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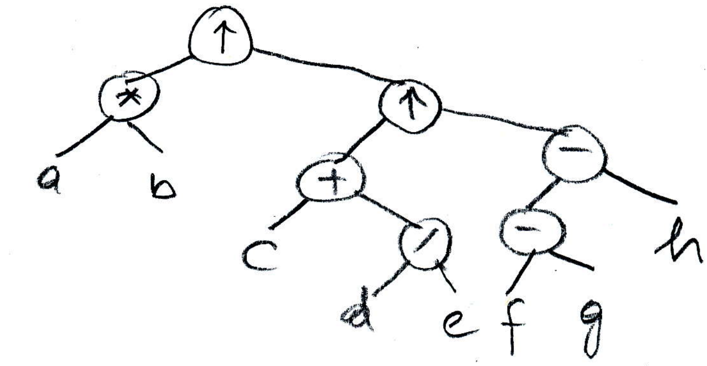
Problem 1 (13 points): Consider the following mathematical expression in **in**fix notation, assuming that each of the binary operators $+, -, *, /, \uparrow$ has two operands, where $\uparrow$ is exponentiation with highest precedence, which is evaluated right-to-left.

$$(a * b) \uparrow (c + d/e) \uparrow (f - g - h) \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).



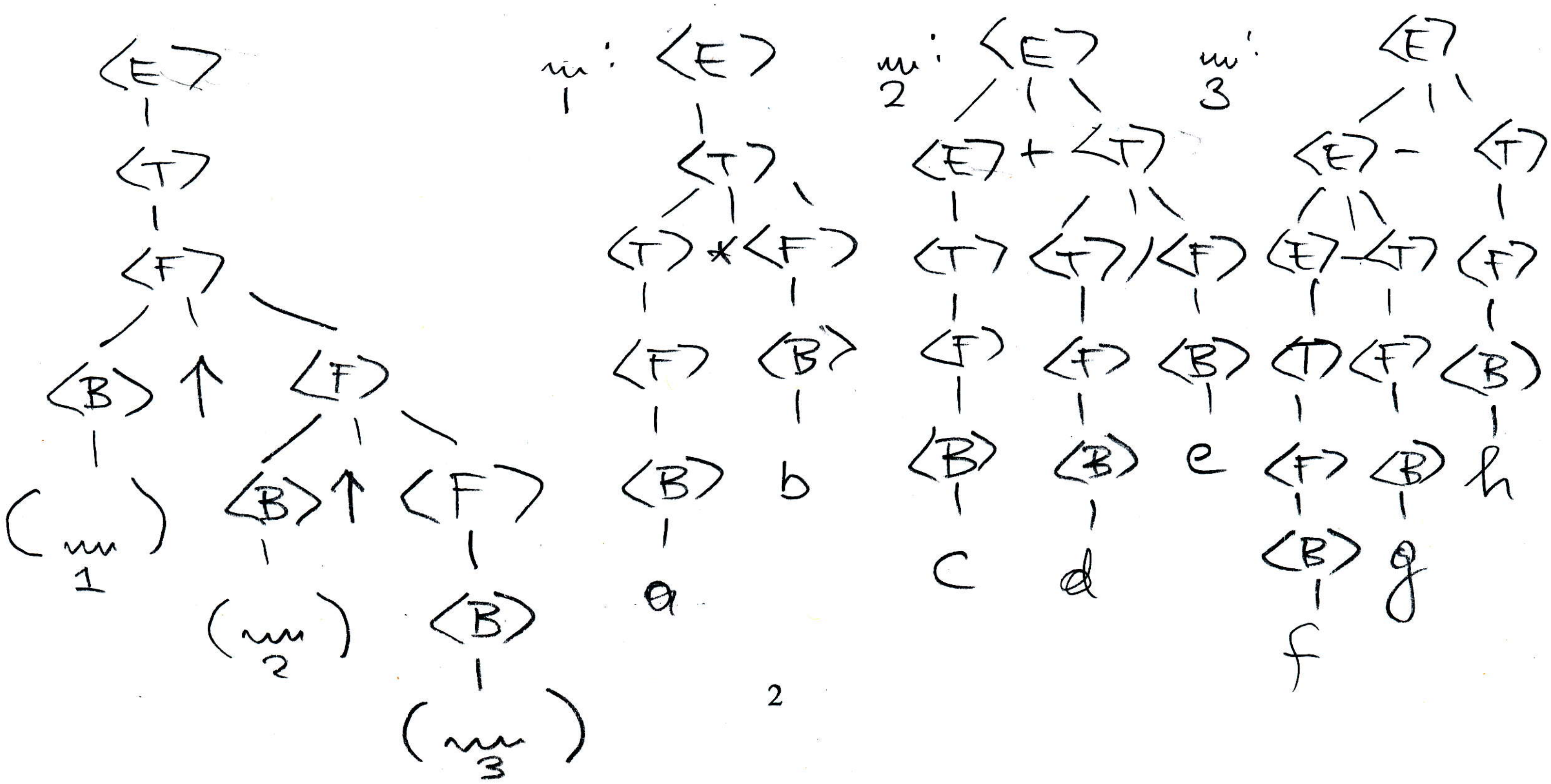
(b, 4pts) Please give both the **pre**fix and the **post**fix representations for the expression (1), both of which only have variables and operators.

