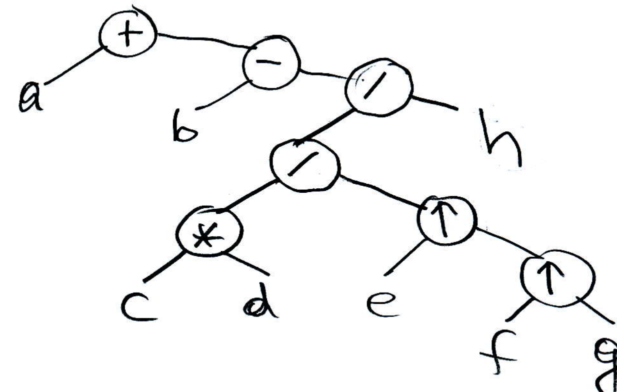
F21 Solution

Problem 1 (13 points): Consider the following mathematical expression in **in**fix notation, assuming that each of the binary operators $+, -, *, /, \uparrow$ has two operands, where $\uparrow$ is exponentiation with highest precedence, which is evaluated right-to-left.

$$a + (b - c * d / e \uparrow f \uparrow g / h) \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).



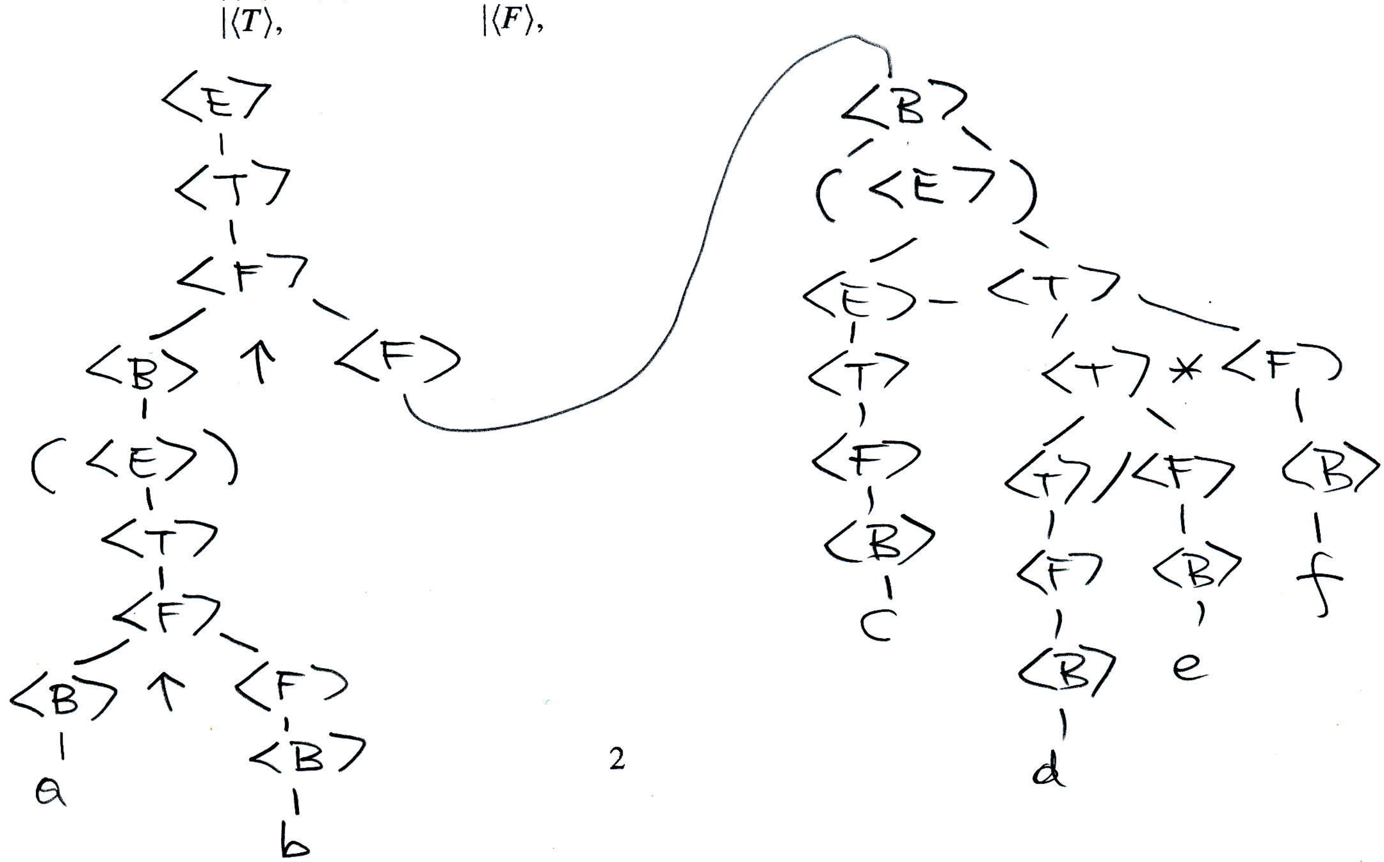
(b, 4pts) Please give both the **pre**fix and the **post**fix representations for the expression (1), both of which only have variables and operators.

