# Umans Complexity Theory Lectures

## Lecture 3b: Nondeterminism: NTIME Hierarchy Theorem

# NTIME Hierarchy Theorem

<u>**Theorem**</u> (Nondeterministic Time Hierarchy Theorem): For every *proper complexity functions* $T(n) \geq n$, and $T'(n)$, where

$$T(n+1)=o(T'(n))$$

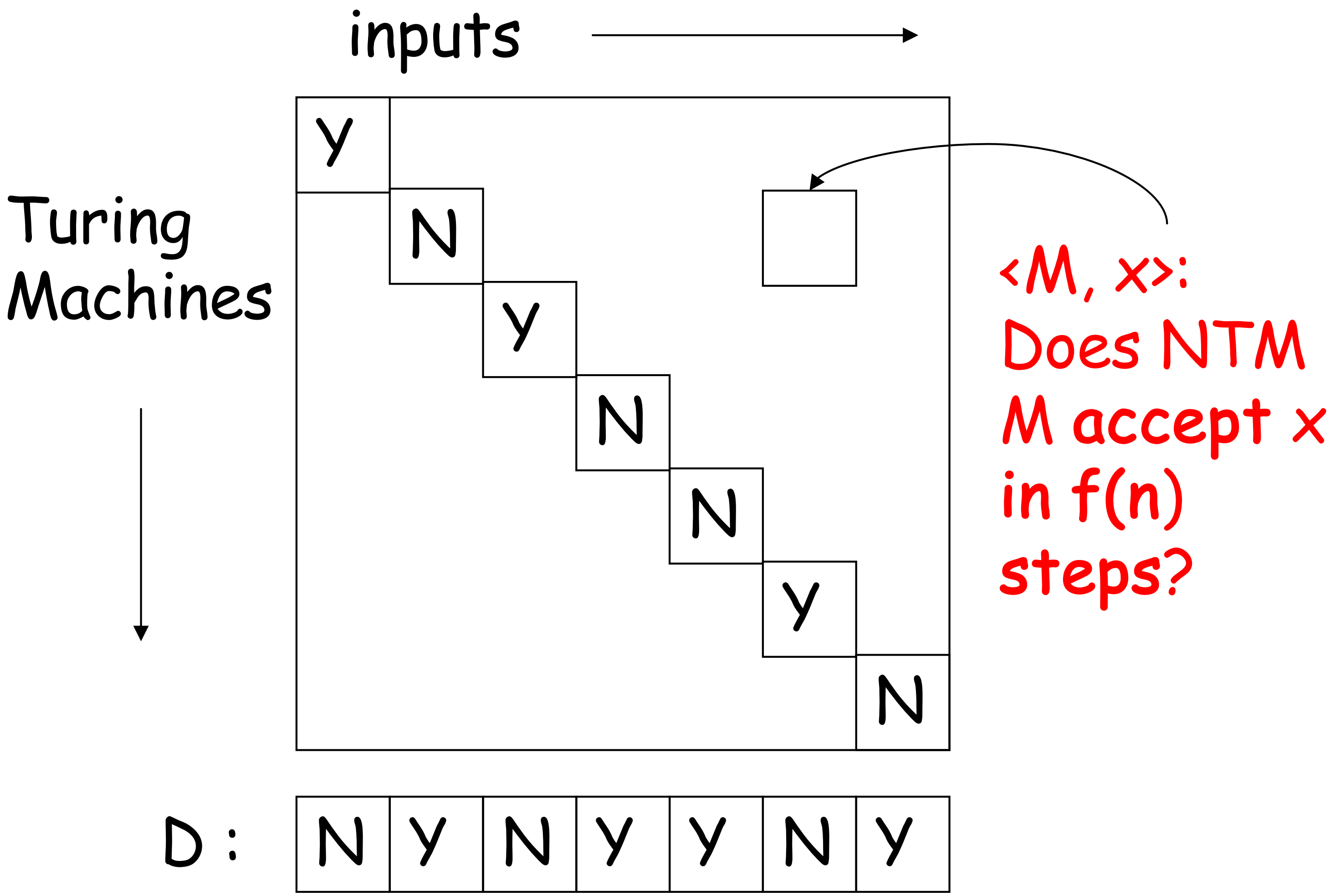
(so $\lim T(n+1)/ T'(n) \rightarrow 0$ as $n \rightarrow \infty$),

we have:

