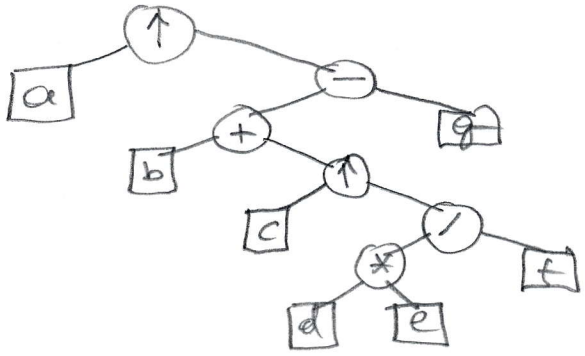
2017

Problem 1 (13 points): Consider the following mathematical expression in **post**fix notation. assuming that each of the operators $+, -, *, /, \uparrow$ has two operands ($\uparrow$ is exponentiation).

$$abcde*f/\uparrow+g-\uparrow \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).



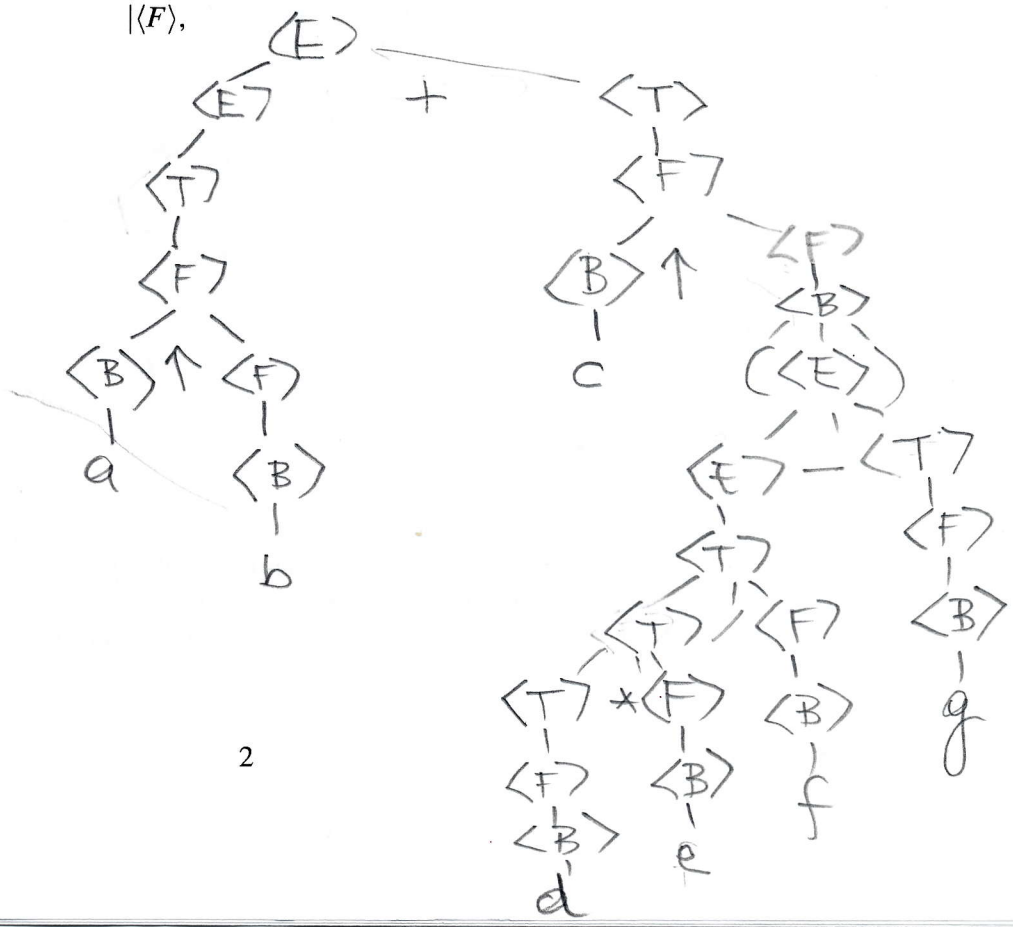
(b, 4pts) Please give both the **minimally parenthesized in**fix and the **pre**fix representations for the expression (1), the latter of which only has variables and operators.

