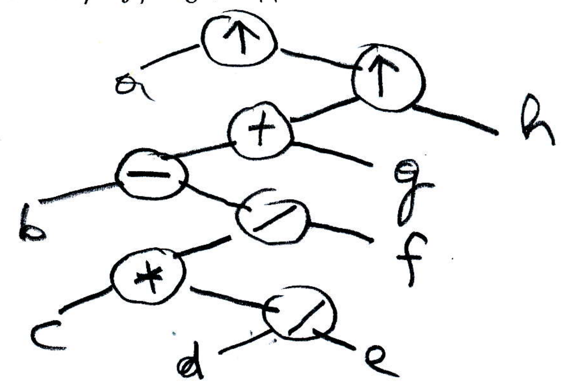
F 18

Problem 1 (13 points): Consider the following mathematical expression in **postfix** notation. assuming that each of the operators $+, -, *, /, \uparrow$ has two operands ($\uparrow$ is exponentiation).

$$abcde/*f/-g+h\uparrow\uparrow \quad (1)$$

(a, 4pts) Please draw the expression tree for (1).



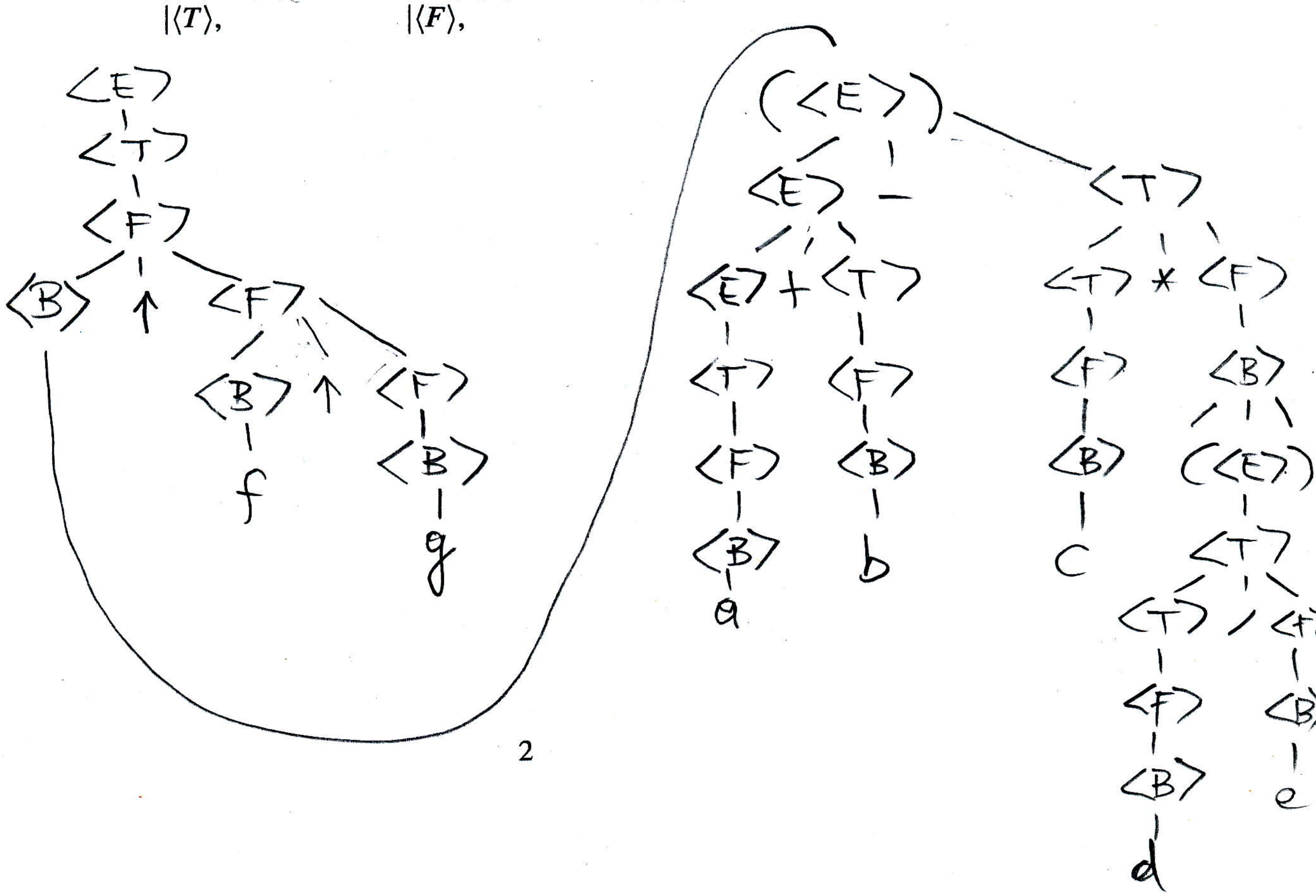
(b, 4pts) Please give both the **minimally parenthesized in**fix and the **pre**fix representations for the expression (1), the latter of which only has variables and operators.

