Due Date: April 30, 2003, 5:00pm

Problem 1: [15pts] Suppose that a Boolean expression Φ has fewer than n^k clauses, each with at least $k \log_2 n$ distinct variables. Show that Φ is satisfiable.

Problem 2: [20pts] A circuit C δ-approximates a function $f : \{0,1\}^n \to \{0,1\}$ if $|\{x \in \{0,1\}^n \mid f(x) = C(x)\}| \geq \delta 2^n$. Show that for any constant $c \geq 0$ and for sufficiently large n, there is a function f such that no circuit of size n^c $(3/4)$-approximates f.

Problem 3: [20pts] (i) Show that $\mathsf{NP} \subseteq \mathsf{coRP}$ implies that $\mathsf{NP} = \mathsf{ZPP}$.

(ii)Show that $\mathsf{NP} \subseteq \mathsf{TIME}(n^{\log n})$, then $\mathsf{PH} \subseteq \mathsf{TIME}(n^{\log^k n})$.

Problem 4: [25pts] Let $T \subseteq 1^*$. Show that if T is NP-Complete, then $\mathsf{NP} = \mathsf{P}$. (**Hint:** *If T is NP-Complete, then there exists a polynomial-time function $g : \Sigma^* \to 1^*$ such that $x \in \mathrm{SAT} \Leftrightarrow g(x) \in T$. Define the equivalence relation $x \sim y \Leftrightarrow g(x) = g(y)$. Use this relation to develop a polynomial-time algorithm for* $3\mathrm{SAT}$.)

Problem 5: [20pts] Show that the language accepted by a verifier that is only allowed to look at one bit in the proof and to use $\log n$ random bits is in P.
(**Hint:** *Show that one has to consider only a single string for the proof, and this string can be computed in polynomial time.*)

Extra Credit: Prove that it is true even if the verifier is allowed to look at two bits.

Instructions:

- You cannot discuss the exam with anyone except me.
- You can refer to the textbook, handouts, and your lecture notes. You can neither search the solutions on the internet, nor can you refer to other sources including books, journals, and conference proceedings.
- Typeset your solution using latex or word.