INFIX (with minimum number of parentheses): $a\uparrow(b+c\uparrow(d*e/f)-g)$

PREFIX: $\uparrow a - + b \uparrow c / * d e f g$

(c, 5pts) Please draw the parse tree for the string $a\uparrow b+c\uparrow(d*e/f-g)$ using the following context-free grammar $G=(N,T,P,s)$ (from class with exponentiation) $N=\{\langle E\rangle,\langle T\rangle,\langle F\rangle,\langle B\rangle\}$; note that $\langle E\rangle$ is an expression, $\langle T\rangle$ is a term, $\langle F\rangle$ is a factor and $\langle B\rangle$ is the base for a power. $T=\{a,b,\ldots,z,(,),+,-,*,/,\uparrow\}$. The start symbol $s=\langle E\rangle$.

$P=\{\langle E\rangle\to\langle E\rangle+\langle T\rangle,\quad \langle T\rangle\to\langle T\rangle*\langle F\rangle,\quad \langle F\rangle\to\langle B\rangle\uparrow\langle F\rangle,\quad \langle B\rangle\to(\langle E\rangle),$
$|\langle E\rangle-\langle T\rangle,\quad |\langle T\rangle/\langle F\rangle,\quad |\langle B\rangle,\quad |a|b|\ldots|z\}.$
$|\langle T\rangle,\quad |\langle F\rangle,$



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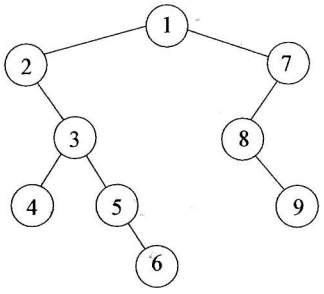
Problem 2 (5 points):
Please consider the binary tree
(with left and right children identified):
please give the parentheses-only string from
class for the tree, labelling each pair
of parentheses with the corresponding vertex



(( ) (( )) ( ) ( )) (( ) ( ))
1 2 2 3 4 4 3 5 5 6 6 1 7 8 8 9 9 7

Problem 3 (6 points): Consider the following graph:

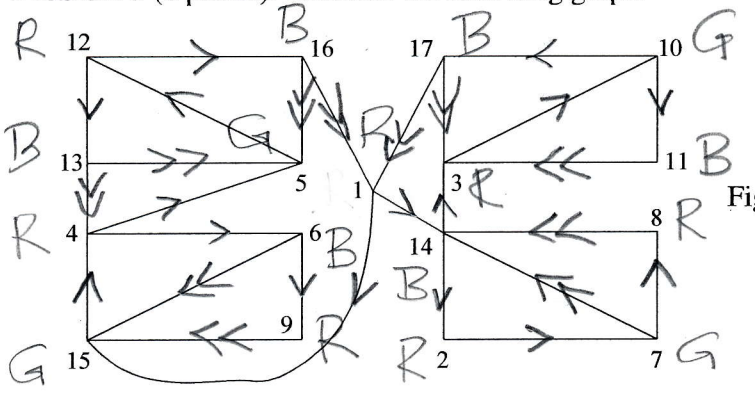
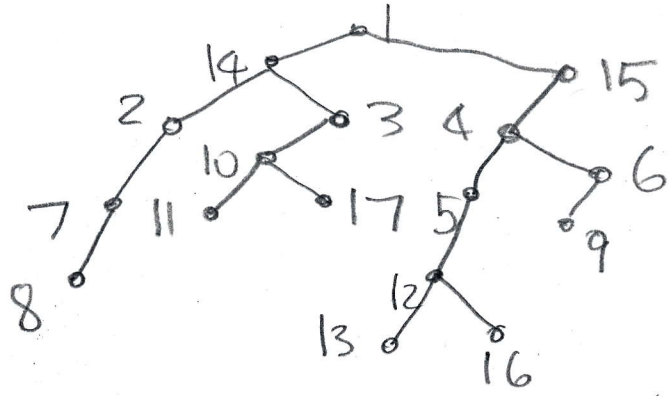


Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.