PREFIX: $+ \; a \; - \; b \; / \; / \; * \; c \; d \; \uparrow \; e \; \uparrow \; f \; g \; h$

POSTFIX: $a \; b \; c \; d \; * \; e \; f \; g \; \uparrow \; \uparrow \; / \; h \; / \; - \; +$

(c, 5pts) Please draw the parse tree for the string $(a \uparrow b) \uparrow (c - d/e * f)$ using the following context-free grammar $G = (N, T, P, s)$ (from class with exponentiation) $N = \{\langle E\rangle, \langle T\rangle, \langle F\rangle, \langle B\rangle\}$; note that $\langle E\rangle$ is an expression, $\langle T\rangle$ is a term, $\langle F\rangle$ is a factor and $\langle B\rangle$ is the base for a power. The terminal symbols are $T = \{a, b, \ldots, z, (,), +, -, *, /, \uparrow\}$. The start symbol is $s = \langle E\rangle$.

$$\begin{aligned} P = \{\langle E\rangle \to \langle E\rangle + \langle T\rangle, \quad & \langle T\rangle \to \langle T\rangle * \langle F\rangle, \quad & \langle F\rangle \to \langle B\rangle \uparrow \langle F\rangle, \quad & \langle B\rangle \to (\langle E\rangle), \\ |\langle E\rangle - \langle T\rangle, \quad & |\langle T\rangle / \langle F\rangle, \quad & |\langle B\rangle, \quad & |a|b|\ldots|z\}. \\ |\langle T\rangle, \quad & |\langle F\rangle, & & \end{aligned}$$



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Problem 2 (5 points): Suppose you divide n representatives over 3 groups of 6000, 6000, 1000 people by Hamilton's Method. For $n = 5$ to $n = 6$ an "Alabama Paradox" observed. Please find a second pair $(n, n+1)$ at which a paradox occurs. Please show your work.

n	$\frac{6}{13}n$	$\frac{1}{13}n$	Rep 1	Rep 2	Rep 3
Ala 5	$2+\frac{4}{13}$	$\frac{5}{13}$	2	2	1
6	$2+\frac{10}{13}$	$\frac{6}{13}$	3	3	0
Ala 7	$3+\frac{3}{13}$	$\frac{7}{13}$	3	3	1
8	$3+\frac{9}{13}$	$\frac{8}{13}$	4	4	0
$18 = 5+13$	$8+\frac{4}{13}$	$1+\frac{5}{13}$	8	8	2
$19 = 6+13$	$8+\frac{10}{13}$	$1+\frac{6}{13}$	9	9	1

Ala

Another one at 7+13, 8+13

Problem 3 (6 points): Consider the following graph:

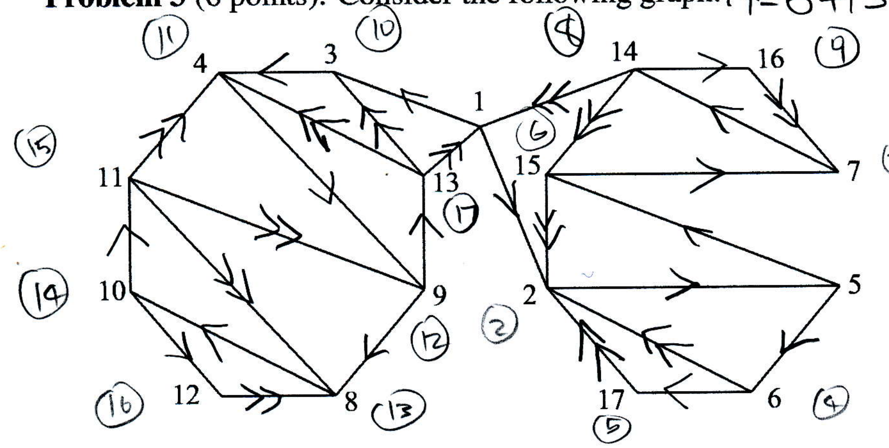
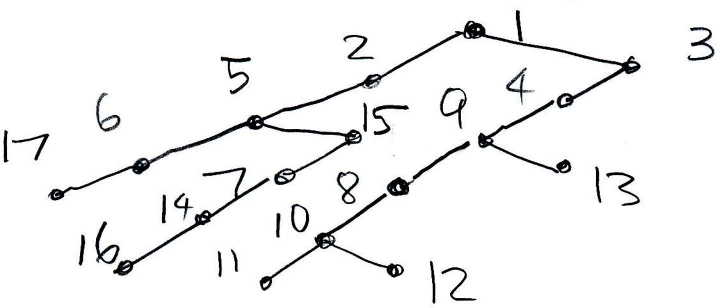


Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.



(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

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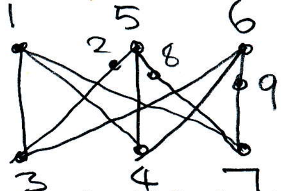
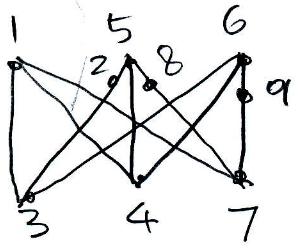
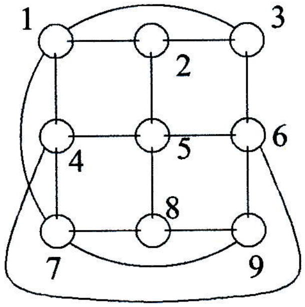
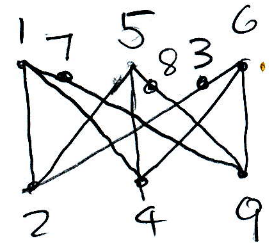


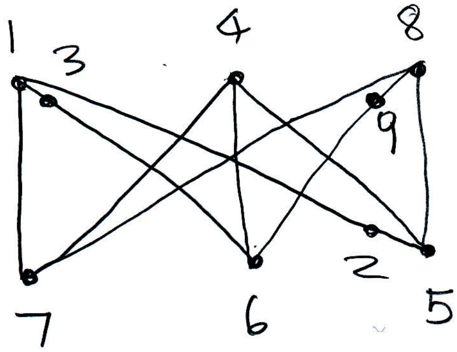
Problem 4 (8 points): Please consider the following 3×3 mesh-like graph with horizontal wrap-around edges $\{1,3\}$, $\{4,6\}$, $\{7,9\}$ and the single vertical wrap-around edge $\{1,7\}$, which causes non-planarity.

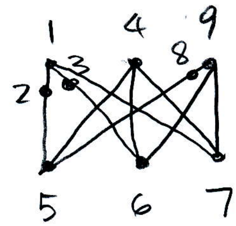


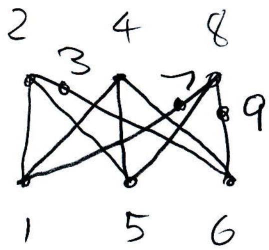


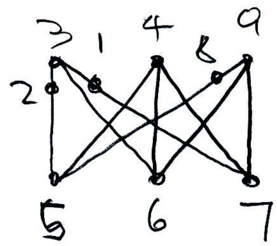


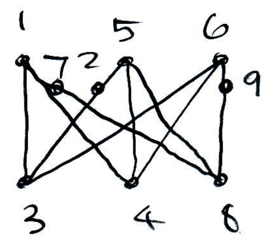
Please draw a subgraph that is homeomorphic to $K_{3,3}$.











