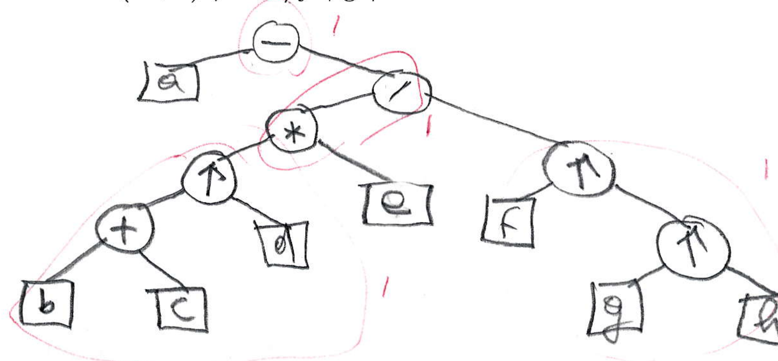
Sol 16

Problem 1 (13 points): Consider the following mathematical expression in **in**fix notation, assuming that each of the binary operators $+, -, *, /, \uparrow$ has two operands, where $\uparrow$ is exponentiation with highest precedence, which is evaluated right-to-left: $a \uparrow b \uparrow c = a \uparrow (b \uparrow c)$:

$$a - (b+c) \uparrow d * e / f \uparrow g \uparrow h. \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).



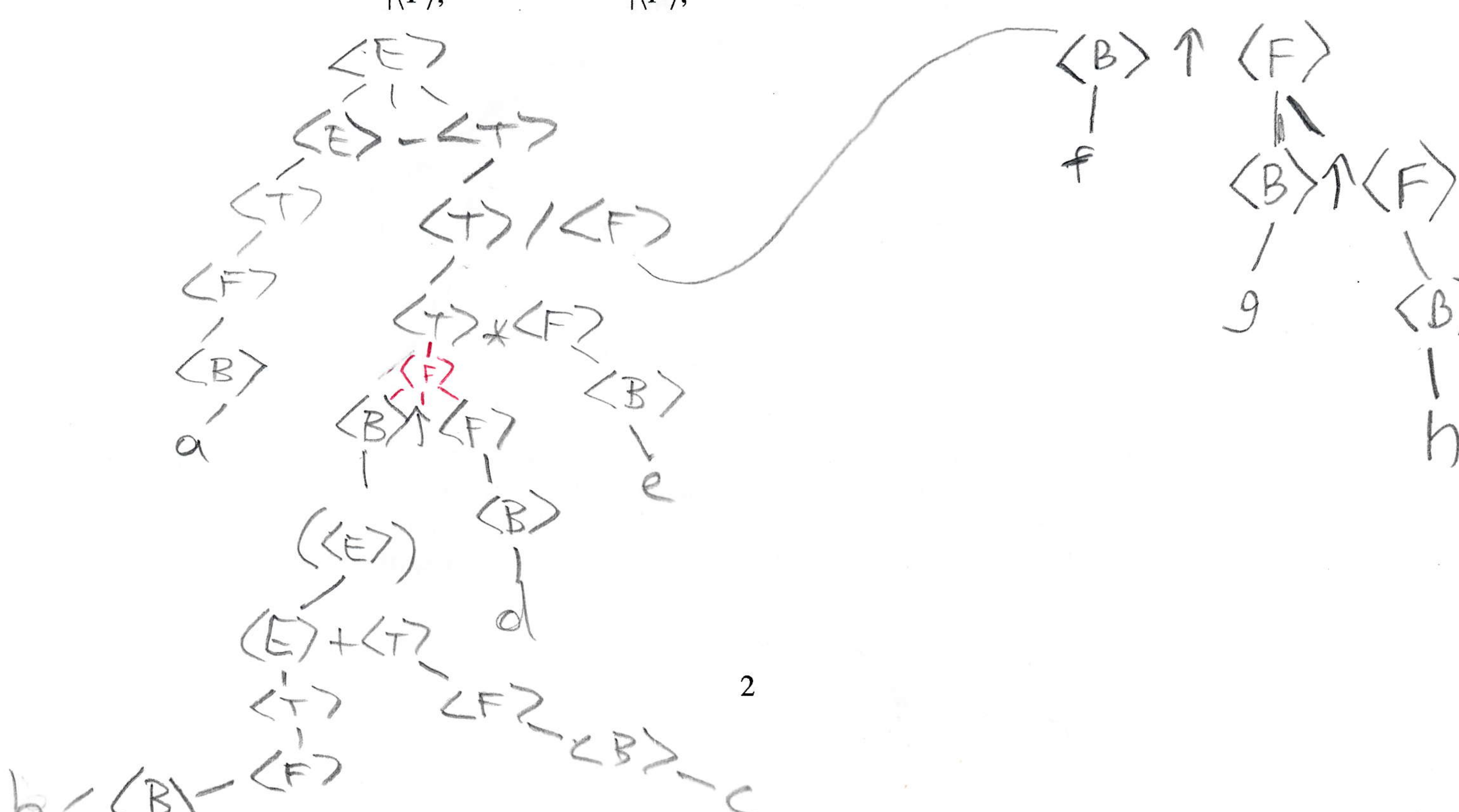
(b, 4pts) Please give both the **pre**fix and the **post**fix representations for the expression (1), both of which only have variables and operators.

