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> restart; with(numtheory);
[GIgcd, bigomega, cfrac, cfracpol, cyclotomic, divisors, factorEQ, factorset, fermat, imagunit,
index, integral_basis, invcfrac, invphi, iscyclotomic, issqrfree, ithrational, jacobi, kronecker,
λ, legendre, mcombine, mersenne, migcdex, minkowski, mipolys, mlog, mobius, mroot,
msqrt, nearestp, nthconver, nthdenom, nthnumer, nthpow, order, pdexpand, φ, π, pprimroot,
primroot, quadres, rootsunity, safeprime, σ, sq2factor, sum2sqr, τ, thue, φ] (1)

> numtheory[index](3,2,11); # discrete log of 2 with prim root 2
modulo 11
8 (2)
> 2 &^ 8 mod 11; # check answer
3 (3)
> numtheory[primroot](11); # get first primitive root mod 11
2 (4)
> # check that it is prim root
for i from 1 to 10 do i, 2&^ i mod 11; od;
1, 2
2, 4
3, 8
4, 5
5, 10
6, 9
7, 7
8, 3
9, 6
10, 1 (5)
> 2*2&^7 mod 11; # sol to  $2 * x^7 \equiv 3 \pmod{11}$ 
3 (6)
> # five 5-th roots of 10 modulo 11
for i from 1 to 9 by 2 do 2 &^ i mod 11, (2 &^ i) &^ 5 mod 11;
od;
2, 10
8, 10
10, 10
7, 10
6, 10 (7)
> g13 := numtheory[primroot](13);
g13 := 2 (8)
> a := 2 &^ 9 mod 13;
a := 5 (9)
> numtheory[index](a,g13,13);
9 (10)
> 2 &^ 9 mod 13;
5 (11)
> b1:= 8; b1 &^ 3 mod 13; # first 3-rd root of 5
b1 := 8
5 (12)
> b2:=2 &^ 7 mod 13; # second 3-rd root of 5
b2 := 11 (13)
> b2 &^ 3 mod 13;

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      5
> b3:=2 &^ 11 mod 13; # third 3-rd root of 5          (14)
      b3 := 7
(15)
> b3 &^ 3 mod 13;
      5
(16)
> 10 &^ 5 mod 11; # another 5-th root of 10 modulo 11
      10
(17)
> # the 5 5-th roots of 10 modulo 11
for i from 1 to 10 by 2 do # all odd indices
  2 &^ i mod 11,(2 &^ i) &^ 5 mod 11; od;
      2,10
      8,10
      10,10
      7,10
      6,10
(18)
> # the 8 8-th roots of 16 modulo 17
for i from 1 to 16 do i,i &^ 8 mod 17; od;
      1,1
      2,1
      3,16
      4,1
      5,16
      6,16
      7,16
      8,1
      9,1
      10,16
      11,16
      12,16
      13,1
      14,16
      15,1
      16,1
(19)
> # all 11 quartic residues modulo 23
# (all residues with even index)
for i from 1 to 22 do i,i &^ 4 mod 23; od;
      1,1
      2,16
      3,12
      4,3
      5,4
      6,8
      7,9
      8,2
      9,6
      10,18
      11,13
      12,13
      13,18
      14,6
      15,2
      16,9

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17, 8
18, 4
19, 3
20, 12
21, 16
22, 1

(20)

> p := 23;
 $p := 23$

(21)

> a := 6; a &^ 11 mod 23; # Legendre symbol of 6 modulo 23
 $a := 6$
1

(22)

> b := 6 &^ ((23+1)/4) mod 23; # squareroot of 6
 $b := 12$

(23)

> b^2 mod 23;
6

(24)

> `?`
`?`

(25)

>