Due Date: April 23, 2003

Problem 1: Let $f : \{0,1\}^n \to \{0,1\}$ be a Boolean function. Show that there exists a circuit of $O(2^n/n)$ size and $O(\log n)$ depth that computes f.
(**Hint:** *Express* f *as* $f(x_1, \ldots, x_n) = \bigvee_i y_{i_1} y_{i_1} \cdots y_{i_k} f_i(x_{k+1}, \ldots, x_n)$, *choose an appropriate value of* k, *and show that the circuit for this expression requires the desired size.*)

Problem 2: Let NC^i (resp. AC^i) denote the set of languages accepted by a $O(\log^i n)$-depth, $n^{O(1)}$-size ciruit, each of whose gate has bounded (resp. unbounded) fain-in.

(i) For any $i \geq 0$, Show that $\mathsf{AC}^i \subseteq \mathsf{NC}^{i+1} \subseteq \mathsf{AC}^{i+1}$.

(ii) Prove that if $\mathsf{NC}_{i+1} = \mathsf{NC}_i$, then $\mathsf{NC} = \mathsf{NC}_i$.

Problem 3: Show that if $\mathsf{NP} \subseteq \mathsf{BPP}$ than $\mathsf{RP} = \mathsf{NP}$. (**Hint:** *Use the ideas from Example 10.3 in the textbook.*)

Problem 4: (i) Show that RP and PP are closed under intersection.

(ii) Show that $\mathsf{DP} \subseteq \mathsf{PP}$