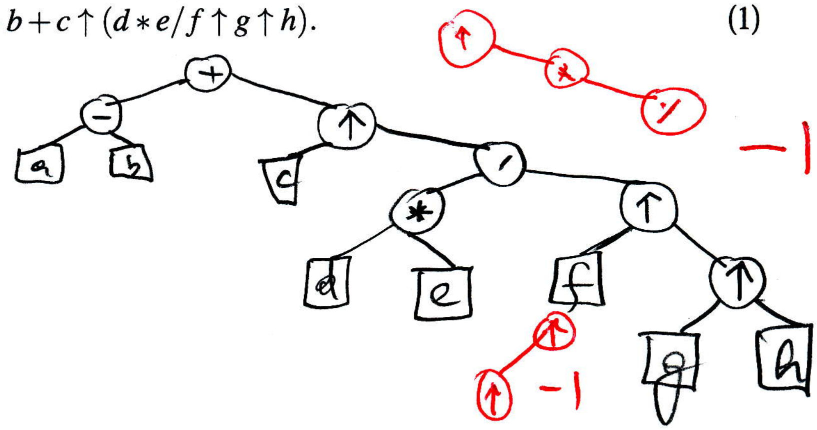
2013

Problem 1 (~~12~~ 3 points): Consider the following mathematical expression in **in**fix notation, assuming that each of the binary operators $+, -, *, /, \uparrow$ has two operands, where $\uparrow$ is exponentiation with highest precedence, which is evaluated right-to-left: $a \uparrow b \uparrow c = a \uparrow (b \uparrow c)$:

$$a - b + c \uparrow (d * e / f \uparrow g \uparrow h). \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).



−1

−1

(b, 4pts) Please give both the **pre**fix and the **post**fix representations for the expression (1), both of which only have variables and operators.

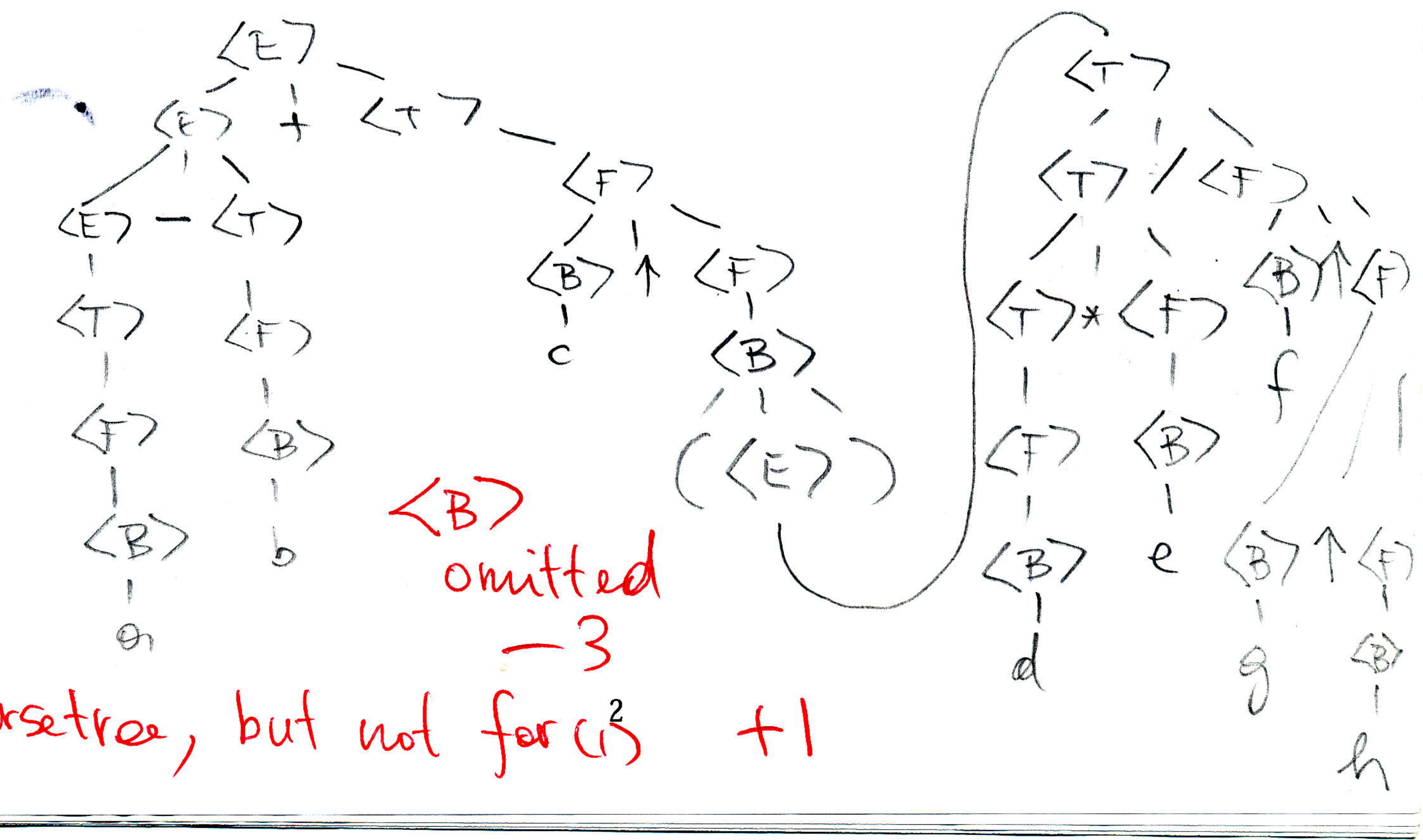
PREFIX: $+ - a\ b \uparrow c / * d\ e \uparrow f \uparrow g\ h$