$$\mathbf{NTIME(T(n))} \subsetneq \mathbf{NTIME(T'(n))}.$$

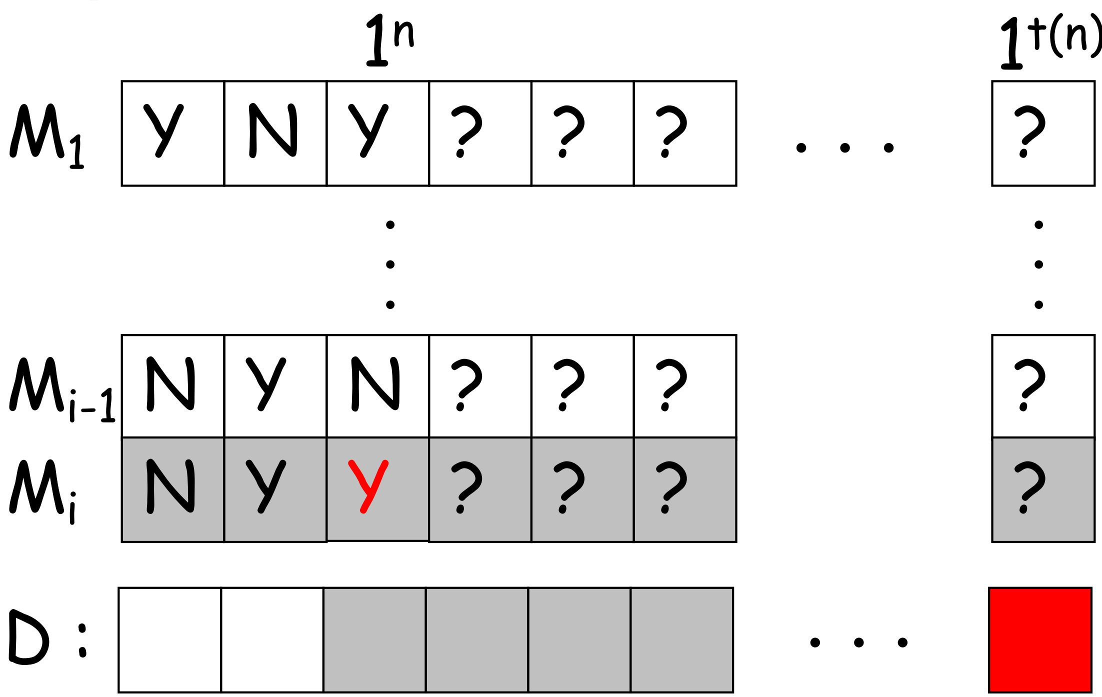
# NTIME Hierarchy Theorem

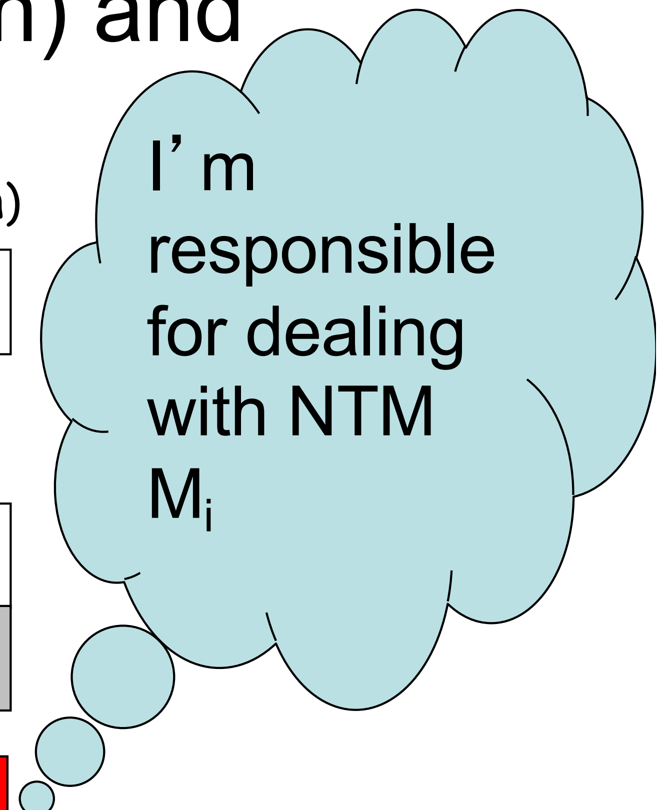
Proof attempt:



# NTIME Hierarchy Theorem

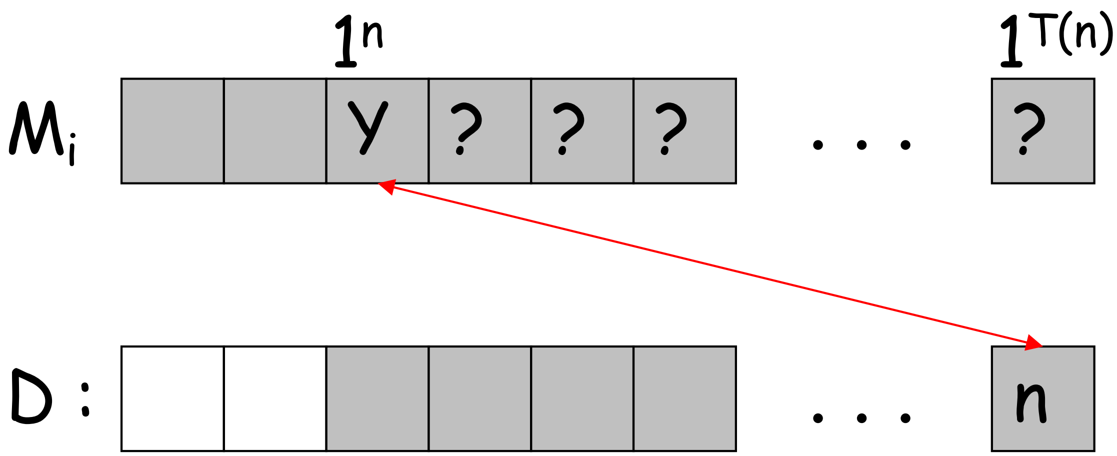
- Let T(n) be large enough so that a nondeterministic TM D can decide in time T(n) if NTM $M_i$ runs in time T(n) and accepts $1^n$.



