CS 547 **Some Useful Formulas** **M. K. Vernon**

1. Stirling's Formula: $n! \approx \sqrt{2\pi n}\ \left(\frac{n}{e}\right)^n$ large n.

2. $e = \lim_{h \to 0^+} (1+h)^{\frac{1}{h}}$.

3. l'Hospital's Rule: If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

4. Maclaurin Series:

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots.$$

Thus,

(a) $\dfrac{1}{1-p} = 1 + p + p^2 + \cdots + p^n + \cdots = \sum_{i=0}^{\infty} p^i \qquad |p| < 1.$

(b) $e^x = 1 + x + \dfrac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{i=0}^{\infty} \dfrac{x^i}{i!} \qquad |x| < \infty.$

5. $\sum_{i=0}^{k} p^i = \dfrac{1-p^{k+1}}{1-p}$ [prove by induction on k].

6. $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$ [prove by induction on n]

7. Binomial Theorem: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$.

[Proof: (a) show $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

(b) prove $(x+y)^n$ by induction on n.]