How to Solve and Write Up Homework Problems

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To many students, expressing ideas in technical prose and solving problems are overwhelming enigmatic challenges. There are, however, simple rules that can guide the student through these important activities. This document offers helpful advice on how to solve and write up homework problems. Even if you do not agree with my advice, I hope you will find it beneficial to think about the issues raised in this document.

1 How to Solve Problems

There are four major steps in doing homework: getting started, carrying out the work, checking your work, and writing up and proofreading your solution. Unfortunately, many people give inadequate attention to one or more of these steps. This section offers several suggestions on how to get started and on how to check your work; Section 2 describes in detail how to write up your work. For additional hints on how to solve problems, read Polya's excellent suggestions given at the end of this handout.[1]

1.1 How to Get Started

Start early! Read over each problem on the day it is handed out so that you can ask questions about the assignment during the next class. By starting early, you will have enough time to do background reading, to digest the problems, to ask questions, and to check your work. Also, starting early avoids the anxiety of last-minute rushing and helps you deal with unexpected difficulties such as computer crashes and personal illness. It is unlikely that you will achieve good results if you wait until the last minute to begin an assignment.

Before beginning any problem, do all of the background reading and review all relevant lecture notes. Be sure to understand all terms used in the problem statement; look up each unfamiliar term. Begin by attempting to understand thoroughly what the problem is asking you to do. In your own words and notations, write down the given and what has to be done. List relevant terms, examples, concepts, theorems, and problem-solving techniques. Draw a picture. Work out an example. Formulate subgoals and list plans of attack. Finally, try out your plans, one at a time.

1.2 How to Check Your Work

Before handing in your solution, you should read, check, revise, and proofread your work. Until you have completed these tasks, you have not finished the assignment. Although many errors can be discovered through a simple checking process, many people do not adequately check their work.

One of the most effective ways to check your work is to read your work carefully and critically as if it were written by someone else. While reading your work you should ask yourself a series of

[1] G. Polya, *How to Solve It: A New Aspect of Mathematical Method*, Princeton University Press (1973), xvi–xvii.

questions, some of which are similar in spirit to those asked by programming language compilers. Whereas the correctness of an answer can often be checked immediately after solving a problem, many people find it helpful to let some time pass between writing up and proofreading a solution.

While checking your work you should ask yourself the following questions. Have you answered all questions in the problem statement? Do your answers make intuitive sense? In particular, do the signs, magnitudes, and units of your answer make sense? (For example, if in computing the mean weight of 100 adult males you arrive at the answer -52,189 miles/second, you probably have errors in the sign, magnitude, and units of your answer.) Can you verify each answer on at least two different examples? Can you verify your answer by a simple calculation (*e.g.* substituting the solution of an equation into the equation); if so, does your answer check? Have you justified every claim? Are there any gaps in your explanation? Is your solution clear and easy to read?

You should also ask yourself the following questions, which can be answered in a mechanical fashion. Is your solution expressed in complete, grammatically correct English sentences? Is each word spelled correctly? Is your solution organized into coherent paragraphs, each with a topic sentence? Are all variables, terms, and notations precisely defined? Is each free variable properly quantified? Have you specified the type of each term? Does each expression "type-check"? (For example, if f is a function from the integers to the integers, and if $S = \{1, 2, 3\}$ is a set of integers, the expression "$f \in S$" has a type error because the set S contains no functions.) Is each theorem precisely stated? For each proof, is the proof technique identified explicitly? Have you explicitly used all of the given information? If so, where have you used this information, and was your use of this information necessary? If not, is some or all of the given information unnecessary? Have you followed all of the guidelines from Section 2?

2 How to Write Up Homework

In writing up homework, it is important to consider what to write, what format to use, and for whom you are writing. In addition, it is important to pay attention to special issues of technical writing. This section addresses these concerns.

2.1 What to Write

Although you need not show all false leads, your solution should show a path of reasoning from the problem statement to the final answer. Show all work and explain all reasoning clearly. Highlight the main ideas in your solution and carefully justify your answers with detailed explanations. Explain both what you did and why you did it. Clearly state all assumptions. Where appropriate, interpret your answers. Subject to these constraints, keep your answers as brief and to-the-point as possible.