PREFIX: $\uparrow * a\, b \uparrow + c / d\, e - - f\, g\, h$

POSTFIX: $a\, b * c\, d\, e / + f\, g - h - \uparrow \uparrow$

(c, 5pts) Please draw the parse tree for (1) above using the following context-free grammar $G = (N, T, P, s)$ (from class with exponentiation) $N = \{\langle E\rangle, \langle T\rangle, \langle F\rangle, \langle B\rangle\}$; note that $\langle E\rangle$ is an expression, $\langle T\rangle$ is a term, $\langle F\rangle$ is a factor and $\langle B\rangle$ is the base for a power.
The terminal symbols $T = \{a, b, \ldots, z, (,), +, -, *, /, \uparrow\}$. The start symbol $s = \langle E\rangle$.

$$P = \{\langle E\rangle \to \langle E\rangle + \langle T\rangle \mid \langle E\rangle - \langle T\rangle \mid \langle T\rangle, \quad \langle T\rangle \to \langle T\rangle * \langle F\rangle \mid \langle T\rangle / \langle F\rangle \mid \langle F\rangle, \quad \langle F\rangle \to \langle B\rangle \uparrow \langle F\rangle \mid \langle B\rangle, \quad \langle B\rangle \to (\langle E\rangle) \mid a \mid b \mid \ldots \mid z\}.$$



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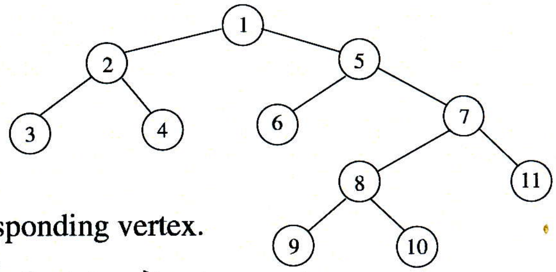
Problem 2 (8 points):
Please consider the binary tree
(with left and right children identified):



(a, 4pts) Please give the parentheses-only string from class for the tree, labelling each pair of parentheses with the corresponding vertex.

((( )) ( )) (( )) ((( )) ( )) ( )
1 2 3 3 2 4 4 1 5 6 6 5 7 8 9 9 8 10 10 7 11 11

(b, 4pts) In the above binary tree of 11 vertices, every non-leaf vertex has 2 children. How may binary trees with 11 vertices have the property that all non-leaf vertices have 2 children? Please explain.

Removing 6 leaves, one obtains an arbitrary binary tree with 5 interior vertices, for each such tree. Therefore, there are $C_5 = \frac{1}{6}\binom{10}{5}$ such trees

Problem 3 (6 points): Consider the following graph:

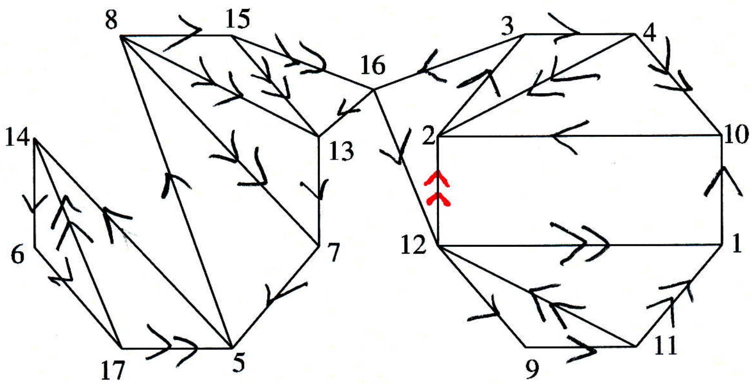
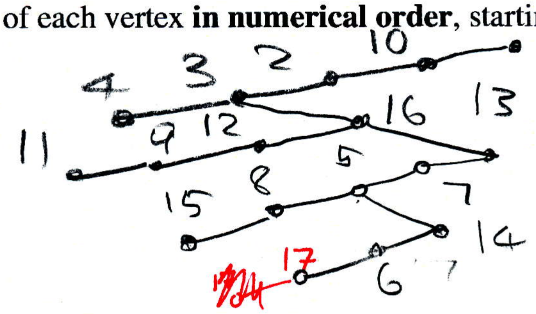


Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.



(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

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Problem 4 (5 points): Please define the Mandelbrot set M_a for $a = 2$, that is, M_2. Please show that $-3 \in M_2$ and $3 \notin M_2$.

