

**NC STATE UNIVERSITY**

MA 351 Intro Discrete Math Models, second mid-semester examination, Thur, Nov 2, 2017  
Prof. Erich Kaltofen <kaltofen@math.ncsu.edu>  
www.math.ncsu.edu/~kaltofen/courses/DiscreteModels/Fall17/index.html (URL)

919.515.8785 (phone)  
919.515.3798 (fax)

Your Name: \_\_\_\_\_

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 6 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of **49 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work**, if necessary. You are allowed to consult **two** 8.5in  $\times$  11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 \_\_\_\_\_

2 \_\_\_\_\_

3 \_\_\_\_\_

4 \_\_\_\_\_

5 \_\_\_\_\_

6 \_\_\_\_\_

Total \_\_\_\_\_

If you are taking the exam later, please sign the following statement:

I, \_\_\_\_\_, affirm that I have no knowledge of the contents of this exam.

\_\_\_\_\_  
Signature

**Problem 1** (13 points):

Consider the following mathematical expression in **postfix** notation. assuming that each of the operators  $+, -, *, /, \uparrow$  has two operands ( $\uparrow$  is exponentiation).

$$abcde * f / \uparrow + g - \uparrow \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).

(b, 4pts) Please give both the **minimally parenthesized infix** and the **prefix** representations for the expression (1), the latter of which only has variables and operators.

INFIX (with minum number of parentheses):

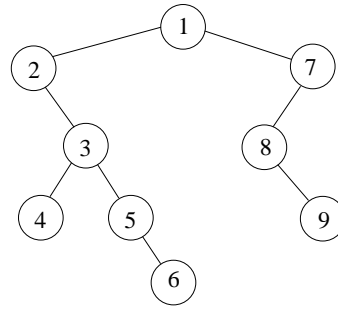
PREFIX:

(c, 5pts) Please draw the parse tree for the string  $a \uparrow b + c \uparrow (d * e / f - g)$  using the following context-free grammar  $G = (N, T, P, s)$  (from class with exponentiation)  $N = \{\langle E \rangle, \langle T \rangle, \langle F \rangle, \langle B \rangle\}$ ; note that  $\langle E \rangle$  is an expression,  $\langle T \rangle$  is a term,  $\langle F \rangle$  is a factor and  $\langle B \rangle$  is the base for a power.  $T = \{a, b, \dots, z, (, ), +, -, *, /, \uparrow\}$ . The start symbol  $s = \langle E \rangle$ .

$$P = \{ \langle E \rangle \rightarrow \langle E \rangle + \langle T \rangle, \quad \langle T \rangle \rightarrow \langle T \rangle * \langle F \rangle, \quad \langle F \rangle \rightarrow \langle B \rangle \uparrow \langle F \rangle, \quad \langle B \rangle \rightarrow (\langle E \rangle), \\ \langle E \rangle - \langle T \rangle, \quad \langle T \rangle / \langle F \rangle, \quad \langle B \rangle, \quad |a|b|\dots|z\}, \\ \langle T \rangle, \quad \langle F \rangle,$$

**Problem 3 (7 points):**

Please consider the binary tree  
(with left and right children identified):  
Please give the parentheses-only string from  
class for the tree, labelling each pair  
of parentheses with the corresponding vertex



**Problem 3 (6 points):** Consider the following graph:

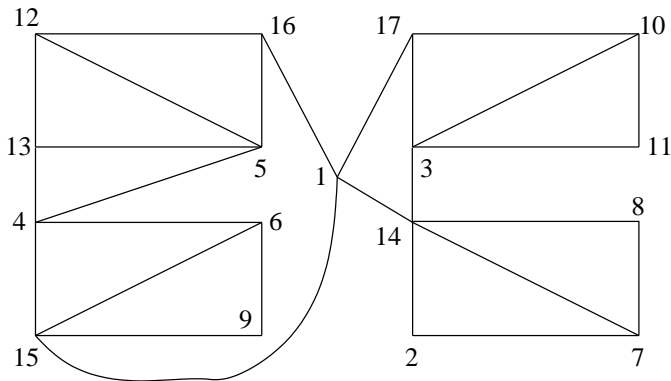


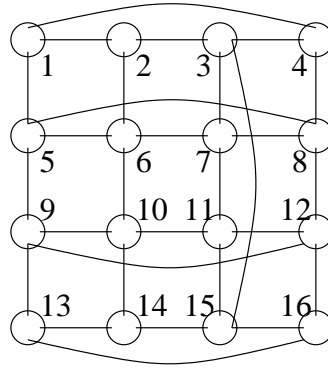
Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

