Quantum Mechanics 812

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Theoretical atomic physics

Parity nonconservation in atoms
High-precision calculations of atomic properties

Quantum computation

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Quantum Mechanics 812

Course will introduce both fundamental concepts and techniques of quantum mechanics and demonstrate their relevance to real-life modern applications
Goals of the course

- Learn fundamentals of quantum mechanics
- Learn how quantum mechanics is relevant to research in various fields and today’s technology
- Aid in student’s research or in selection of field research
- Aid in preparation to qualifying exam
- ...

Learning

The objective of the course is not to cover certain set of topics but to provide a base of fundamentals concepts and skills as well as to demonstrate examples of their applications which will facilitate further interest, learning, and thinking.
Syllabus. Part 1: review

The hydrogenic atom

Review

One-valence electron systems

Applications

Special hydrogenic systems: positronium, muonium, antihydrogen, muonic and hadronic atoms; Rydberg atoms

Doping of semiconductors

Syllabus. Part 1: review

Physical symmetries & conservation laws

Angular momentum and the addition of angular momenta; 3j and 6j symbols and their use
Syllabus. Part 1-2: review and “new” topics

Identical particles & second quantization

Applications
  Phonons (lattice vibrations)
  Plasmons (collective electron vibrations)
  Magnons (spin waves)

Classical and quantum statistics
  Metals, insulators, and semiconductors
  Normal and superconducting states
  Ordinary and superfluid liquid helium

Syllabus. Part 1-2: review and “new” topics

Approximation methods

Time-independent problems
Ammonia molecules & masers
Atomic clock

Time-dependent problems

Radiative transitions & how to calculate atomic transition properties
Syllabus. Part 2
More fundamental topics

Collisions and scattering

Relativistic quantum mechanics

- Dirac equation
- Relativity and fine structure
- Lamb shift

Self-consistent fields

- Hartree-Fock equations
- Atoms with one and two valence electrons

Syllabus. Part 3
Applications of quantum mechanics

- Quantum dots and artificial atoms
- Masers and lasers
- Atomic clocks & their applications
- Laser cooling and trapping
- Bose-Einstein condensation
- Atom lasers
- Josephson junctions
- Magnetic effects
- Quantum communication & quantum cryptography
- Quantum computation
- Atom interferometry and its applications
- Decay of K-mesons (CP violation)
- Parity violation in atoms
Syllabus. Part 4

Measurement and interpretation

Hidden variables?
The Einstein-Podolsky-Rosen paradox
Bell’s theorem and its consequences
The problem of measurement

Textbooks

- *Quantum Mechanics: Fundamentals & Applications to Technology*, by Jasprit Singh
- *Lecture notes on Atomic Physics*, by Walter Johnson; available online at [http://www.nd.edu/~johnson](http://www.nd.edu/~johnson) under “Unpublished material”.

A number of books are ON RESERVE at the Physics Library
Homework

- Homework is assigned ones a week, on Thursday
- It is due in one week, next Thursday
- Exception: week of the Thanksgiving
- Late homework policy: it is best to always return it **on time**
- **MAXIMUM** number of late homeworks: 3
  (no more than a **week** late)
- No explanation or notification is needed
- Homework will be graded
- For certain topics an alternative (more research-like)
  assignment may be given

Quizzes

- Occasional quizzes will be given throughout the semester
- They will not be graded and will not affect the grade
- They are, however, required
- The purpose of the quizzes is to access the familiarity
  of the class with specific topics and the progress of the course
- Student input about the course is always very welcome
  (too fast, too slow, too elementary, too complicated, etc.)
Active listening

• Students are extremely encouraged to ask questions
• There will be a five minute break around the middle of the lecture to think about questions and to discuss them with other students before asking if needed
• Summary of the material will be given throughout the lecture, especially at the end of the topic
• Conclusion of each lecture will be given and main points will be summarized
• Ideas to facilitate active listening are very welcome

Exams and grades

• One midterm exam (open book): October 23
• Final exam
• The total grade is determined from the homework, midterm and final exam grades.
• Homework 50%
• Midterm 20%
• Final 30%
**Quantum Computing**