INFIX (with minimum number of parentheses): $a\uparrow(b-c*(d/e)/f+g)\uparrow h$

PREFIX: $\uparrow a \uparrow + - b / * c / d e f g h$

(c, 5pts) Please draw the parse tree for the string $(a+b-c*(d/e))\uparrow f \uparrow g$ using the following context-free grammar $G = (N,T,P,s)$ (from class with exponentiation) $N = \{\langle E\rangle, \langle T\rangle, \langle F\rangle, \langle B\rangle\}$; note that $\langle E\rangle$ is an expression, $\langle T\rangle$ is a term, $\langle F\rangle$ is a factor and $\langle B\rangle$ is the base for a power. $T = \{a, b, \ldots, z, (,), +, -, *, /, \uparrow\}$. The start symbol $s = \langle E\rangle$.

$$P = \{\langle E\rangle \to \langle E\rangle + \langle T\rangle \mid \langle E\rangle - \langle T\rangle \mid \langle T\rangle, \quad \langle T\rangle \to \langle T\rangle * \langle F\rangle \mid \langle T\rangle / \langle F\rangle \mid \langle F\rangle, \quad \langle F\rangle \to \langle B\rangle \uparrow \langle F\rangle \mid \langle B\rangle, \quad \langle B\rangle \to (\langle E\rangle) \mid a \mid b \mid \ldots \mid z\}.$$



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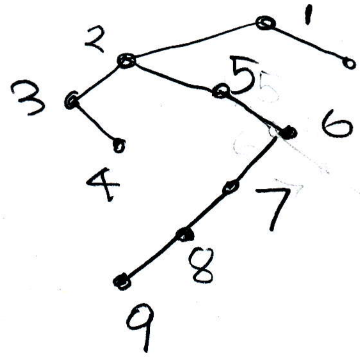
Problem 2 (6 points): Consider binary trees in which each node has either 0 children, or one left or one right child, or both. Such a tree with 10 nodes has been linearized by our method from class to ((( ) ( )) ( ) (((( ))))) ( ) with matchings indicated by numbers below the parentheses.
1 2 3 3 4 4 2 5 5 6 7 8 9 9 8 7 6 1 10 10
Please draw the tree writing in each node the corresponding parentheses number. How many such binary trees with 10 nodes are there?



$\frac{1}{11}\binom{20}{10}$ such trees

Problem 3 (6 points): Consider the following graph:

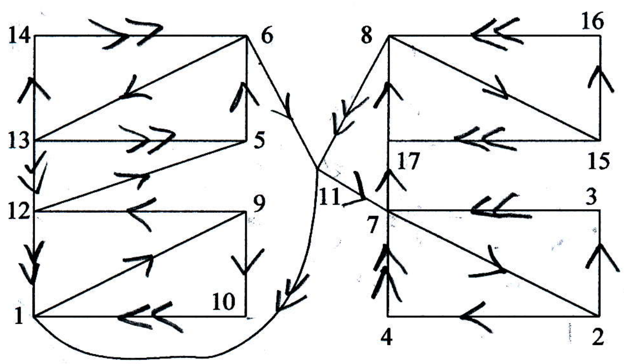
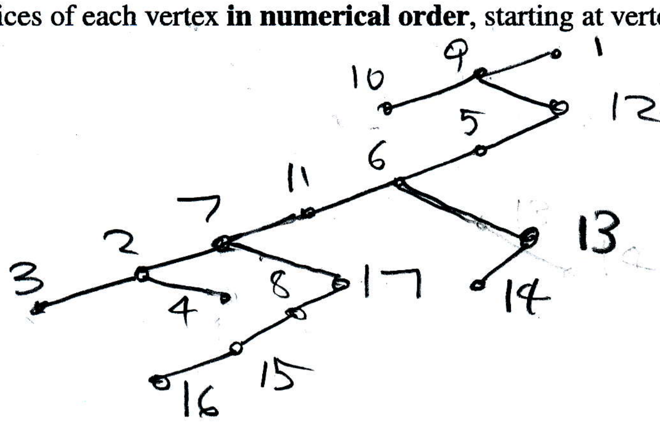


Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.



(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

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Problem 4 (5 points): Consider the following variant of Fibonacci's rabbits problem: *A super-fertile pair after* 1 *month of maturing gives birth to* 3 *pairs of rabbits, while a fertile pair after* 1 *month of maturing gives birth to* 2 *pairs of rabbits. Of the* 3 *pairs of newly born rabbits of the super-fertile pair,* 1 *is super-fertile,* 1 *is fertile, and* 1 *is infertile. The infertile pair matures in one month, but then has no offsprings. Of the* 2 *pairs of newly born rabbits of the fertile pair,* 1 *is super-fertile and* 1 *is fertile.* Please (a) model the variant by a Lindenmayer system, annotating each variable by what type of pair it represents, and (b) give the first 5 new generations of the system, starting at generation 0 with a single pair of newly born super-fertile rabbits.

