North Carolina State University is a land-grant university and a constituent institution of The University of North Carolina

Department of Mathematics

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Nov 4, 1999

919.515.8785 (phone)
919.515.3798 (fax)
kaltofen@math.ncsu.edu (email)
www.math.ncsu.edu/~kaltofen/courses/DiscreteModels/Fall99/ (URL)

Your Name: SOLUTION

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 4 problems, which are subdivided into 10 questions, where each question counts for the explicitly given number of points, adding to a total of **44 points**. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult **two** 8.5in $\times$ 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

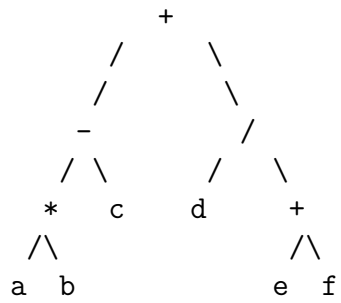
4 _____

Total _____

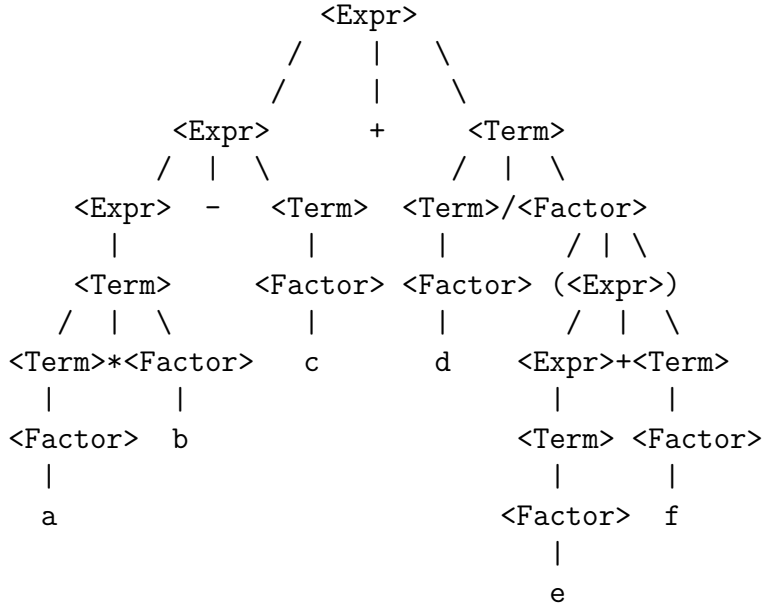
Problem 1 (14 points) Consider the following mathematical formula:

$$a \cdot b - c + d/(e + f). \tag{1}$$

(a, 5pts) Please draw an expression tree for (1) that complies with the usual operator precendence rules and left-to-right tie-breaking for operators of equal precendence.



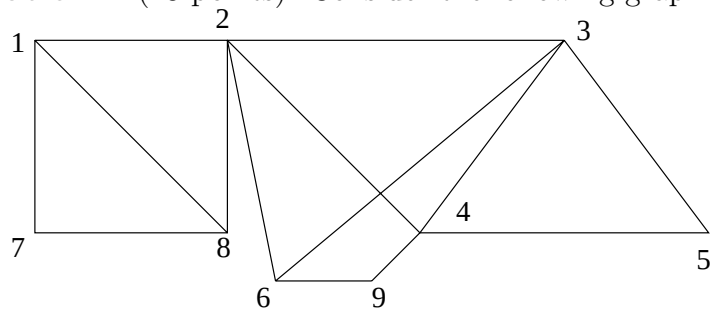
(b, 5pts) Please draw the parse tree for (1) using the context-free grammar given in class.



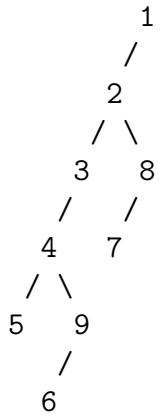
(c, 4pts) Please give a prefix string of operators and variables, but with no parentheses, that represents the tree given under part (a).

$$+ - \cdot abc/d + ef$$

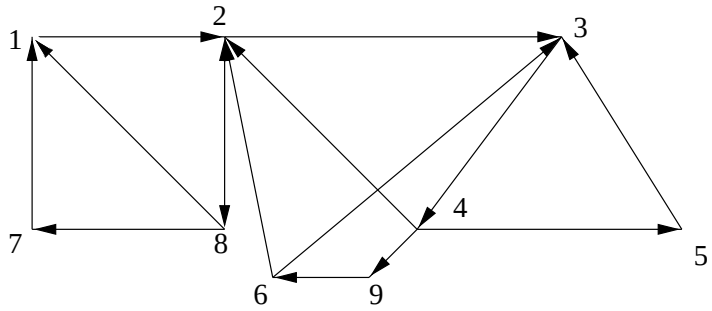
Problem 2 (13 points): Consider the following graph:



(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex in numerical order, starting at vertex 1.