Write in a clear, understandable, and well-organized style. Be kind to your reader. Understandability is as important as correctness. Take as your audience another student in the class. Organize long solutions into meaningful sections. Where appropriate, begin each solution with a brief summary paragraph identifying the main ideas and techniques in your solution and stating the outline of your write-up.

If you cannot solve a problem, show all partial work. State what methods you tried and explain why these methods failed. Work out an example or try to solve a special case of the problem. At the very least, explain in your own words what the problem is asking, and explain how you spent your time. I do not expect that every student will be able to solve every problem.

Whenever you believe a problem is incorrectly or unclearly stated, explain why you believe so. Make reasonable assumptions, state your assumptions, and proceed.

Your solutions will be graded on the basis of what is written on the papers you hand in. Do not leave explanations up to the reader's imagination. You will be graded on the accuracy, clarity, precision, thoroughness, insightfulness, and originality of your answers.

If your solution involves any experimental work, you should include with your answer documentation of the experimental work. For example, if your solution involves any computer programming, you should include commented source listings. Often, this documentation is best supplied as a separate appendix to the written solution.

Acknowledge all help received. In particular, you should acknowledge any help received from other people, books, articles, or machines.

2.2 Format

Please adhere to each of the following guidelines, which are designed to facilitate the grading process and, more importantly, to make your work easier to understand. The instructor reserves the right not to read any work that does not meet these guidelines.

1. *Hand in your solution to each problem separately* so that the problems can be graded separately. Begin your solution to each problem on a separate sheet of paper. If your write-up to a single problem takes more than one sheet, then staple the sheets together (do not use glue, tape, paper clips, string, nor the notorious "fold-and-tear" technique). To staple two or more sheets of paper together, place one staple in the upper left corner of the sheets. Do not staple together solutions of different problems. (Note: If the problem has multiple parts, there is no need to begin your solution to each part on a separate page; you should staple your solutions to all of the parts together.)

2. Use standard-sized ($8\frac{1}{2} \times 11$ inch) white or light-colored paper. *Do not hand in paper that has been torn out of a spiral-ring notebook unless the frayed edge is first trimmed.* Do not use paper that has already been written on.

3. To make your solutions easier to read, write only on one side of each sheet.

4. *On each and every page you hand in, clearly write your name and the title of the work.* As shown in Figure 1, in the upper right corner of each page you hand in, write the following information: name, assignment number, course number, and semester. Because papers sometimes become separated, it is important to write your name and title of work on every page you hand in. Write the problem number on the top center of each page.

5. Number each page, separately numbering the pages for each problem. Specifically, in the upper right corner of each page, write p/N, where p is the current page number and N is the total number of pages handed in for the current problem.

6. Write neatly and legibly.

7. Do not write in red ink.

8. Write in a linear fashion. *Do not hand in a "mosaic" of prose and math symbols.* Where appropriate, however, figures are helpful and encouraged.[2]

9. Leave generous margins on each page (each margin should be at least 1 inch wide). Moreover, let your solution "breathe" by leaving enough space between lines, figures, and equations. Single spacing is fine, but do not crowd your solutions.

10. Clearly identify your final answers.

11. Do not turn in excessively long solutions unless there is compelling reason to do so. Most problems can be solved in one or two pages. The instructor reserves the right to stop reading after the first five pages of any solution.

2.3 General Advice on Writing

In most cases, whatever makes for good expository writing also makes for good technical writing. Please adhere to the following important general rules.

1. *Write in complete, grammatically correct English sentences.* Begin each sentence with a capital letter. End each sentence with an appropriate punctuation mark. Each sentence must have a subject and verb, and the verb must agree with the subject.

2. *Organize your writing into coherent paragraphs, each with a topic sentence.* Each paragraph should have exactly one main idea, and this idea should be summarized in the topic sentence.

3. Omit needless words.

4. Use active verbs and avoid the passive voice. For example, instead of the passive "The graphs *were drawn by* Acme Arts," write "Acme Arts *drew* the graphs."

