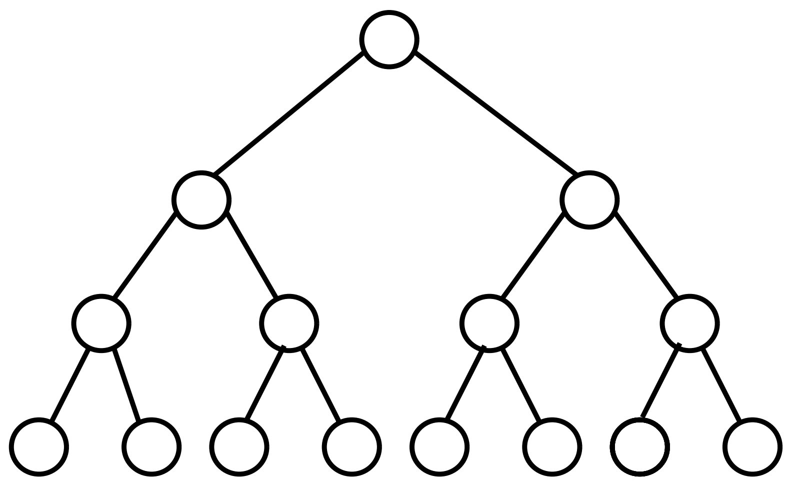
7.1-2: Show that an n-element heap has height $\lfloor \lg n \rfloor$.

Since it is balanced binary tree, the height of a heap is clearly $O(\lg n)$, but the problem asks for an exact answer.

The height is defined as the number of edges in the longest simple path from the root.



The number of nodes in a complete balanced binary tree of height h is $2^{h+1} - 1$.

Thus the height increases only when $n = 2^{\lg n}$, or in other words when $\lg n$ is an integer.

7.1-5 Is a reverse sorted array a heap?

In a heap, each element is greater than or equal to each of its descendants.

In the array representation of a heap, the descendants of the ith element are the $2i$th and $(2i+1)$th elements.

If A is sorted in reverse order, then $A[i] \geq A[j]$ if $i \leq j$.

Since $2i > i$ and $2i + 1 > i$ then $A[2i] \leq A[i]$ and $A[2i + 1] \leq A[i]$.

Thus by definition A is a heap!