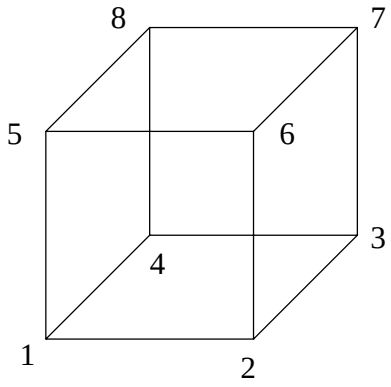
(b, 5pts) Using the tree in part (a), find a one-way street assignment for the above graph, i.e., orient the edges so that the resulting digraph is strongly connected.



(c, 3pts) What is the size of the maximum clique in the above graph? Please explain.

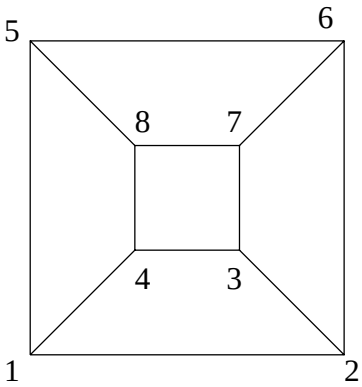
The maximum clique size is 3 (vertices 1, 2, and 8)
Explanation: There is not square with diagonals in the graph.

Problem 3 (12 points):
Consider the following graph:



(a, 4pts) Is the above "cube" graph planar? Please explain.

Yes:



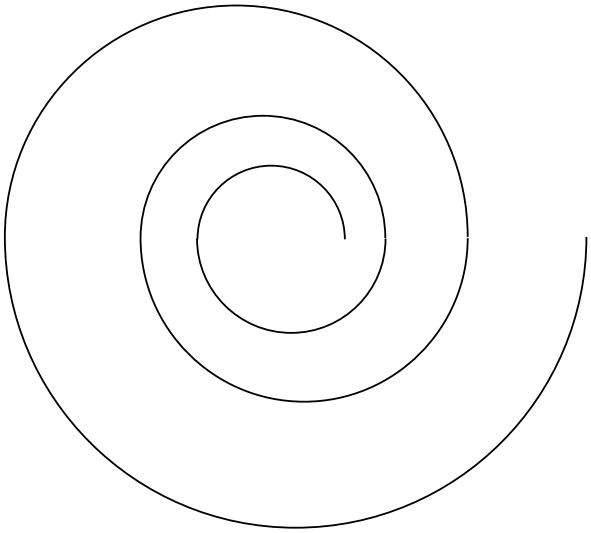
(b, 4pts) What is the chromatic number of the above "cube" graph? Please justify your answer.

$\chi = 2$*: Red vertices: 1, 6, 8, 3; green vertices: 2, 5, 4, 7*
$\chi > 1$ *because vertex 1 and 2 are adjacent and must have different colors.*

(c, 4pts) True or false: any non-planar graph must have a chromatic number that is ≥ 5. Please justify your answer.

False. $K_{3,3}$ *can be colored with 2 colors, but is not planar.*

Problem 4 (5 points): Consider the following fractal that looks like a snail.



Here you start out with a semicircle at the bottom, and then place another semicircle on the top such that the left end-points coincide and the second semicircle has a radius that is 3/4th of the first. The next semicircle is on the bottom, now sharing the right endpoint with the previous, again 3/4th in radius of the previous. And so on.

Please determine the length of the line when this fractal is drawn to infinity, assuming that the radius of the first semicircle is 1 unit in length.

$$\begin{aligned} L &= 1 \cdot \pi + \frac{3}{4}\pi + \frac{3}{4} \cdot \frac{3}{4}\pi + \cdots \\ &= \pi \cdot \sum_{i=0}^{\infty} \left(\frac{3}{4}\right)^i = \pi \frac{1}{1 - \frac{3}{4}} = 4\pi. \end{aligned}$$