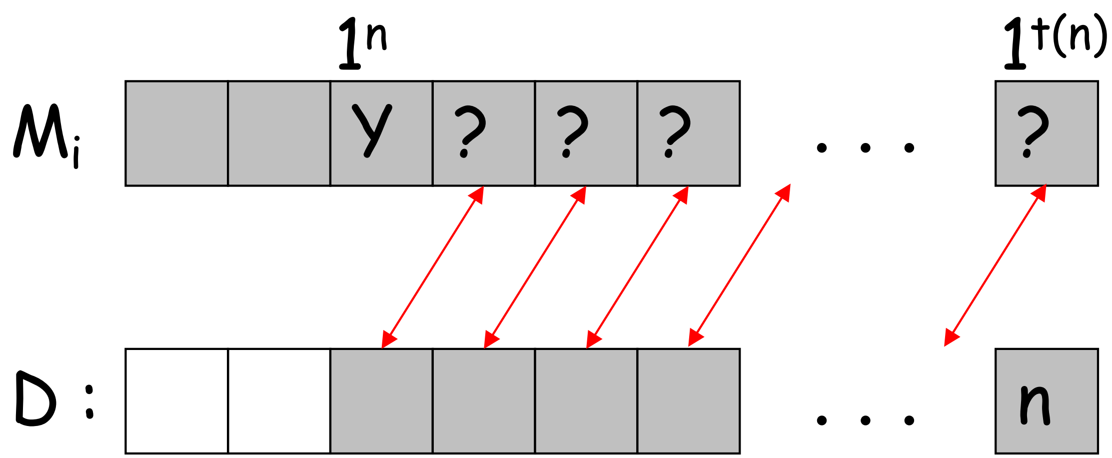
# NTIME Hierarchy Theorem

- Need T(n) to grow fast enough (exponential in n) so have enough time on input $1^{T(n)}$ to do the *opposite* of $M_i(1^n)$:



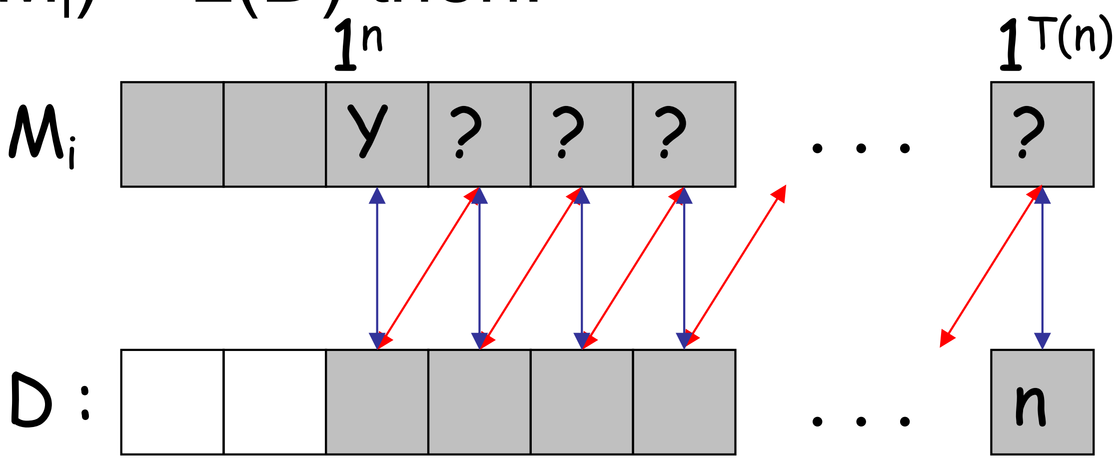
# NTIME Hierarchy Theorem

- For k in [n,n+1,…,T(n)] can to do *same* as $M_i(1^{k+1})$ on input $1^k$



# NTIME Hierarchy Theorem

- Machine D can be used to diagonalize against $M_i$:
  - if $L(M_i) = L(D)$ then:




  - equality along all arrows.
  - contradiction.

# NTIME Hierarchy Theorem

- General scheme:
  - interval [1...T(1)] kills $M_1$
  - interval [T(1)… T(T(1))] kills $M_2$
  - interval [$T^{i-1}(1)$… $T^i(1)$] kills $M_i$
- Running time of D on $1^n$ is:

  T(n+1) + (time to compute interval containing n)

- conclude **if** $T(n+1)=o(T'(n))$

then D in **NTIME(T'(n))** **QED**