$M_2 = \{ c \in \mathbb{C} \mid \{ z_1 = 2,\ z_2 = z_1^2 + c,\ z_3 = z_2^2 + c, \dots \}$ is bounded in absolute value $\}$

$(\exists B : \forall i : |z_i| \leq B)$

$-3 \in M_2$: $z_1 = 2,\ z_2 = z_1^2 - 3 = -1,\ z_3 = z_2^2 - 3 = -2,\ z_4 = z_3^2 - 3 = -1$ periodic, hence bnd.

$3 \notin M_2$: $z_1 = 2,\ z_2 = z_1^2 + 3 = 7,\ z_3 = z_2^2 + 3 = 52, \dots, |z_i| \geq 2^i$

Problem 5 (5 points): Consider the following Lindenmayer system:

Variables:	X	P	Y	r	F	Z	o	f	K
Right-sides:	PYZ	P	rFZ	r	ofZ	KPrF	o	f	K

Please write down the first 4 new generations of strings starting with X.

$X \to PYZ$

$\to P\underbrace{rFZ}_{Y}\underbrace{KPrF}_{Z}$

$\to Pr\underbrace{ofZ}_{F}\underbrace{KPrF}_{Z}KPr\underbrace{ofZ}_{F}$

$\to Prof\underbrace{KPrF}_{Z}KPr\underbrace{ofZ}_{F}KProf\underbrace{KPrF}_{Z}$

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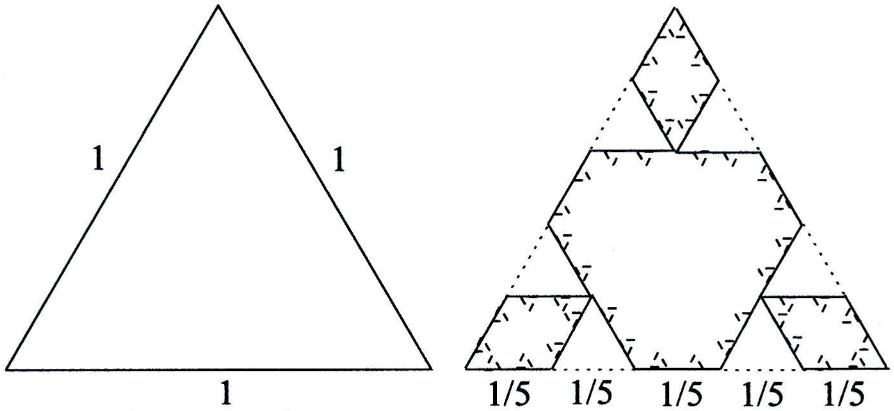
Problem 6 (10 points): Please consider the following "inverted" Koch-like snowflake fractal:



Here one starts at the 1st-iteration with an equilateral triangle with side length 1. At the 2nd-iteration 2 equilateral triangles of side length $1/5$ are pushed into the triangle at equal spaced intervals on each of the 3 sides. The interior is now 4 polygons connected at 3 shared vertices. At the 3rd-iteration again 2 triangles of side length $1/25$ are pushed in on each of the 21 line segments of length $1/5$. They are shown above with dashed sides. And so on.

(a, 5 pts) Please give the sum L_i of the lengths of the boundaries of all polygons at iteration i.

$$L_i = 3 + 3\cdot 2\cdot\frac{1}{5} + 3\cdot 2\cdot 7\cdot\frac{1}{5^2} + \cdots + 3\cdot 2\cdot 7^{i-2}\frac{1}{5^{i-1}} = 3 + 3\left(\left(\frac{7}{5}\right)^{i-1} - 1\right)$$

$$= 3\cdot\left(\frac{7}{5}\right)^{i-1}$$

or $L_i = 3\cdot\left(\frac{7}{5}\right)^{i-1}$

(b, 5 pts) Please give the remaining area, namely, the sum P_i of the areas of all polygons at iteration i (note: not at infinity). Note that $P_1 = \sqrt{3}/4$.

$$A - 6\cdot\frac{A}{25} - 3\cdot 2\cdot 7\cdot\frac{A}{25^2} - \cdots - 3\cdot 2\cdot 7^{i-2}\frac{A}{25^{i-1}}$$

$$= A - A6\cdot\frac{1}{25}\cdot\frac{1-\left(\frac{7}{25}\right)^{i-1}}{1-\frac{7}{25}} = A\left(1 - \frac{1}{3}\left(1-\left(\frac{7}{25}\right)^{i-1}\right)\right)$$

$$= \frac{\sqrt{3}}{4}\left(\frac{2}{3} + \frac{1}{3}\left(\frac{7}{25}\right)^{i-1}\right)$$

$$= \frac{\sqrt{3}}{6} + \frac{\sqrt{3}}{12}\left(\frac{7}{25}\right)^{i-1}$$