PREFIX: $- \; a \; / \; * \uparrow + \; b \; c \; d \; e \uparrow f \uparrow g \; h$

POSTFIX: $a \; b \; c \; + \; d \uparrow e \; * \; f \; g \; h \uparrow \uparrow / \; -$

(c, 5pts) Please draw the parse tree for (1) above using the following context-free grammar $G = (N, T, P, s)$, which is the one in class extended by exponentiation: $N = \{\langle E\rangle, \langle T\rangle, \langle F\rangle, \langle B\rangle\}$; note that $\langle E\rangle$ is an expression, $\langle T\rangle$ is a term, $\langle F\rangle$ is a factor and $\langle B\rangle$ is the base for a power. $T = \{a, b, \ldots, z, (,), +, -, *, /, \uparrow\}$. The start symbol $s = \langle E\rangle$.

$$\begin{aligned} P = \{ \langle E\rangle &\to \langle E\rangle + \langle T\rangle, & \langle T\rangle &\to \langle T\rangle * \langle F\rangle, & \langle F\rangle &\to \langle B\rangle \uparrow \langle F\rangle, & \langle B\rangle &\to (\langle E\rangle), \\ &| \langle E\rangle - \langle T\rangle, & &| \langle T\rangle / \langle F\rangle, & &| \langle B\rangle, & &| a | b | \ldots | z \}. \\ &| \langle T\rangle, & &| \langle F\rangle, \end{aligned}$$



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Problem 2 (5 points): Suppose you divide $n = 9, 10, 11, \ldots$ representatives over 3 groups of $300, 300, 100$ people by Hamilton's Method. At which n is an "Alabama Paradox" observed. Please show your work.

$\frac{3}{7} \cdot 9 = 3 + \frac{6}{9} : 4 \qquad \frac{3}{7} \cdot 10 = 4 + \frac{2}{7} : 4 \qquad \frac{3}{7} \cdot 11 = 4 + \frac{5}{7} : 5$

$3 + \frac{6}{9} : 4 \qquad 4 + \frac{2}{7} : 4 \qquad 4 + \frac{5}{7} : 5$

$\frac{1}{7} \cdot 9 = 1 + \frac{2}{9} : 1 \qquad \frac{1}{7} \cdot 10 = 1 + \frac{3}{7} : 2 \qquad \frac{1}{7} \cdot 11 = 1 + \frac{4}{7} : 1$

At $n=11$, the 100 group loses a representative

Problem 3 (6 points): Consider the following graph:

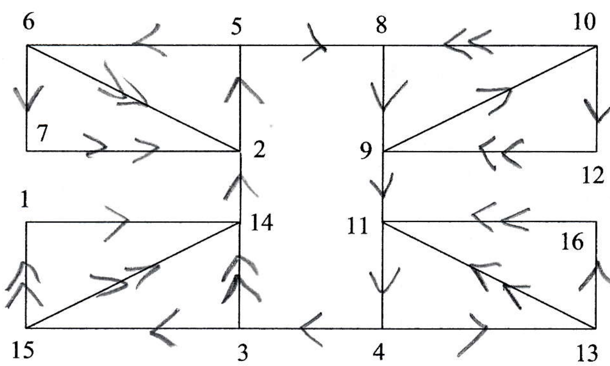
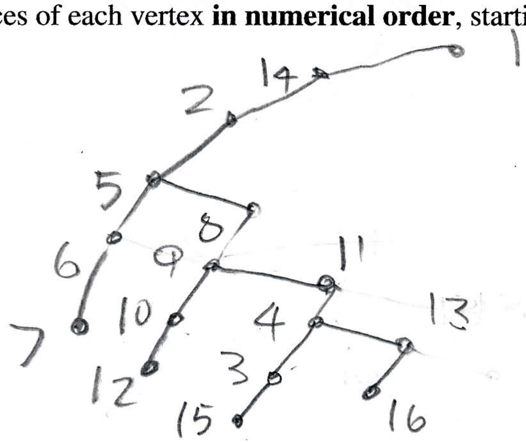


Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.



