

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Tue, Nov 8, 2022

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<https://users.cs.duke.edu/~elk27/courses/DiscreteModels/Fall22/index.html> (URL)

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There are 6 problems on this exam for a total of 47 points. Please take a photo/scan of your solution of each problem done on paper (not necessarily the printed exam) and upload the problem on the Moodle course web page on wolfware.ncsu.edu.

By taking the exam, you agree that you will not consult with others about the solution. You can consult your notes/book/the Internet such as <https://www.wolframalpha.com>.

You have until 10am today, Tuesday, November 8, to upload the photos/scans. I will be on Zoom from 8:30am-9:45am to answer questions about the exam.

Good luck!

Solution Fall 2022

Problem 1 (13 points): Consider the following mathematical expression in **prefix** notation, assuming that each of the operators $+$, $-$, $*$, $/$, \uparrow has two operands (\uparrow is exponentiation).

$$\uparrow / * abc + d \uparrow e - f - gh \tag{1}$$

(a, 4pts) Please draw the expression tree for (1).

(b, 4pts) Please give both the **minimally parenthesized infix** and the **postfix** representations for the expression tree of Part (a).

INFIX (with minimum number of parentheses):

POSTFIX (operators and variables only):

(c, 5pts) Please draw the parse tree for the string $a \uparrow (b - c + d) \uparrow e / f$ using the following context-free grammar $G = (N, T, P, s)$ (from class with exponentiation), $N = \{\langle E \rangle, \langle T \rangle, \langle F \rangle, \langle B \rangle\}$; note that $\langle E \rangle$ is an expression, $\langle T \rangle$ is a term, $\langle F \rangle$ is a factor and $\langle B \rangle$ is the base for a power. The terminal symbols are $T = \{a, b, \dots, z, (,), +, -, *, /, \uparrow\}$. The start symbol is $s = \langle E \rangle$.

$$P = \{ \langle E \rangle \rightarrow \langle E \rangle + \langle T \rangle, \quad \langle T \rangle \rightarrow \langle T \rangle * \langle F \rangle, \quad \langle F \rangle \rightarrow \langle B \rangle \uparrow \langle F \rangle, \quad \langle B \rangle \rightarrow (\langle E \rangle), \\ | \langle E \rangle - \langle T \rangle, \quad | \langle T \rangle / \langle F \rangle, \quad | \langle B \rangle, \quad | a | b | \dots | z \}, \\ | \langle T \rangle, \quad | \langle F \rangle, \quad | \langle B \rangle, \quad | a | b | \dots | z \}.$$

Solution Fall 2022

Problem 2 (5 points): Please define (for a quadratic recurrence) Gaston Julia's Set J_c for $c = -2$, that is, J_{-2} . Please show that $-2 \in J_{-2}$ and $-3 \notin J_{-2}$.

Problem 3 (6 points): Consider the following graph:

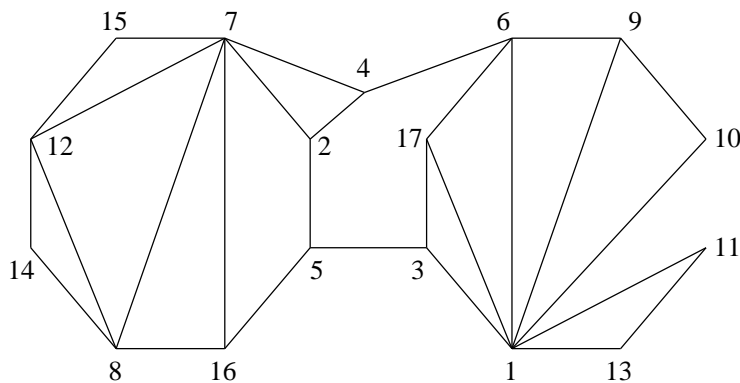


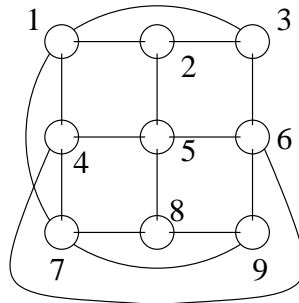
Figure 1.

(a, 4pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex **1**.

(b, 2pts) Using the DFS tree in part (a), find a one-way street assignment for the graph in Figure 1 on page 3, i.e., please orient the edges so that the resulting digraph is strongly connected. Please draw your orientation of each edge in Figure 1, using a different arrow head for those arcs that correspond to edges in the DFS tree.

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Problem 4 (8 points): Please consider the following 3×3 mesh-like graph with horizontal wrap-around edges $\{1, 3\}$, $\{4, 6\}$, $\{7, 9\}$ and the single vertical wrap-around edge $\{1, 7\}$, which causes non-planarity.



Please draw a subgraph that is homeomorphic to K_5 .

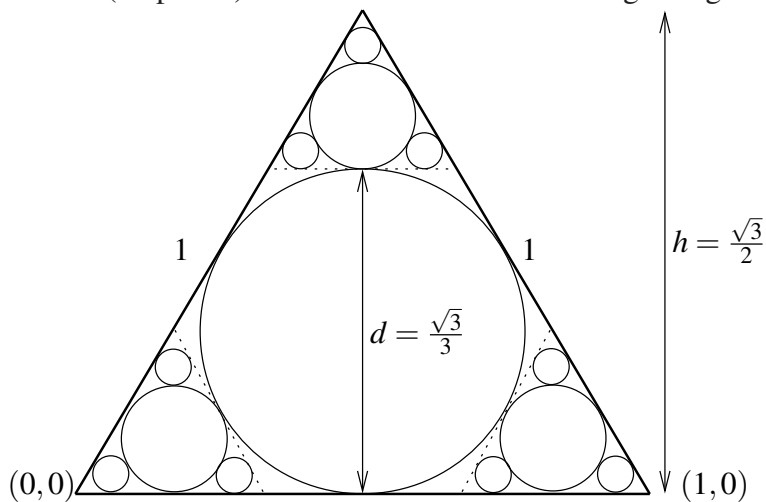
Problem 5 (5 points): Consider the following Lindenmayer system:

<i>Variables:</i>	L	M	a	m	3	5	1	#
<i>Right-sides:</i>	M3	L#ma	a	m	35	1#ma35	1	#

Please write down the first 4 new generations of strings starting with L.

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Problem 6 (10 points): Please consider the following triangular “gasket” fractal.



Here one starts with the inscribed circle of an equilateral triangle of side length 1. The dashed tangent lines parallel to the three sides cut off 3 smaller equilateral triangles with inscribed circles. The process iterates to 9 triangles which are cut off in those 3 triangles. Note that six inscribed circles touch the dashed lines and not the large circle.

- (a, 5 pts) Please compute the factor by which the smaller triangles shrink in each iteration, given the height of the first triangle and the diameter of the inscribed circle. Next, please compute the circumference of the first $1 + 3 + 9 + \dots + 3^i$ circles.
- (b, 5 pts) Please compute the remaining area of the triangle if the areas of all circles are removed up to infinity.