POSTFIX: $a\ b - c\ d\ e * f\ g\ h \uparrow \uparrow / \uparrow +$

no penalty if OK for (a)

(c, ~~4~~ 5 pts) Please draw the parse tree for (1) above using the following context-free grammar $G = (N, T, P, s)$, which is the one in class extended by exponentiation: $N = \{\langle E\rangle, \langle T\rangle, \langle F\rangle, \langle B\rangle\}$; note that $\langle E\rangle$ is an expression, $\langle T\rangle$ is a term, $\langle F\rangle$ is a factor and $\langle B\rangle$ is the base for a power. $T = \{a, b, \ldots, z, (,), +, -, *, /, \uparrow\}$. The start symbol $s = \langle E\rangle$.

$$\begin{aligned} P = \{ \langle E\rangle &\to \langle E\rangle + \langle T\rangle, & \langle T\rangle &\to \langle T\rangle * \langle F\rangle, & \langle F\rangle &\to \langle B\rangle \uparrow \langle F\rangle, & \langle B\rangle &\to (\langle E\rangle), \\ &| \langle E\rangle - \langle T\rangle, & &| \langle T\rangle / \langle F\rangle, & &| \langle B\rangle, & &| a | b | \ldots | z \}. \\ &| \langle T\rangle, & &| \langle F\rangle, & & & & \end{aligned}$$



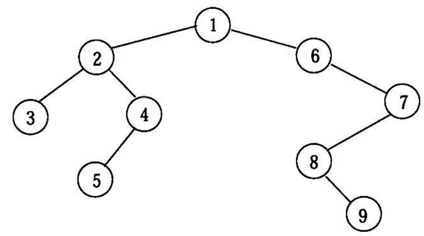
<B> omitted −3

parsetree, but not for (1) +1

2013

only

Problem 2 (4 points):
Please consider the binary tree
(with left and right children identified):
Please give the parenthesis string from
class for the tree, labelling each pair of
parentheses with the corresponding vertex



((( )) (( ))) ( ) (( ) ( ))
1 2 3 3 2 4 5 5 4 1 6 6 7 8 8 9 9 7

Problem 3 (6 points): Consider the following graph:

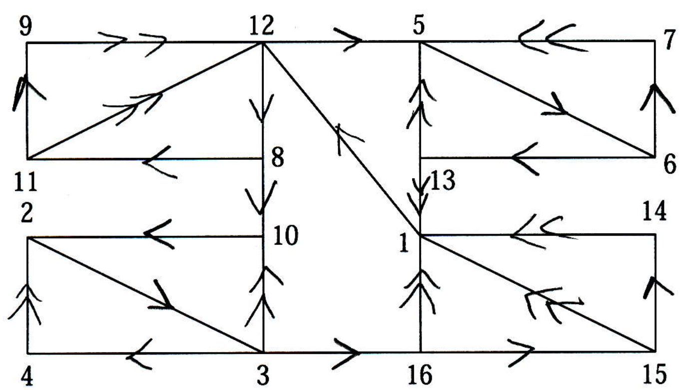
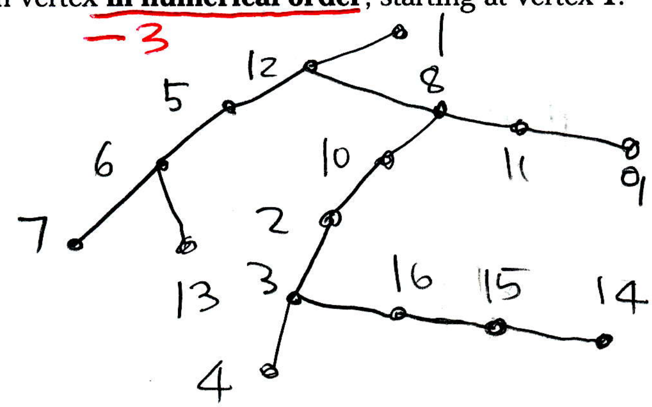


Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

−3



one branch wrong −2

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1 using a different arrow head for those arcs that correspond to edges in the DFS tree.

−1

2013

Problem 4 (6 points): Consider the following variant of Fibonacci's rabbits problem: *Each pair takes 1 or 2 months to mature, and then after every additional month gives birth to 2 pairs of rabbits. Of those, one pair takes 1 month to mature while the other pair takes 2 months to mature.* Please (a) model the variant by a Lindenmayer system, annotating each variable by what type of pair it represents, and (b) give the first ~~6~~ 5 new generations of the system, starting at generation 0 with a single pair of newly born rabbits that takes 1 month to mature.

3 Vars A (new, 1m) B −2 C (new, 2m) E F

r.s. B $\boxed{B}$AC E F FAC

3 $A \to \underset{1}{B} \to \underset{2}{BAC} \to \underset{3}{BACBE} \to \underset{4}{\underbrace{BAC}_{B}}\ B\ E\ BACF$

$\to \underbrace{BAC}_{B}\ B\ E\ \underbrace{BAC}_{B}\ F\ BACBE\ FAC$

Problem 5 (5 points): Please define the Julia set J_c for $c = -1$, that is, J_{-1}. Please show that $-1 \in J_{-1}$ and $2 \notin J_{-1}$.

−1

3 $J_{-1} = \{ b \in \mathbb{C} \mid \exists B \in \mathbb{R}_{>0} : \forall i \geq 2.\ z_i = z_{i-1}^2 \boxed{-1}$ and $z_1 = b \Rightarrow |z_i| \leq B \}$

1 $b = -1$: $z_2 = b^2 - 1 = 0$, $z_3 = z_2^2 - 1 = -1$, $z_4 = 0$, $z_5 = -1, \dots$
Choose $B = 1$.

1 $b = 2$: $z_2 = b^2 - 1 = 3$, $z_3 = 3^2 - 1 = 8$, $z_4 = 8^2 - 1 = 63, \dots$

$z_i \to \infty$

Problem 6 (4 points): The "butterfly effect" is used as a ficticious state in a system that is chaotic. Please describe how the butterfly effect metaphor explains chaos.