Vars	A	B	C	D	I_0	I_1
Right sides	B	$BACI_0$	D	DAC	I_1	I_1

A s.-f. baby, B s.f. adult, C f. baby, D f. adult

I_0 inf. baby, I_1 inf. adult

$A \to B \to BACI_0 \to \underbrace{BACI_0}_{B} \underline{B} D I_1$

$\to \underbrace{BACI_0}_{B} \underbrace{B}_{A} \underbrace{D}_{C} \underbrace{I_1}_{I_0} \underbrace{BACI_0}_{B} \underbrace{DAC}_{D} I_1$

$\to BACI_0BDI_1 \; BACI_0 \; DACI_1 \; BACI_0BDI_1 \; DACBDI_1$

Problem 5 (5 points): Please describe a natural event whose occurrence exhibits a chaotic state. Please state why the event has chaotic properties.

The cone of sand in the hour glass:

One grain causes the cone to collapse: instability.

When the collapse happens is unpredictable

Averlanche in the mountains

Cancer cell in a human

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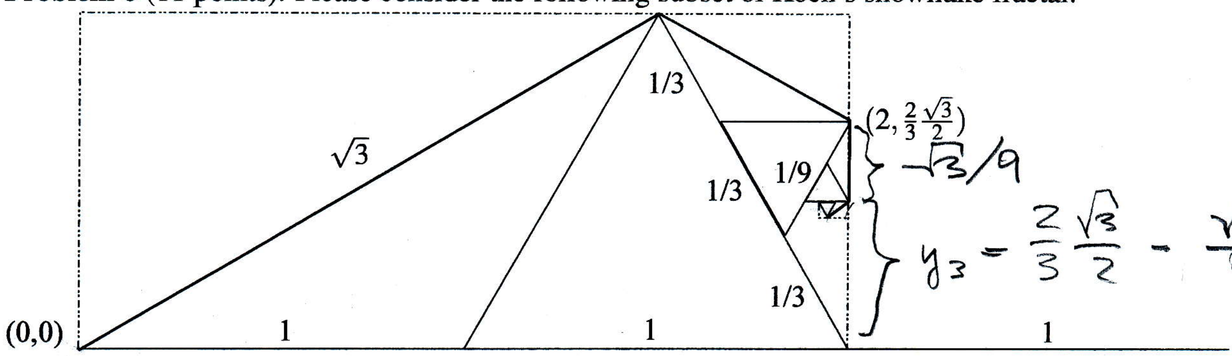
Problem 6 (11 points): Please consider the following subset of Koch's snowflake fractal:



$\sqrt{3}/9$

$y_3 = \frac{2}{3}\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{9}$

Here one starts at iteration 1 with an equilateral triangle with side length 1. At iteration 2 an equilateral triangle of side length $1/3$ is placed on the middle of the right side which goes up from the base side. At iteration 3 an equilateral triangle of side length $1/9$ is placed on the middle of the right side that goes out from the base line. And so on.

(a, 5 pts) At iteration i one draws a straight-line segment from the left vertex on the side of the previous triangle to the tip of the newly placed triangle. At iteration 1 the line segment has length $L_1 = 2\cos(\pi/6) = \sqrt{3}$. Please give the length L_i at iteration i and $\sum_{i=1}^{\infty} L_i$.

$$L_i = \sqrt{3}\left(\frac{1}{3}\right)^{i-1}$$

$$\sum_{i=1}^{\infty} L_i = \sqrt{3}\,\frac{1}{1-1/3} = \frac{3\sqrt{3}}{2}$$

(b, 6 pts) Please compute the x-y-coordinates of the tip of the extruded triangle at ∞ where the origin and the coordinates of the second tip are shown in the figure. Hint: Note that the triangle placed at iteration 7 again has a horizontal base line with a tip straight above it. The small dashed-dotted rectangle which encloses the 4-th triangle is a shrunk upside-down version of the large dash-dotted rectangle which encloses the triangle at iteration 1.

$$x = 2 - \frac{2}{27} + \frac{2}{27^2} - \frac{2}{27^3} \pm \cdots = 2 \cdot \frac{1}{1+\frac{1}{27}} = 1.9286$$

$$y = \underbrace{\sqrt{3}\cdot\frac{2}{9}}_{y_3} - \frac{y_3}{27} + \frac{y_3}{27^2} \pm \cdots = y_3 \underbrace{\frac{1}{1+\frac{1}{27}}}_{\frac{27}{28}}$$

≈ 0.3712