**Problem 4** (8 points):

Consider the subgraph of the  $4 \times 4$  toric mesh (with the given vertex labeling); note that the edges  $\{1, 13\}$ ,  $\{2, 14\}$  and  $\{4, 16\}$  are missing.

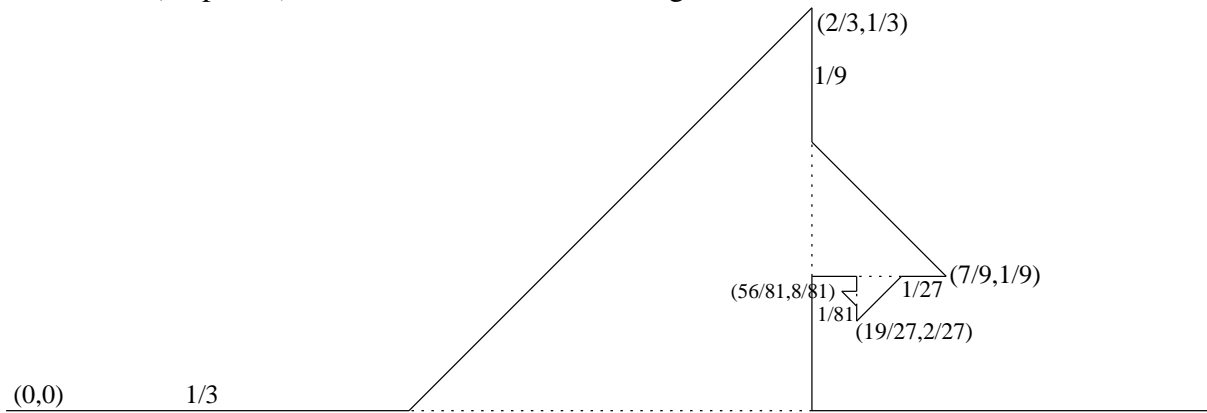


- (a, 6pts) Please draw a subgraph that is homeomorphic to  $K_{3,3}$ . [Hint: choose for the top vertex set  $\{5, 7, 10\}$  and for the bottom vertex set  $\{11\}$  and two other vertices.]

- (b, 2pts) Please 3-color the graph in Figure 1 on page 3.

**Problem 5** (4 points): Consider the following Lindenmayer system:  $T \rightarrow L(M)R, L \rightarrow M, M \rightarrow T, R \rightarrow L, (\rightarrow, (, ) \rightarrow)$ . Here the parentheses ( and ) are constant symbols. Please write down the first 5 new generations of strings starting with  $T$ .

**Problem 6** (10 points): Please consider the following “hook” fractal.



Here one starts at iteration 1 with three line segments of length  $1/3$  arranged on a base line, and extrudes a right triangle upwards in the middle segment with its hypotenuse being the left extruded side. In the subsequent iterations, one repeats the process on the shorter “leg-”sides of lengths  $1/3, 1/9, 1/27, \dots$ , placing the new hypotenuses nearer to the previous tip.

- (a, 5 pts) Please give the area of the extruded right triangle at iteration  $i$ , with  $A_1 = 1/18$ . Finally, please compute  $\sum_{i=1}^{\infty} A_i$ .

- (b, 5 pts) Please compute the x-y-coordinates of the tip of the extruded triangle at  $\infty$ . Hint: note that after 4 iterations the leg side of length  $1/81$  is again aligned like the initial base line, but its x-y-coordinates have moved from  $(0,0)$  to  $(56/81, 8/81)$ . At  $\infty$  the left point of the base line is the tip.