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Department of Mathematics

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Thur, Nov 7, 2019

Prof. Erich Kaltofen <kaltofen@math.ncsu.edu>
www.math.ncsu.edu/~kaltofen/courses/DiscreteModels/Fall19/index.html (URL)

919.515.8785 (phone)
919.515.3798 (fax)

Your Name: _____

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 6 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of **46 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **two** 8.5in $\times$ 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

5 _____

6 _____

Total _____

Problem 1 (13 points): Consider the following mathematical expression in **in**fix notation, assuming that each of the binary operators $+, -, *, /, \uparrow$ has two operands, where $\uparrow$ is exponentiation with highest precedence, which is evaluated right-to-left.

$$(a * b) \uparrow (c + d/e) \uparrow (f - g - h) \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).

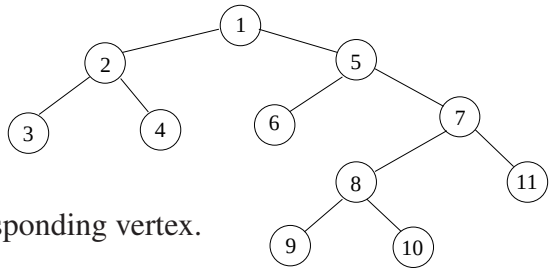
(b, 4pts) Please give both the **pre**fix and the **post**fix representations for the expression (1), both of which only have variables and operators.

PREFIX:

POSTFIX:

(c, 5pts) Please draw the parse tree for (1) above using the following context-free grammar $G = (N, T, P, s)$ (from class with exponentiation) $N = \{\langle E\rangle, \langle T\rangle, \langle F\rangle, \langle B\rangle\}$; note that $\langle E\rangle$ is an expression, $\langle T\rangle$ is a term, $\langle F\rangle$ is a factor and $\langle B\rangle$ is the base for a power.
The terminal symbols $T = \{a, b, \ldots, z, (,), +, -, *, /, \uparrow\}$. The start symbol $s = \langle E\rangle$.
$P = \{\langle E\rangle \to \langle E\rangle + \langle T\rangle, \quad \langle T\rangle \to \langle T\rangle * \langle F\rangle, \quad \langle F\rangle \to \langle B\rangle \uparrow \langle F\rangle, \quad \langle B\rangle \to (\langle E\rangle),$
$|\langle E\rangle - \langle T\rangle, \quad |\langle T\rangle / \langle F\rangle, \quad |\langle B\rangle, \quad |a|b|\ldots|z\}.$
$|\langle T\rangle, \quad |\langle F\rangle,$

Problem 3 (8 points):
Please consider the binary tree
(with left and right children identified):



(a, 5pts) Please give the parentheses-only string from class
for the tree, labelling each pair of parentheses with the corresponding vertex.

(b, 3pts) In the above binary tree of 11 vertices, every non-leaf vertex has 2 children. How may binary trees with 11 vertices have that property: all non-leaf vertices have 2 children? Please explain.

Problem 3 (6 points): Consider the following graph:

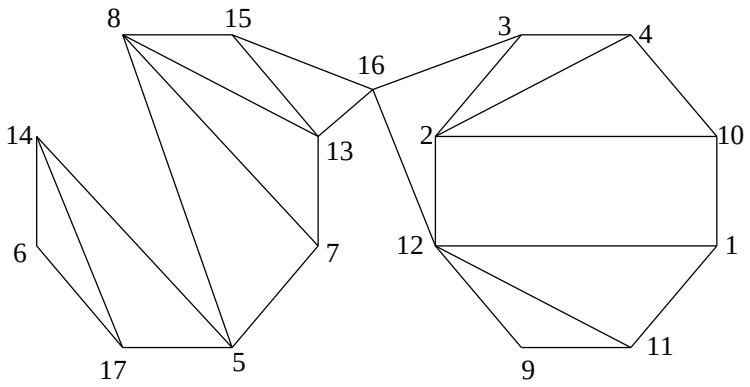


Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

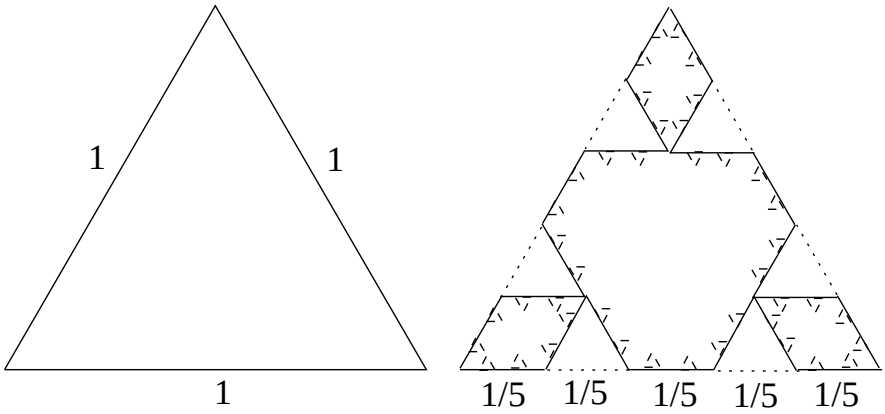
Problem 4 (5 points): Consider the following variant of Fibonacci's rabbits problem: *A super-fertile pair after* 1 *month of maturing gives birth to* 3 *pairs of rabbits, while a fertile pair after* 1 *month of maturing gives birth to* 2 *pairs of rabbits. Of the* 3 *pairs of newly born rabbits of the super-fertile pair,* 1 *is super-fertile,* 1 *is fertile, and* 1 *is infertile. The infertile pair matures in one month, but then has no offsprings. Of the* 2 *pairs of newly born rabbits of the fertile pair,* 1 *is super-fertile and* 1 *is fertile.* Please (a) model the variant by a Lindenmayer system, annotating each variable by what type of pair it represents, and (b) give the first 5 new generations of the system, starting at generation 0 with a single pair of newly born super-fertile rabbits.

Problem 5 (5 points): Consider the following Lindenmayer system:

Var's:	X	P	Y	r	F	Z	o	f	K
Right-sides:	PYZ	P	rFZ	r	ofZ	KPrF	o	f	K

Please write down the first 4 new generations of strings starting with X.

Problem 6 (10 points): Please consider the following "inverted" Koch-like snowflake fractal:



Here one starts at 1st- iteration 1 with an equilateral triangle with side length 1. At the 2nd-iteration 2 equilateral triangles of side length $1/5$ are pushed into the triangle at equal spaced intervals on each of the 3 sides. The interior is now 3 polygons connected at 3 points. At the 3rd-iteration again 2 triangles of side length $1/25$ are pushed in on each of the 21 sides. They are shown above with dashed sides. And so on.

(a, 5 pts) Please give the sum L_i of the lengths of the boundaries of all polygons at iteration i.

(b, 5 pts) Please give the remaining area, namely, the sum A_i of the areas of all polygons, at iteration i. Note that $A_1 = \sqrt{3}/4$.