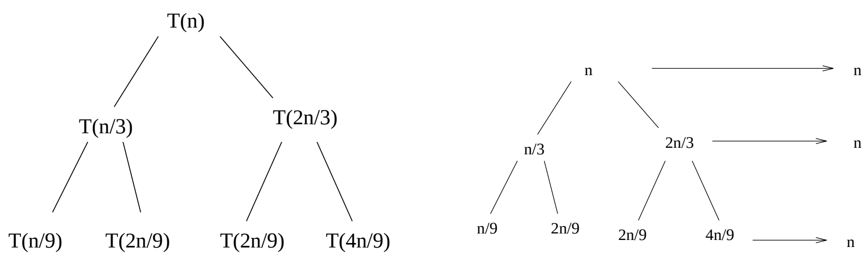
4.2-2 Argue the solution to

$$T(n) = T(n/3) + T(2n/3) + n$$

is $\Omega(n \lg n)$ by appealing to the recursion tree.

Draw the recursion tree.



How many levels does the tree have? This is equal to the longest path from the root to a leaf.

The shortest path to a leaf occurs when we take the heavy branch each time. The height k is given by $n(1/3)^k \leq 1$, meaning $n \leq 3^k$ or $k \geq \lg_3 n$.

The longest path to a leaf occurs when we take the light branch each time. The height k is given by $n(2/3)^k \leq 1$, meaning $n \leq (3/2)^k$ or $k \geq \lg_{3/2} n$.

The problem asks to show that $T(n) = \Omega(n \lg n)$, meaning we are looking for a lower bound

On any *full* level, the additive terms sums to n. There are $\log_3 n$ full levels. Thus $T(n) \geq n \log_3 n = \Omega(n \lg n)$

4.2-4 Use iteration to solve where $a \geq 1$ is a constant.

Note iteration is backsubstitution.

$$\begin{aligned}
T(n) &= T(n-a) + T(a) + n \\
&= (\mathrm{T(n-2a) + T(a) + n - a) + T(a) + n} \\
&= (\mathrm{T(n-3a) + T(a) + n - 2a) + 2T(a) + 2n - 3a} \\
&\vdots \\
&\approx \sum_{i=0}^{n/a} T(a) + \sum_{i=0}^{n/a} n - ia \\
&\approx (n/a)T(a) + \sum_{i=0}^{n/a} n - a \sum_{i=0}^{n/a} i \\
&\approx (n/a)T(a) + n \sum_{i=0}^{n/a} 1 - a \sum_{i=0}^{n/a} i \\
&\approx (n/a)T(a) + n(n/a) - a(n/a)^2/2
\end{aligned}$$