**Quantum teleportation**

**Bits & Qubits**

- **Fundamental building blocks of classical computers:**
  - **BITS**

<table>
<thead>
<tr>
<th>STATE: Definitely</th>
<th>0 or 1</th>
</tr>
</thead>
</table>

- **Fundamental building blocks of quantum computers:**
  - **Quantum bits**
  - **or**
  - **Qubits**

Basis states: $|0\rangle$ and $|1\rangle$

- **Superposition:**

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
**Bits & Qubits:**

**Primary Differences**

- **Superposition**
  \[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]

- **Measurement**
  - Classical bit: we can find out if it is in state 0 or 1 and the measurement will **not** change the state of the bit.
  - Qubit: we cannot just measure \( \alpha \) and \( \beta \) and thus determine its state! We get either \( |0\rangle \) or \( |1\rangle \) with corresponding probabilities \( |\alpha|^2 \) and \( |\beta|^2 \).
  \[ |\alpha|^2 + |\beta|^2 = 1 \]
  - The measurement **changes** the state of the qubit!

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**Hilbert space is a big place!**

- Carlton Caves

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**Multiple qubits**

- Two bits with states 0 and 1 form four definite states 00, 01, 10, and 11.
- Two qubits: can be in superposition of four computational basis set states.
  \[ |\psi\rangle = \alpha |00\rangle + \beta |01\rangle + \gamma |10\rangle + \delta |11\rangle \]

<table>
<thead>
<tr>
<th>Number of Qubits</th>
<th>Amplitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 qubits</td>
<td>4</td>
</tr>
<tr>
<td>3 qubits</td>
<td>8</td>
</tr>
<tr>
<td>10 qubits</td>
<td>1024</td>
</tr>
<tr>
<td>20 qubits</td>
<td>1 048 576</td>
</tr>
<tr>
<td>30 qubits</td>
<td>1 073 741 824</td>
</tr>
<tr>
<td><strong>500 qubits</strong></td>
<td>More amplitudes than our estimate of number of atoms in the Universe!!!</td>
</tr>
</tbody>
</table>
**Entanglement**

\[ |\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \]

- Bell state
- or EPR state/pair

\[ |\psi\rangle \neq |\alpha\rangle \otimes |\beta\rangle \rightarrow \text{Entangled states} \]

**Results of measurement**

<table>
<thead>
<tr>
<th></th>
<th>First qubit</th>
<th>Second qubit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**Logic gates**

### Classical NOT gate

<table>
<thead>
<tr>
<th>A</th>
<th>NOT A</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

- The only non-trivial single bit gate

### Quantum NOT gate (X gate)

\[ |\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow X \rightarrow \alpha|1\rangle + \beta|0\rangle \]

**Matrix form representation**

\[
X = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]

\[
X \begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} = \begin{bmatrix}
\beta \\
\alpha
\end{bmatrix}
\]
More single qubit gates

Any unitary matrix U will produce a quantum gate!

\[
Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \alpha |0\rangle + \beta |1\rangle \quad Z \quad \alpha |0\rangle - \beta |1\rangle
\]

Hadamard gate:

\[
H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
\]

\[
\alpha |0\rangle + \beta |1\rangle \quad H \quad \frac{\alpha |0\rangle + |1\rangle}{\sqrt{2}} + \frac{\beta |0\rangle - |1\rangle}{\sqrt{2}}
\]

Two-bit/qubit gates

Classical AND gate

Quantum CNOT gate

<table>
<thead>
<tr>
<th>Classical AND gate</th>
<th>Quantum CNOT gate</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A \text{ AND } B</td>
</tr>
<tr>
<td>B</td>
<td></td>
</tr>
</tbody>
</table>
| \begin{tabular}{c|c|c}
0 & 0 & 0 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
1 & 1 & 1 \\
\end{tabular} | \begin{tabular}{c|c}
|A\rangle & |A\rangle \\
|B\rangle & |B'\rangle \\
|AB\rangle & |AB'\rangle \\
|00\rangle & |00\rangle \\
|01\rangle & |01\rangle \\
|10\rangle & |11\rangle \\
|11\rangle & |10\rangle \\
\end{tabular} |

Irreversible! \quad \text{reversible!}
From gates to circuits

Example: swap circuit

Differences with classical circuits

- No loops - no feedback from one part of circuit to another.
- No wires joined together since it is not reversible.
- No “copy a qubit” operation (forbidden by quantum mechanics).

