Umans Complexity Theory Lectures

Lecture 1c: Turing Machine Halting Problem

Halting Problem for Turing Machines

- Amazing fact: there exist (natural) **undecidable** problems

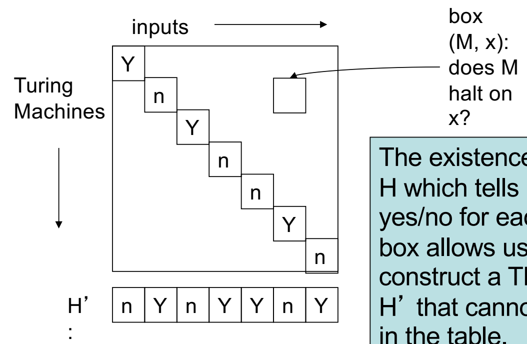
HALT = { (M, x) : M halts on input x }

- Theorem: HALT is undecidable.

Turing Machines

- Proof:
  - Suppose TM H decides HALT
  - Define new TM H’: on input <M>
    - if H accepts (M, <M>) then loop
    - if H rejects (M, <M>) then halt
  - Consider H’ on input <H’>:
    - if it halts, then H rejects (H’, <H’>), which implies it cannot halt
    - if it loops, then H accepts (H’, <H’>) which implies it must halt
  - contradiction.

Diagonalization



The existence of H which tells us yes/no for each box allows us to construct a TM H’ that cannot be in the table.