Problem 5 (5 points): Consider the following Lindenmayer system:

Variables:	E	m	T	t	F	e	ℓ	r
Right-sides:	E m T	m	T t F	t	ℓ E r e F	e	ℓ	r

Please write down the first 4 new generations of strings starting with E.

$E \rightarrow EmT$

$\rightarrow EmT\ m\ TtF$

$\rightarrow \underbrace{EmTmTtF}_{EmT}\ m\ \underbrace{TtFt\ell EreF}_{TtF}$

$\rightarrow \underbrace{EmTmTtFmTtFt\ell EreF}_{EmTmTtF}$

$m\ \underbrace{TtFt\ell EreF}_{TtF}\ t\ \underbrace{\ell EmTr}_{\ell Er}\ e\ \underbrace{\ell EreF}_{F}$

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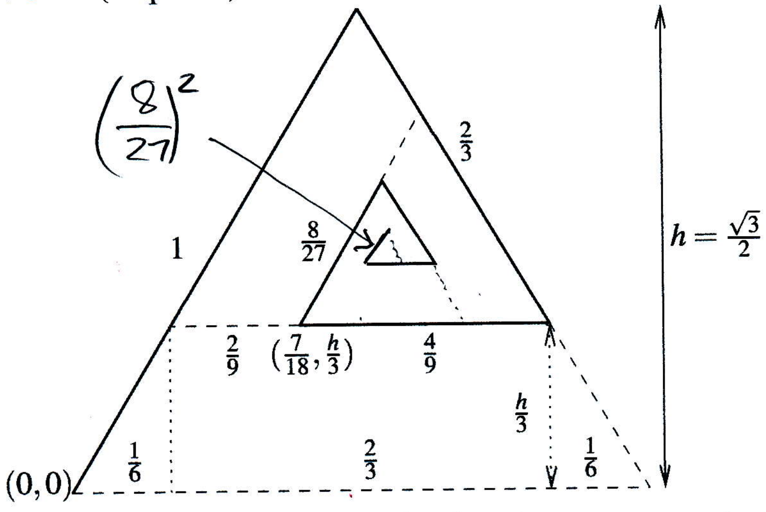
Problem 6 (10 points): Please consider the following zig-zag line with 60° turns.



Here one starts at x-y-coordinate $(0,0)$ and goes up at angle 60° one unit. Then down at angle 60° shrinking the length by a factor 2/3. Then back at angle 60° shrinking the length again be a factor 2/3 to length 4/9. The new x-y-coordinate is $(7/18, \sqrt{3}/6)$, as indicated. One continues the up-down-back's, always shrinking the line segment lengths by a factor 2/3. Only one more is shown.

(a, 5 pts) Please give the sum of the lengths of the line segments after i segments, with $L_1 = 1$ and $L_2 = 1 + 2/3$.

$$L_i = 1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \cdots + \left(\frac{2}{3}\right)^{i-1} = \frac{1 - \left(\frac{2}{3}\right)^i}{1 - \frac{2}{3}}$$

$$= 3\left(1 - \left(\frac{2}{3}\right)^i\right)$$

(b, 5 pts) Please give the x-y-coordinate of the limit point after infinitely many line segments are drawn. After 3 segments the point is $(7/18, h/3)$.

$$x_\infty = \frac{7}{18} + \frac{8}{27} \cdot \frac{7}{18} + \left(\frac{8}{27}\right)^2 \frac{7}{18} + \cdots$$

$$= \frac{7}{18} \frac{1}{1 - \frac{8}{27}} = \frac{7}{\cancel{18}_{2}} \cdot \frac{\cancel{27}^{3}}{19} = \frac{21}{38} = 0.5526$$

$$y_\infty = \frac{h}{3} + \frac{8}{27} \frac{h}{3} + \left(\frac{8}{27}\right)^2 \frac{h}{3} = \frac{h}{\cancel{3}} \frac{27}{19} = \frac{9\sqrt{3}}{38} = 0.4102$$

$$x_\infty = \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^i \cos\left((1+4i)\frac{\pi}{3}\right), \quad y_\infty = \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^i \sin\left((1+4i)\frac{\pi}{3}\right)$$