5. Avoid awkward noun phrases: utilization of nounification is abominable pontification.

6. Avoid contractions in formal writing: do not write "don't."

7. *To unnecessarily split* an infinitive is *to split* an infinitive *unnecessarily.*

8. Effective *writers place* verbs near their subjects *intentionally*; weak *writers unnecessarily separate* them.

9. To end any sentence with a preposition is a heinous crime to engage *in.* (better: "... crime *in which* to engage.")

2.4 Specific Advice on Technical Writing

In addition to general principals of effective writing, there are special concerns of technical writing. These concerns include writing in a clear and precise fashion, using appropriate punctuation and typography, and adopting an effective technical style. Strive to make it easy for your reader to understand your message by adhering to the following special guidelines.

[2]This rule is not intended to exclude concept maps, hypertext, or other acceptable forms of text intentionally designed to be nonlinear.

2.4.1 Be Clear, Precise, and Informative

1. Write in simple and reasonably short sentences. Do not try to say too much in any one sentence.

2. Whenever possible, use simple and easily recognized words. Sesquipedalianism fecundates catachresis, not pellucidity. For example, some authors might try to impress their readers by writing, "This research will fill the *lacuna* in our understanding of real-time systems." In this example, the same thought could be expressed more clearly using the easily recognized word "gap" instead of the pretentious and less familiar word "lacuna." Choose words that express exactly what you want to say, but do not put your vocabulary on display.

3. Define each term and symbol; each definition should include a type specification. For example, do not leave it up to the reader's imagination whether you mean "for all *complex numbers* x" or "for all *positive odd integers* x."

4. Quantify each free variable. For example, do not write "for x", which leaves it up to the reader's imagination whether you intend the free variable x to be universally or existentially quantified; instead, write "for *each* integer x" or "for *some* integer x".

5. Do not use vague or content-free language. For example, avoid describing something as "good" unless you explain in what sense it is good. Also, eliminate the phrases "it is obvious that" and "clearly", or replace them with concrete explanations.

6. Avoid "weasel words" such as "might," "may," "maybe," and "perhaps." For example, do not write: "Probably these data seem to suggest that smoking might be a possible risk factor for sometimes developing cancer." Instead, write "These data indicate that smoking is a risk factor for developing cancer"—provided your data support this conclusion. Even when describing experimental uncertainty, effective technical writing has clear, exact, and persuasive logic.

7. Make antecedents explicit. Instead of writing "*This* will work," write "*This approach* will work."

8. Be informative. For example, instead of "The digits of π were calculated to many places," write, "In 1968, George Foo calculated the first 1 million digits of π using an IBM 360 computer [give reference]."

9. Use the conjunction "that" to introduce clauses that are restrictive or definitional; use "which" to introduce nonrestrictive clauses, which simply add additional information.

10. To make your writing easy to understand, whenever using words with multiple commmon meanings, use such words only for their most common meanings. For example, do not use the word "while" in the sense of "although," and do not use the word "as" in the sense of "because." Similarly, avoid phrases with multiple common meanings. For example, avoid the phrase "in general" because it can mean either "always" or "usually."

2.4.2 Use Appropriate Punctuation and Typography

1. Number every "display equation" you refer to, as is done in Equation 1. I prefer to number *every* equation, so that the reader can refer to every equation, including those to which the author does not refer.

   Everyone should learn Stirling's approximation

   $$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \left(1 + O\left(\frac{1}{n}\right)\right), \tag{1}$$

   which gives a useful way to approximate $n!$ for any positive integer n.[3]

2. Give a number and a title to each table, graph, and figure. Refer to display equations, tables, graphs, and figures by their numbers. Clearly label all important parts of all tables, graphs, and figures—these important parts include table headings and graph axes.

3. To make it easier for the reader to determine where each sentence begins, do not begin any sentence with a math symbol. This rule is especially important when the preceding sentence ends with a math symbol.

4. If a sentence includes an equation, add punctuation to the end of the equation when appropriate (see Equation 1).

5. For clarity, write a zero in front of all decimal expressions. Thus, write "0.35" rather than ".35".

2.4.3 Adopt an Effective Technical Style

1. Use meaningful symbol names, and, whenever possible, perform intermediate calculations with these meaningful symbols rather than with actual numbers. Substitute actual numbers for these symbols only in the last step. Following this rule simplifies algebra, makes your solution easier to understand and check, and renders your solution more general and more meaningful.