(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

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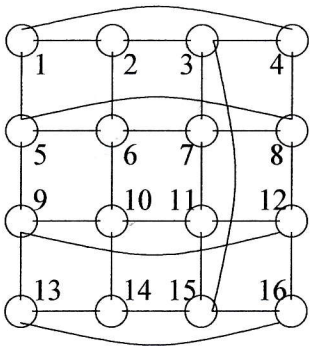
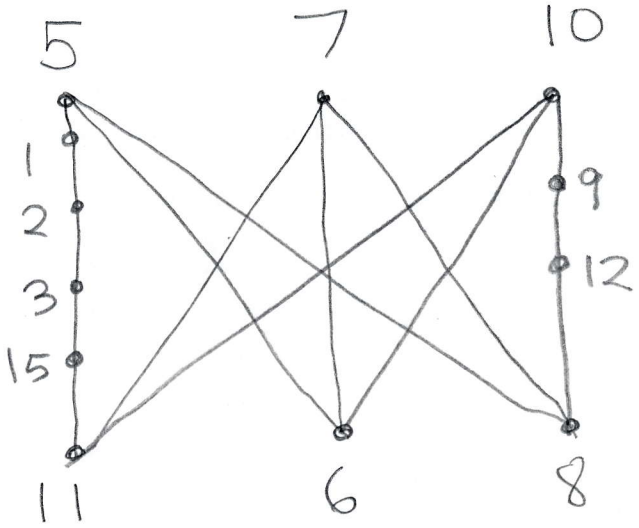
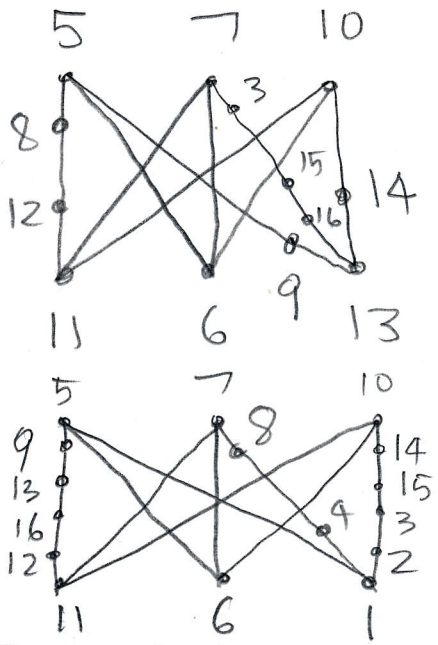
Problem 4 (8 points):
Consider the subgraph of the 4×4 toric mesh (with the given vertex labeling); note that the edges $\{1,13\}$, $\{2,14\}$ and $\{4,16\}$ are missing.



(a, 6pts) Please draw a subgraph that is homeomorphic to $K_{3,3}$. [Hint: choose for the "top" vertex set $\{5,7,10\}$ and for the "bottom" vertex set $\{11\}$ and two other vertices.]





(b, 2pts) Please 3-color the graph in Figure 1 on page 3 by placing R, G or B next to each vertex.

Problem 5 (4 points): Consider the following Lindenmayer system: $B \to bL$, $L \to laH$, $H \to hXB$, $X \to xLx$, $b \to b$, $l \to l$, $a \to a$, $h \to h$, $x \to x$. Please write down the first 5 new generations of strings starting with B.