Quantum teleportation

What is it?

Technique for moving quantum states around, even in the absence of quantum communication channel.
The problem

\[ |\beta_{\infty}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \]

- Alice and Bob generated an EPR pair together.
- They moved to different places and each took one qubit of the EPR pair.
- Alice must deliver qubit \( |\psi\rangle \) to Bob
  - She does not know the state of the qubit
  - She can use only classical channels

How does it work?

\[ |\beta_{\infty}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \]

- Alice interacts qubit \( |\psi\rangle \) with half of her EPR pair and then makes a measurement on two qubits which she has.
- She can get one out of four possible results: 00, 01, 10, and 11
- Alice reports this information to Bob
- Bob performs one of four operations on his half of the EPR pair
- Amazingly, he can recover the original state \( |\psi\rangle \)!
Teleportation scheme \( |\beta_w\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \)

Alice
\[ |\psi\rangle \]
Bob
\[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]

\[ |\psi_0\rangle \]
\[ |\psi_1\rangle \]
\[ |\psi_2\rangle \]
\[ |\psi_3\rangle \]
\[ |\psi_4\rangle \]

\[ X_{M_1} |\psi\rangle \]
\[ Z_{M_1} |\psi\rangle \]

Teleportation scheme \( |\beta_w\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \)

\[ |\psi\rangle \]
\[ |\beta_w\rangle \]

\[ |\psi_0\rangle = |\psi\rangle |\beta_w\rangle = \frac{1}{\sqrt{2}} \left[ \alpha |0\rangle (|00\rangle + |11\rangle) + \beta |1\rangle (|00\rangle + |11\rangle) \right] \]

\[ = \frac{1}{\sqrt{2}} \left[ \alpha |000\rangle + \alpha |011\rangle + \beta |100\rangle + \beta |111\rangle \right] \]
Teleportation scheme

\[ |\psi_o\rangle = \frac{1}{\sqrt{2}} \left[ \alpha |000\rangle + \alpha |011\rangle + \beta |100\rangle + \beta |111\rangle \right] \]

\[ |\psi_i\rangle = \frac{1}{\sqrt{2}} \left[ \alpha |000\rangle + \alpha |011\rangle + \beta |110\rangle + \beta |101\rangle \right] \]

| Teleportation scheme |

\[ |\psi_2\rangle = \frac{1}{2} \left[ (00)(\alpha |0\rangle + \beta |1\rangle) + (01)(\alpha |1\rangle + \beta |0\rangle) + (10)(\alpha |0\rangle - \beta |1\rangle) + (11)(\alpha |1\rangle - \beta |0\rangle) \right] \]
\[ \alpha |0\rangle + \beta |1\rangle \rightarrow X \rightarrow \alpha |1\rangle + \beta |0\rangle \]

**Teleportation scheme**

\[ \alpha |0\rangle + \beta |1\rangle \rightarrow Z \rightarrow \alpha |0\rangle - \beta |1\rangle \]

\[ |\psi\rangle \rightarrow Z \rightarrow X \rightarrow H \rightarrow |\psi\rangle \]

\[ |\psi\rangle \rightarrow |\psi\rangle \rightarrow |\psi\rangle \rightarrow |\psi\rangle \rightarrow |\psi\rangle \]

\[ |00\rangle \rightarrow (\alpha |0\rangle + \beta |1\rangle) \quad |01\rangle \rightarrow (\alpha |0\rangle + \beta |1\rangle) \quad [X] \]

\[ |10\rangle \rightarrow (\alpha |0\rangle - \beta |1\rangle) \quad [Z] \quad |11\rangle \rightarrow (\alpha |1\rangle - \beta |0\rangle) \quad [XZ] \]