2. When to spell out phrases in English and when to use math symbols and abbreviations is a delicate matter of style. Where appropriate, use math symbols and abbreviations. However, avoid using awkward symbols when words would make the sentence more readable, and avoid using awkward words when symbols would be clearer. For example, instead of "$\exists x$ s.t. x squared is greater than twenty," write "There exists x such that $x^2 > 20$." Also keep the following guidelines in mind. Use standard symbols. Avoid inconsistent mixtures of prose and math symbols. Remember to write in complete sentences; math symbols are simply abbreviations for words and should be used accordingly (every sentence with a math symbol must still have a subject and verb).

3. Do not begin any abstract with the boring phrase "This paper ...."

[3] William Feller, *An Introduction to Probability Theory and its Applications*, vol. I, John Wiley (New York, 1968), 52–54.

4. As a check of the clarity and smoothness of your writing, try reading your work replacing each math symbol and equation with "bla" or your favorite grunting sound—the resulting utterances should be pleasing and understandable. One tool for achieving this goal is to add redundancy, where appropriate, by preceding each math symbol with its type. For example, instead of writing "Gauss approximated f by L" it is clearer to write "Gauss approximated the nonlinear function f by the linear transformation L."

5. Clearly isolate each technical argument from the higher level explanations, and summarize each technical argument intuitively. For example, before (and after) each theorem or lengthy calculation, intuitively summarize what the theorem or calculation says and explain how the theorem or calculation fits into the rest of the work. Highlight the main idea of the theorem or calculation. One way to isolate a technical argument is to put it in a separate paragraph; for theorems, use the delimiting words and symbol "Theorem", "Proof", and " $\Box$ ", as is illustrated below.

   Theorem. *All horses are black.*
   Proof (by induction on the number of horses N).
   $\langle$Proof goes here.$\rangle$ Therefore, all horses are black. $\Box$

   For each proof, help the reader by explicitly identifying the proof technique. Following these rules enables the reader to skip detailed technical arguments without losing the main ideas.

6. One controversial issue that often arises is whether or not to use the pronouns "we" and "I". Although this issue is largely a matter of style which you must decide for yourself, there are some guidelines to follow.

   First, avoid the pronouns "we" and "I" whenever they can can be avoided without harm. For example, instead of writing "I believe that blue laws are unconstitutional," it is better to write, "Blue laws are unconstitutional." The phrase "I believe" is unnecessary because it is implicit that the statement comes from the author. One exception to this rule is when the author of a textbook states a personal opinion that he does not want interpreted as fact.

   Second, there are legitimate circumstances in which the author needs to describe something that the author did. In these circumstances, use "I" if there is exactly one author and "we" if there are two or more authors. For example, the author might write, "I proved Foo's theorem in 1964." Alternately, some authors refer to themselves by their last name or by the phrases "this author" or "the first author"; I prefer "I". However, for multi-authored documents in which it is necessary to refer to individual authors (such as in grant proposals), I prefer to refer to individual authors in the third person using last names.

   Third, many authors use the pronoun "we" to avoid the passive voice. For example, instead of using the passive construction "It is proven that," some authors write the active "We prove that" or the anthropomorphic "Chapter 1 proves that." Some authors use the pronoun "we" in the sense of "we the authors" (this usage is illogical when there is only one author); others use this pronoun in the sense of "we the author(s) and the reader" (I dislike this usage because it suggests that the reader took part in the writing). When the offending sentence cannot be conveniently reworded in the active voice to avoid using "we", I prefer to use the active voice over the passive voice. When there is exactly one author, however, I prefer to use "I" rather than "we". Unfortunately, many journals disallow the use of "I" for this purpose, and some journals eliminate all occurrences of "we", even at the cost of using the passive voice.

1/2

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CMSC–441 Fall 90
Assignment 6

Problem 3

a) ⟨Solution to part a goes here.⟩

b) ⟨Solution to part b goes here.⟩

c) ⟨Solution to part c goes here.⟩

Figure 1: Sample homework page.