(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

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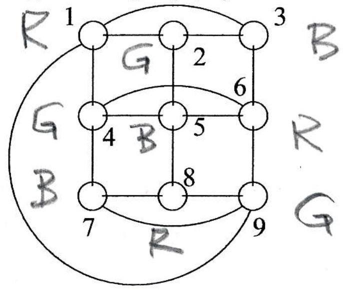
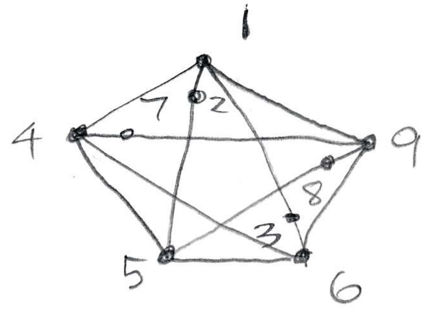
Problem 4 (10 points): Please consider the following 3×3 mesh-like graph with wrap-around edges $\{1,3\}$, $\{4,6\}$, $\{7,9\}$ and the diagonal edge $\{1,9\}$.



(a, 8pts) Please draw a subgraph that is homeomorphic to K_5, which denotes the complete graph with 5 vertices. [Hint: the vertices of the K_5 must have degree 4.]



(b, 2pts) What is the chromatic number of the above graph? Please explain.

3 Because  is a 3-clique one needs 3 colors, but 3 colors suffice (see above)

Problem 5 (5 points): Consider the following Lindenmayer system: $T \rightarrow L(M)R$, $L \rightarrow M$, $M \rightarrow T$, $R \rightarrow L$, $(\rightarrow ($, $) \rightarrow)$. Here the parentheses (and) are constant symbols. Please write down the first 5 new generations of strings starting with T.

$T \rightarrow L(M)R$

$\rightarrow M(T)L \rightarrow T(L(M)R)M$

$\rightarrow L(M)R(M(T)L)T$

$\rightarrow M(T)L(T(L(M)R)M)\ L(M)R$

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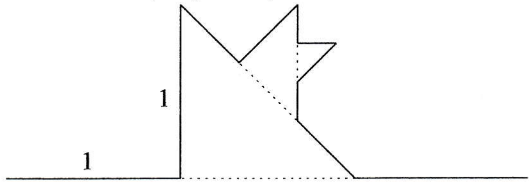
Problem 6 (10 points): Please consider the following "saw tooth" fractal.



Here one starts at iteration 1 with three line segments of length 1 arranged on a base line, and extrudes a right triangle upwards in the middle segment with its hypotenuse being the right extruded side. In the subsequent iterations, one repeats the process on the hypotenuse of the last extruded right triangle.

(a, 5 pts) Please give the length L_i, namely the two extruded sides minus the dashed side, that gets added to the saw tooth at iteration i, where $L_1 = \sqrt{2}$ and $L_2 = 2/3$. Then please compute $\sum_{i=1}^{\infty} L_i$.

$$L_i = \sqrt{2}\left(\frac{\sqrt{2}}{3}\right)^{i-1}$$

$$= \sqrt{2}\sum_{i=0}^{\infty}\left(\frac{\sqrt{2}}{3}\right)^{i} = \sqrt{2}\left(\frac{1}{1-\frac{\sqrt{2}}{3}}\right)$$

$$= \sqrt{2}\left(\frac{1}{\left(\frac{3-\sqrt{2}}{3}\right)}\right) = \frac{3\sqrt{2}}{3-\sqrt{2}}$$

(b, 5 pts) Please give the area of the right triangle that is added at iteration i: note $A_1 = 1/2$ and $A_2 = 1/9$. Finally, please compute $\sum_{i=1}^{\infty} A_i$.

$$A_i = \frac{1}{2}\left(\frac{2}{9}\right)^{i-1}$$

$$\sum_{i=1}^{\infty} A_i = \frac{1}{2}\sum_{i=0}^{\infty}\left(\frac{2}{9}\right)^{i} = \frac{1}{2}\left(\frac{1}{1-\frac{2}{9}}\right) = \frac{1}{2}\left(\frac{1}{\left(\frac{7}{9}\right)}\right)$$

$$= \frac{1}{2}\left(\frac{9}{7}\right) = \frac{9}{14}$$