B → bL

→ blaH

→ blahXB

→ blahxLxbL

→ blahxlaHxblaH

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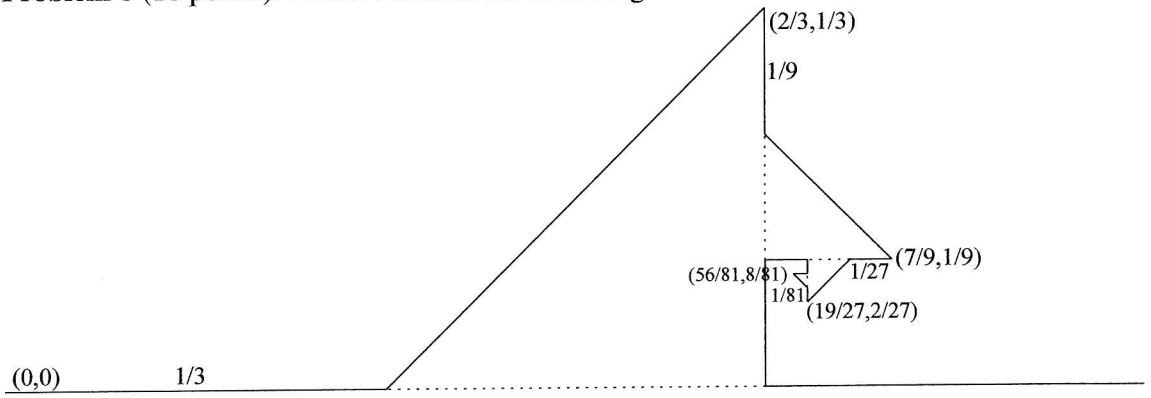
Problem 6 (10 points): Please consider the following "hook" fractal.



Here one starts at iteration 1 with three line segments of length $1/3$ arranged on a base line, and extrudes a right triangle upwards in the middle segment with its hypotenuse being the left extruded side. In the subsequent iterations, one repeats the process on the shorter "leg-"sides of lengths $1/3, 1/9, 1/27, \ldots$, placing the new hypotenuses nearer to the previous tip.

(a, 5 pts) Please give the area of the extruded right triangle at iteration i, with $A_1 = 1/18$. Finally, please compute $\sum_{i=1}^{\infty} A_i$.

$$A_2 = \left(\frac{1}{3}\right)^2 A_1, \quad A_i = \left(\frac{1}{9}\right)^{i-1} A_1 = \frac{1}{2}\left(\frac{1}{9}\right)^i$$

$$\sum_{i=1}^{\infty} A_i = \frac{1}{18}\sum_{i=0}^{\infty}\left(\frac{1}{9}\right)^i = \frac{1}{18}\,\frac{1}{1-\frac{1}{9}}$$

$$= \frac{1}{18}\,\frac{9}{8} = \frac{1}{16}$$

(b, 5 pts) Please compute the x-y-coordinates of the tip of the extruded triangle at ∞. Hint: note that after 4 iterations the leg side of length $1/81$ is again aligned like the intial base line, but its x-y-coordinates have moved from $(0,0)$ to $(56/81, 8/81)$. At ∞ the left point of the base line is the tip.

After 8 iterations: $x = \frac{56}{81} + \frac{56}{81^2}$

$$y = \frac{8}{81} + \frac{8}{81^2}$$

$$x_\infty = \frac{56}{81}\cdot\sum_{i=0}^{\infty}\left(\frac{1}{81}\right)^i = \frac{56}{81}\,\frac{1}{1-\frac{1}{81}} = \frac{56}{80}$$

$$y_\infty = \frac{8}{81}\sum_{i=0}^{\infty}\left(\frac{1}{81}\right)^i = \frac{8}{81}\,\frac{1}{1-\frac{1}{81}} = \frac{1}{10}$$