Umans
Complexity Theory Lectures

Lecture 18: Course Review

Course summary

- Time and space
  - hierarchy theorems
  - FVAL in **L**
  - CVAL **P**-complete
  - QSAT **PSPACE**-complete
  - succinct CVAL **EXP**-complete

Course summary

- Non-determinism
  - NTIME hierarchy theorem
  - "NP-intermediate" problems (Ladner's Theorem)
  - unary languages (likely) not NP-complete
  - Savitch's Theorem
  - Immerman-Szelepcsényi Theorem
  - *sparse* languages (likely) not NP-complete

Course summary

- Non-uniformity
  - formula lower bound (Andreev, Hastad)
  - monotone circuit lower bound (Razborov)
  - Barrington's Theorem
  - formula lower bound for parity

Course summary

- Randomness
  - polynomial identity testing + Schwartz-Zippel
  - unique-SAT (Valiant-Vazirani Theorem)
  - Blum-Micali-Yao PRG
  - Nisan-Wigderson PRG
  - worst-case hardness) average-case hardness
  - Trevisan extractor
  - Goldreich-Levin hard bit

Course summary

- Alternation
  - $QSAT_i$ complete for levels of the **PH**
  - Karp-Lipton theorem
  - **BPP** in **PH**
  - approximate counting + sampling with an **NP**-oracle
  - **VC**-dimension is Σ_3-complete
  - the class S_2^P (final)

Course summary

- Counting
  - #matching is **#P**-complete
  - permanent is **#P**-complete
  - Toda's theorem: **PH** $\subseteq$ $\mathbf{P}^{\#\mathbf{P}}$

Course summary

- Interaction
  - **IP** = **PSPACE**
  - GI in **NP** $\cap$ **coAM**
  - using NW PRG for **MA**, variant for **AM**
  - hardness of approximation , PCPs
  - elements of the PCP theorem
  - BLR linearity test
  - Clique hard to approximate to within N^{δ}

Course summary

- Barriers to progress
  - oracles rule out relativizing proofs
  - "natural proofs" rule out many circuit lower bound techniques

Course summary

- Time and space **L, P, PSPACE, EXP**
- Non-determinism **NL, NP, coNP, NEXP**
- Non-uniformity **NC, P/poly**
- Randomness **RL, ZPP, RP, coRP, BPP**
- Alternation **PH, PSPACE**
- Counting **#P**
- Interaction **IP, MA, AM, PCP[log n, 1]**

The big picture

- All classes on previous slide are probably distinct, except:
  - **P, ZPP, RP, coRP, BPP** (probably all equal)
  - **L, RL** (probably all equal; **NL**?)
  - **NP, MA, AM** (probably all equal)
  - **IP = PSPACE**
  - **PCP[log n, 1] = NP**
- Only real separations we know separate classes delimiting same resource:
  - e.g. **L ≠ PSPACE, NP ≠ NEXP**

The big picture

Remember:

possible explanation for failure to prove conjectured separations…

…is that they are false

The big picture

- Important techniques/ideas:
  - simulation and diagonalization
  - reductions and completeness
  - self-reducibility
  - encoding information using low-degree polynomials
  - randomness
  - others…

The big picture

- I hope you take away:
  - an ability to extract the essential features of a problem that make it hard/easy…
  - knowledge and tools to connect computational problems you encounter with larger questions in complexity
  - background needed to understand current research in this area

The big picture

  - background to *contribute* to current research in this area
    - many open problems
    - young field
    - try your hand…