**Problem 1** (13 points): Consider the following mathematical expression in *infix* notation, assuming that each of the binary operators $+,-,\times,\div$ has two operands, where $\uparrow$ is exponentiation with highest precedence, which is evaluated right-to-left.

\[(a \times b) \uparrow (c + d/e) \uparrow (f - g - h)\] (1)

(a, 4pts) Please draw the expression tree for (1).

(b, 4pts) Please give both the *prefix* and the *postfix* representations for the expression (1), both of which only have variables and operators.

**PREFIX:**  
\[\uparrow \times a \ b \ \uparrow + c \ / \ d \ e \ \uparrow - f \ \uparrow - g \ \uparrow - h\]

**POSTFIX:**  
\[a \ b \ \times \ c \ \div \ d \ e \ \uparrow - f \ \uparrow - g \ \uparrow - h \ \uparrow\]

(c, 5pts) Please draw the parse tree for (1) above using the following context-free grammar $G = (N, T, P, s)$ (from class with exponentiation) $N = \{<E>, <T>, <F>, <B>\}$; note that $<E>$ is an expression, $<T>$ is a term, $<F>$ is a factor and $<B>$ is the base for a power.

The terminal symbols $T = \{a, b, \ldots, z, (,), +, -, \times, \div, \uparrow\}$. The start symbol $s = <E>$.

$P = \{ <E> \rightarrow <E> + <T>, \ <T> \rightarrow <T> \times <F>, \ <F> \rightarrow <B> \uparrow <F>, \ <B> \rightarrow (<E>),\ <(E)> \rightarrow <(T)>\}$.  

\[<E> \rightarrow <T> \} / \{<F> \} \ 
\ <T> \rightarrow <F> \} / \{<B> \} \ 
\ <F> \rightarrow <B> \} / \{ <(E)> \} \ 
\ <B> \rightarrow <E> \} / \{ <(T)> \} \ 
\}

\[<E> \rightarrow <F> \} / \{<B> \} \ 
\ <F> \rightarrow <E> \} / \{ <(B)> \} \ 
\ <B> \rightarrow <E> \} / \{ <(E)> \} \ 
\]

\[<E> \rightarrow <F> \} / \{<B> \} \ 
\ <F> \rightarrow <E> \} / \{ <(B)> \} \ 
\ <B> \rightarrow <E> \} / \{ <(E)> \} \ 
\]
Problem 2 (8 points):
Please consider the binary tree
(with left and right children identified):
(a, 4pts) Please give the parentheses-only string from class
for the tree, labelling each pair of parentheses with the corresponding vertex.

(b, 4pts) In the above binary tree of 11 vertices, every non-leaf vertex has 2 children. How may
binary trees with 11 vertices have the property that all non-leaf vertices have 2 children? Please
explain.

Problem 3 (6 points): Consider the following graph:

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring ver-
tices of each vertex in numerical order, starting at vertex 1.

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1
on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected.
Please draw your orientation of each edge in Figure 1, using a different arrow head for those
arcs that correspond to edges in the DFS tree.
Problem 4 (5 points): Please define the Mandelbrot set $M_a$ for $a = 2$, that is, $M_2$. Please show that $-3 \in M_2$ and $3 \notin M_2$.

\[ M_2 = \{ c \in \mathbb{C} \mid \{ z_0 = 2, z_1 = 2 + c, z_2 = z_1^2 + c, \ldots \} \text{ is bounded in absolute value} \} \]

(EB: $\forall i: |z_i| \leq R$)

-3 \in M_2: $z_1 = 2, z_2 = 2^2 - 3 = -1, z_3 = z_2^2 - 3 = -2, z_4 = z_2^2 - 3 = -1$

$3 \notin M_2: z_1 = 2, z_2 = 2^2 + 3 = 7, z_3 = z_2^2 + 3 = 52, \ldots, |z_i| > 2$

Problem 5 (5 points): Consider the following Lindenmayer system:

| Variables: | X | P | Y | r | F | Z | o | f | K |
| Right-sides: | PYZ | P | rFZ | r | ofZ | KPrF | o | f | K |

Please write down the first 4 new generations of strings starting with $X$.

\[
X \rightarrow PYZ \\
\rightarrow PFZKPrF \\
\rightarrow ProfZKPrF KProfZ \\
\rightarrow ProfKPrF KProfZ KProfKPrF
\]
Problem 6 (10 points): Please consider the following “inverted” Koch-like snowflake fractal:

Here one starts at the 1st-iteration with an equilateral triangle with side length 1. At the 2nd-iteration 2 equilateral triangles of side length 1/5 are pushed into the triangle at equal spaced intervals on each of the 3 sides. The interior is now 4 polygons connected at 3 shared vertices. At the 3rd-iteration again 2 triangles of side length 1/25 are pushed in on each of the 21 line segments of length 1/5. They are shown above with dashed sides. And so on.

(a, 5 pts) Please give the sum $L_i$ of the lengths of the boundaries of all polygons at iteration $i$.

\[
L_i = 3 + 3 \cdot 2 \cdot \frac{1}{5} + 3 \cdot 2 \cdot 7 \cdot \frac{1}{5^2} + \cdots + 3 \cdot 2 \cdot 7^{i-2} \cdot \frac{1}{5^{i-1}} = 3 + 3 \left( \frac{7}{5} \right)^{i-1} - 1
\]

or

\[
L_i = 3 \cdot \left( \frac{7}{5} \right)^{i-1}
\]

(b, 5 pts) Please give the remaining area, namely, the sum $P_i$ of the areas of all polygons at iteration $i$ (note: not at infinity). Note that $P_1 = \sqrt{3}/4$.

\[
A = 6 \cdot \frac{A}{25} - 3 \cdot 2 \cdot 7 \cdot \frac{A}{25^2} - \ldots - 3 \cdot 2 \cdot 7^{i-2} \cdot \frac{A}{25^{i-1}}
\]

\[
= A - A \cdot 6 \cdot \frac{1}{25} \cdot \frac{1 - \left( \frac{7}{25} \right)^{i-1}}{1 - \frac{7}{25}}
\]

\[
= A \left( 1 - \frac{1}{3} \left( 1 - \left( \frac{7}{25} \right)^{i-1} \right) \right)
\]

\[
= \frac{\sqrt{3}}{4} \left( \frac{2}{3} + \frac{1}{3} \left( \frac{7}{25} \right)^{i-1} \right)
\]

\[
= \frac{\sqrt{3}}{6} + \frac{\sqrt{3}}{12} \cdot \left( \frac{7}{25} \right)^{i-1}
\]