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Department of Mathematics

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Tue, Nov 2, 2021
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https://users.cs.duke.edu/~elk27/courses/DiscreteModels/Fall21/index.html (URL)

There are 6 problems on this exam for a total of 47 points. Please take a photo/scan of your solution of each problem done on paper (not necessarily the printed exam) and upload the problem on the Moodle course web page on wolfware.ncsu.edu.

By taking the exam, you agree that you will not consult with others about the solution. You can consult your notes/book/the Internet such as https://www.wolframalpha.com.

You have until 11am today, Tuesday, November 2, to upload the photos/scans. I will be on Zoom from 8:30am-9:45am to answer questions about the exam.

Good luck!

If you are taking the exam later, please sign the following statement:

I, _____, *affirm that I have no knowledge of the contents of this exam.*

_____

Signature

Problem 1 (13 points): Consider the following mathematical expression in **in**fix notation, assuming that each of the binary operators $+, -, *, /, \uparrow$ has two operands, where $\uparrow$ is exponentiation with highest precedence, which is evaluated right-to-left.

$$a + (b - c * d / e \uparrow f \uparrow g / h) \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).

(b, 4pts) Please give both the **pre**fix and the **post**fix representations for the expression (1), both of which only have variables and operators.

PREFIX:

POSTFIX:

(c, 5pts) Please draw the parse tree for the string $(a \uparrow b) \uparrow (c - d/e * f)$ using the following context-free grammar $G = (N, T, P, s)$ (from class with exponentiation) $N = \{\langle E\rangle, \langle T\rangle, \langle F\rangle, \langle B\rangle\}$; note that $\langle E\rangle$ is an expression, $\langle T\rangle$ is a term, $\langle F\rangle$ is a factor and $\langle B\rangle$ is the base for a power. The terminal symbols are $T = \{a, b, \ldots, z, (,), +, -, *, /, \uparrow\}$. The start symbol is $s = \langle E\rangle$.

$$P = \{\langle E\rangle \to \langle E\rangle + \langle T\rangle, \quad \langle T\rangle \to \langle T\rangle * \langle F\rangle, \quad \langle F\rangle \to \langle B\rangle \uparrow \langle F\rangle, \quad \langle B\rangle \to (\langle E\rangle),$$
$$|\langle E\rangle - \langle T\rangle, \quad |\langle T\rangle / \langle F\rangle, \quad |\langle B\rangle, \quad |a|b|\ldots|z\}.$$
$$|\langle T\rangle, \quad |\langle F\rangle,$$

Problem 2 (5 points): Suppose you divide n representatives over 3 groups of $6000, 6000, 1000$ people by Hamilton's Method. For $n = 5$ to $n = 6$ an "Alabama Paradox" observed. Please find a second pair $(n, n+1)$ at which a paradox occurs. Please show your work.

Problem 3 (6 points): Consider the following graph:

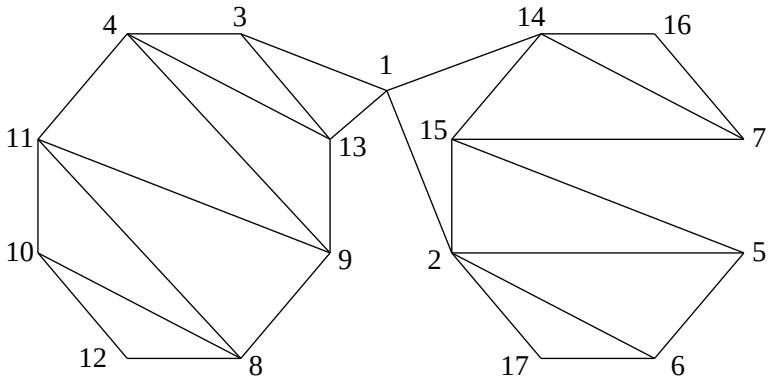
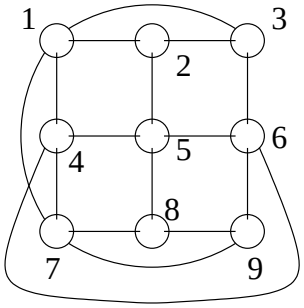


Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

Problem 4 (8 points): Please consider the following 3×3 mesh-like graph with horizontal wrap-around edges $\{1,3\}$, $\{4,6\}$, $\{7,9\}$ and the single vertical wrap-around edge $\{1,7\}$, which causes non-planarity.



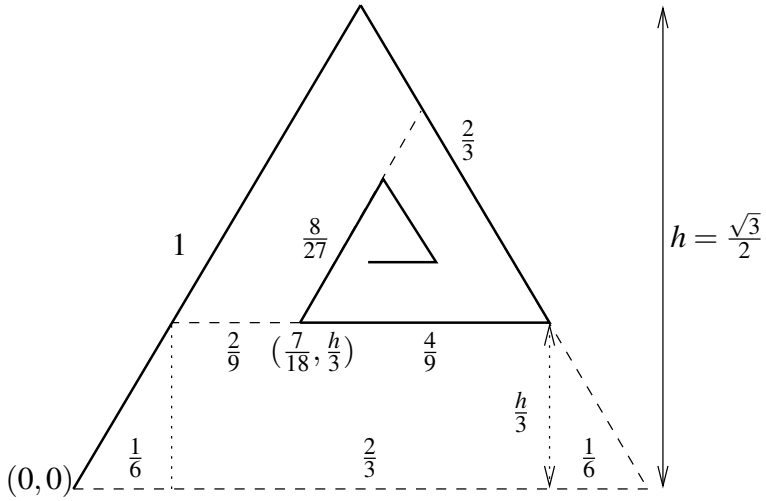
Please draw a subgraph that is homeomorphic to $K_{3,3}$.

Problem 5 (5 points): Consider the following Lindenmayer system:

Variables:	E	m	T	t	F	e	ℓ	r
Right-sides:	E m T	m	T t F	t	ℓ E r e F	e	ℓ	r

Please write down the first 4 new generations of strings starting with E.

Problem 6 (10 points): Please consider the following zig-zag line with 60° turns.



Here one starts at x-y-coordinate $(0,0)$ and goes up at angle 60° one unit. Then down at angle 60° shrinking the length by a factor $2/3$. Then back at angle 60° shrinking the length again be a factor $2/3$ to length $4/9$. The new x-y-coordinate is $(7/18, \sqrt{3}/6)$, as indicated. One continues the up-down-back's, always shrinking the line segment lengths by a factor $2/3$. Only one more is shown.

(a, 5 pts) Please give the sum of the lengths of the line segments after i segments, with $L_1 = 1$ and $L_2 = 1 + 2/3$.

(b, 5 pts) Please give the x-y-coordinate of the limit point after infinitely many line segments are drawn. After 3 segments the point is $(7/18, h/3)$.