Your Name: __________________________
For purpose of anonymous grading, please do not write your name on the subsequent pages.

This examination consists of 6 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of 46 points. Please write your answers in the spaces indicated, or below the questions, using the back of the sheets for completing the answers and for all scratch work, if necessary. You are allowed to consult two 8.5in × 11in sheets with notes, but not your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have 75 minutes to do this test.

Good luck!

Problem 1

2

3

4

5

6

Total
**Problem 1** (13 points): Consider the following mathematical expression in *infix* notation, assuming that each of the binary operators $+, -, *, /, \uparrow$ has two operands, where $\uparrow$ is exponentiation with highest precedence, which is evaluated right-to-left.

$$(a \ast b) \uparrow (c + d / e) \uparrow (f - g - h) \quad (1)$$

**(a, 4pts)** Please draw the expression tree for (1).

**(b, 4pts)** Please give both the **prefix** and the **postfix** representations for the expression (1), both of which only have variables and operators.

PREFIX:

POSTFIX:

**(c, 5pts)** Please draw the parse tree for (1) above using the following context-free grammar $G = (N, T, P, s)$ (from class with exponentiation) $N = \{\langle E \rangle, \langle T \rangle, \langle F \rangle, \langle B \rangle\}$; note that $\langle E \rangle$ is an expression, $\langle T \rangle$ is a term, $\langle F \rangle$ is a factor and $\langle B \rangle$ is the base for a power.

The terminal symbols $T = \{a, b, \ldots, z, (, ) , +, -, *, /, \uparrow\}$. The start symbol $s = \langle E \rangle$.

$P = \{\langle E \rangle \rightarrow \langle E \rangle + \langle T \rangle, \quad \langle T \rangle \rightarrow \langle T \rangle \ast \langle F \rangle, \quad \langle F \rangle \rightarrow \langle B \rangle \uparrow \langle F \rangle, \quad \langle B \rangle \rightarrow (\langle E \rangle), \quad |\langle E \rangle - \langle T \rangle, \quad |\langle T \rangle / \langle F \rangle, \quad |\langle B \rangle, \quad |a|b|\ldots|z\}$.
**Problem 3** (8 points):
Please consider the binary tree
(with left and right children identified):
(a, 5pts) Please give the parentheses-only string from class
for the tree, labelling each pair of parentheses with the corresponding vertex.

(b, 3pts) In the above binary tree of 11 vertices, every non-leaf vertex has 2 children. How may binary trees with 11 vertices have that property: all non-leaf vertices have 2 children? Please explain.

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**Problem 3** (6 points): Consider the following graph:

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex in numerical order, starting at vertex 1.

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.
**Problem 4** (5 points): Consider the following variant of Fibonacci’s rabbits problem: A super-fertile pair after 1 month of maturing gives birth to 3 pairs of rabbits, while a fertile pair after 1 month of maturing gives birth to 2 pairs of rabbits. Of the 3 pairs of newly born rabbits of the super-fertile pair, 1 is super-fertile, 1 is fertile, and 1 is infertile. The infertile pair matures in one month, but then has no offsprings. Of the 2 pairs of newly born rabbits of the fertile pair, 1 is super-fertile and 1 is fertile. Please (a) model the variant by a Lindenmayer system, annotating each variable by what type of pair it represents, and (b) give the first 5 new generations of the system, starting at generation 0 with a single pair of newly born super-fertile rabbits.

**Problem 5** (5 points): Consider the following Lindenmayer system:

<table>
<thead>
<tr>
<th>Var’s:</th>
<th>X</th>
<th>P</th>
<th>Y</th>
<th>r</th>
<th>F</th>
<th>Z</th>
<th>o</th>
<th>f</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right-sides:</td>
<td>PYZ</td>
<td>P</td>
<td>rFZ</td>
<td>r</td>
<td>ofZ</td>
<td>KPrF</td>
<td>o</td>
<td>f</td>
<td>K</td>
</tr>
</tbody>
</table>

Please write down the first 4 new generations of strings starting with X.
Problem 6 (10 points): Please consider the following “inverted” Koch-like snowflake fractal:

Here one starts at 1st-iteration 1 with an equilateral triangle with side length 1. At the 2nd-iteration 2 equilateral triangles of side length 1/5 are pushed into the triangle at equal spaced intervals on each of the 3 sides. The interior is now 3 polygons connected at 3 points. At the 3rd-iteration again 2 triangles of side length 1/25 are pushed in on each of the 21 sides. They are shown above with dashed sides. And so on.

(a, 5 pts) Please give the sum $L_i$ of the lengths of the boundaries of all polygons at iteration $i$.

(b, 5 pts) Please give the remaining area, namely, the sum $A_i$ of the areas of all polygons, at iteration $i$. Note that $A_1 = \sqrt{3}/4$. 