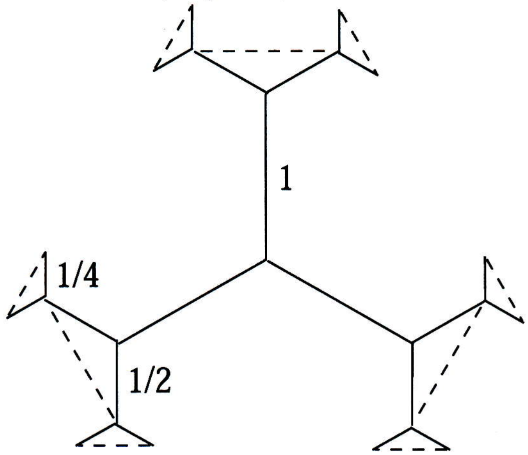
2 Butterfly flaps wing $\Rightarrow$ hurricane:
unstable state

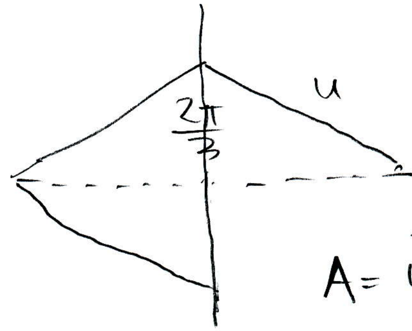
2 Butterfley in Amazon $\Rightarrow$ hurricane
in Atlantic
unpredictable instability

2013

Problem 7 (10 points): Please consider the following (Steiner) tree fractal.

explain





$A = u^2 \frac{\sqrt{3}}{4}$

Here one starts at iteration 1 with three line segements of length 1 arranged at a root point with angle $2\pi/3$ (120 degrees). At each tip of the 3 segments, away from the root, one adds at iteration 2 two line segments of length $1/2$, again at an angle of $2\pi/3$ to the longer already drawn first segment. At iteration $i = 3$, one adds at each of the 6 tips a total of 12 segments of lenght $1/4$, which is the iteration shown above.

(a, 5 pts) Please give the total length L_i of all line segments, drawn above as solid lines, in the tree after i iterations, where $L_1 = 3$.

+2

$$3 + 3\cdot 2\tfrac{1}{2} + 3\cdot 2\cdot 2\tfrac{1}{4} + \dots\ 3\cdot 2^{i-1}\frac{1}{2^{i-1}} = 3\cdot i$$

$L_i \Rightarrow \infty$ +3 Σ no penalty

(b, 5 pts) Please give the total area of all obtuse isosceles triangles with dashed base lines and obtuse angle $2\pi/3$ that are added at iteration i: note $A_1 = 0$ and $A_2 = 3\times\sqrt{3}/16$. Finally, please compute $\sum_{i=1}^{\infty} A_i$.

$\sum_{i=1}^{\infty}\frac{1}{2^i} = 1$

$$A_3 = 2\cdot 3\cdot\frac{1}{4}\cdot\frac{\sqrt{3}}{16}, \quad A_4 = 2\cdot 2\cdot 3\cdot\frac{1}{4^2}\frac{\sqrt{3}}{16}$$

$$A_i = 2^{i-2}\cdot 3\cdot\frac{1}{4^{i-2}}\frac{\sqrt{3}}{16} = \frac{3\cdot\sqrt{3}}{2^{i+2}}$$

$\sqrt{3/16}$

-2

$$\frac{2^{i-2}}{2^{2i-4}\cdot 2^4} \qquad \sum_{i=2}^{\infty}\frac{3\sqrt{3}}{2^{i+2}} = \frac{3\sqrt{3}}{16}\sum_{j=0}^{\infty}\frac{1}{2^j}$$

$\sum_{i=2}^{\infty}\frac{1}{2^i} = 2$

-1

$$= \frac{3\sqrt{3}}{8}$$