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Department of Mathematics

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Thur, Nov 8, 2018

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www.math.ncsu.edu/~kaltofen/courses/DiscreteModels/Fall17/index.html (URL)

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Your Name: _____

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 6 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of **46 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **two** 8.5in $\times$ 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1	_____
2	_____
3	_____
4	_____
5	_____
6	_____
Total	_____

Problem 1 (13 points): Consider the following mathematical expression in **post**fix notation. assuming that each of the operators $+, -, *, /, \uparrow$ has two operands ($\uparrow$ is exponentiation).

$$abcde/*f/-g+h\uparrow\uparrow \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).

(b, 4pts) Please give both the **minimally parenthesized in**fix and the **pre**fix representations for the expression (1), the latter of which only has variables and operators.

INFIX (with minimum number of parentheses):

PREFIX:

(c, 5pts) Please draw the parse tree for the string $(a+b-c*(d/e))\uparrow f\uparrow g$ using the following context-free grammar $G=(N,T,P,s)$ (from class with exponentiation) $N=\{\langle E\rangle,\langle T\rangle,\langle F\rangle,\langle B\rangle\}$; note that $\langle E\rangle$ is an expression, $\langle T\rangle$ is a term, $\langle F\rangle$ is a factor and $\langle B\rangle$ is the base for a power. $T=\{a,b,\ldots,z,(,),+,-,*,/,\uparrow\}$. The start symbol $s=\langle E\rangle$.

$$\begin{aligned}
P=\{&\langle E\rangle\to\langle E\rangle+\langle T\rangle, && \langle T\rangle\to\langle T\rangle*\langle F\rangle, && \langle F\rangle\to\langle B\rangle\uparrow\langle F\rangle, && \langle B\rangle\to(\langle E\rangle),\\
&\quad|\langle E\rangle-\langle T\rangle, && \quad|\langle T\rangle/\langle F\rangle, && \quad|\langle B\rangle, && \quad|a|b|\ldots|z\}.\\
&\quad|\langle T\rangle, && \quad|\langle F\rangle, && &&
\end{aligned}$$

Problem 2 (6 points): Consider binary trees in which each node has either 0 children, or one left or one right child, or both. Such a tree with 10 nodes has been linearized by our method from class to

$$\begin{array}{cccccccccccccccccccc} (& (& (&) & (&) &) & (&) & (& (& (& (&) &) &) &) &) & (&) \\ 1 & 2 & 3 & 3 & 4 & 4 & 2 & 5 & 5 & 6 & 7 & 8 & 9 & 9 & 8 & 7 & 6 & 1 & 10 & 10 \end{array}$$

with matchings indicated by numbers below the parentheses. Please draw the tree writing in each node the corresponding parentheses number. How many such binary trees with 10 nodes are there?

Problem 3 (6 points): Consider the following graph:

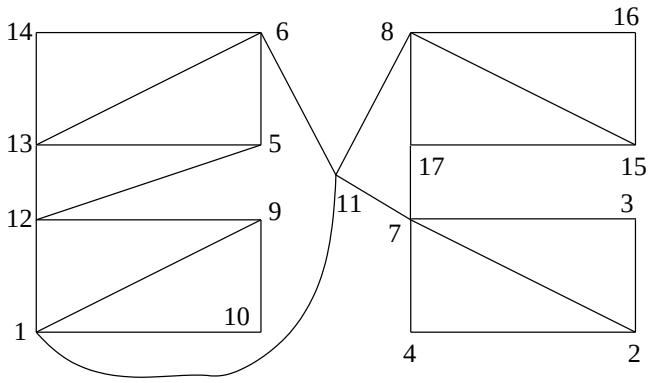


Figure 1.

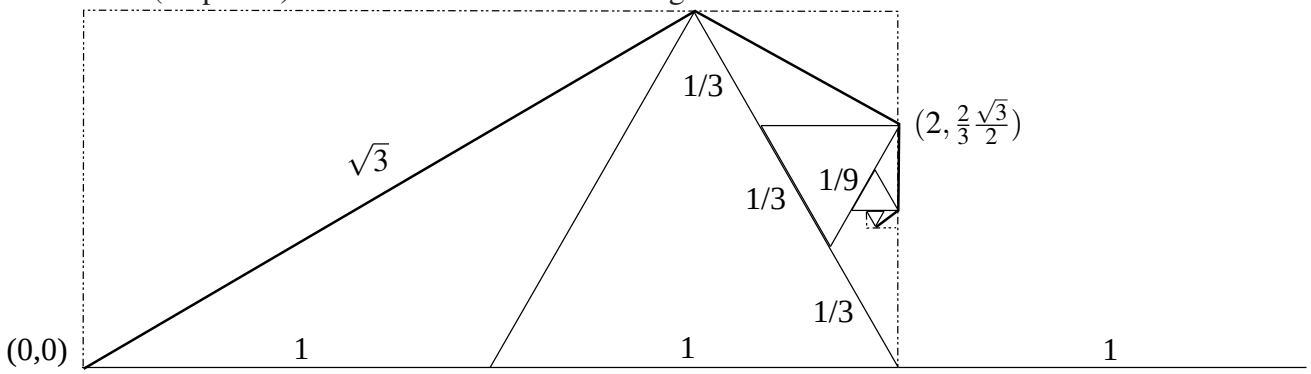
(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

Problem 4 (5 points): Consider the following variant of Fibonacci's rabbits problem: *A super-fertile pair after* 1 *month of maturing gives birth to* 3 *pairs of rabbits, while a fertile pair after* 1 *month of maturing gives birth to* 2 *pairs of rabbits. Of the* 3 *pairs of newly born rabbits of the super-fertile pair,* 1 *is super-fertile,* 1 *is fertile, and* 1 *is infertile. The infertile pair matures in one month, but then has no offsprings. Of the* 2 *pairs of newly born rabbits of the fertile pair,* 1 *is super-fertile and* 1 *is fertile.* Please (a) model the variant by a Lindenmayer system, annotating each variable by what type of pair it represents, and (b) give the first 5 new generations of the system, starting at generation 0 with a single pair of newly born super-fertile rabbits.

Problem 5 (5 points): Please describe a natural event whose occurrence exhibits a chaotic state. Please state why the event has chaotic properties.

Problem 6 (11 points): Please consider the following subset of Koch's snowflake fractal:



Here one starts at iteration 1 with an equilateral triangle with side length 1. At iteration 2 an equilateral triangle of side length $1/3$ is placed on the middle of the right side which goes up from the base side. At iteration 3 an equilateral triangle of side length $1/9$ is placed on the middle of the right side that goes out from the base line. And so on.

(a, 5 pts) At iteration i one draws a straight-line segment from the left vertex on the side of the previous triangle to the tip of the newly placed triangle. At iteration 1 the line segment has length $L_1 = 2\cos(\pi/6) = \sqrt{3}$. Please give the length L_i at iteration i and $\sum_{i=1}^{\infty} L_i$.

(b, 6 pts) Please compute the x-y-coordinates of the tip of the extruded triangle at ∞ where the origin and the coordinates of the second tip are shown in the figure. Hint: Note that the triangle placed at iteration 7 again has a horizontal base line with a tip straight above it. The small dashed-dotted rectangle which encloses the 4-th triangle is a shrunk upside-down version of the large dash-dotted rectangle which encloses the triangle at iteration 1.