

Brownian Motor

In his famous lectures Richard Feynman discussed the impossibility to violate the second law of thermodynamics by a *ratchet* mechanism. The simplest model for a ratchet is an overdamped Brownian particle in an asymmetric but spatially periodic potential (with asymmetry α and period L). Due to the fluctuating force caused by the pushing molecules of the surrounding fluid or gas the Brownian particle may overcome the potential barrier moving to the left or to the right. The probabilities for both directions are equal. Thus on average the particle does not move. Hence building a motor which turns thermal energy into mechanical work from a *single* heat bath is impossible.

But the ratchet can be turned into a so-called a *Brownian motor* that seems to violate the second law of thermodynamics. The idea is to turn the ratchet potential periodically on and off with a frequency $1/2$

. Under certain circumstances this may yield directed motion even against an applied force f . It is indeed a device doing work. Below you

can play with a Java applet which simulates such a system. Vary the parameters and figure out the condition for a drift from right to left even for $f > 0$. If you want to know why a Brownian motor isn't a *perpetuum mobile* of the second kind click on [here](#).

Sorry, no Java applet!

There are two possibilities why you don't see a Java applet:

1. Your browser is able to run Java applets but this feature is currently switched off.
2. Your browser is not able to run Java applets.

Instructions

- Parameters can be changed either by manipulating the corresponding scroll bar or by putting directly a number into the number field. There are the following parameters:
 N = number of particles.
 kT = thermal energy kT in units of the potential barrier.
 α = asymmetry α of the ratchet potential.
 τ = switching time τ .
 f = applied force f in units of the potential barrier divided by L .
- The start/stop button starts or stops the animation in the animation area.
- The reset button resets the particle positions.
- The animation area shows:
 1. *The result of the simulation.* If the number of particle is less than or equal 20 the particles are shown by circles. For a larger ensemble the particle positions are shown as a function of time (time direction is pointing upwards). The black line shows the averaged particle position. In the upper left corner the time and the averaged velocity (assuming $L = 1$) are shown.
 2. *The potential.* The actual potential is shown in black. When the saw-toothed part is switch off it is still shown in light gray. The potential is tilted due to the external force f .
 3. *The potential energy.* The averaged potential energy is shown as a green bar.

Why is a Brownian motor not a *perpetuum mobile* of the second kind?

As long as the ratchet potential is off the particle will move diffusively according to a (biased) random walk, leading to a variance in position of $\Delta x = \sqrt{2D\tau}$ and a mean position of $\langle x \rangle = f\tau/\gamma$, where $D = kT /$

γ is the diffusion constant. When the ratchet potential is switched on, the particle gets trapped in one of the potential minima. If

$\alpha L \approx \Delta x \approx (1 - \alpha)L$ for the variance holds, the particle on average gets trapped into the minimum left to the starting point. The maximum flux is obtained if the switching time τ is large enough to assure that the particle can adjust in the trapping minimum ('adiabatic adjustment time') and also is small enough to fulfill the above requirement for the variance. Roughly one can say that a netto flux to the left always occurs, when thermal energy is significantly smaller than the potential maximum, the external force is chosen not too big and the driving frequency matches the adiabatic adjustment time needed for the particle to move into a potential minima.

Where does the energy come from leading to a drift against the external force? The energy does not come from the heat bath but from the ratchet potential when it is switched on. At that moment the potential energy of the particle will be suddenly increased. In the simulation this can be seen by a sudden increase of the energy bar. But most of the energy pushed into the system will be just dissipated into the heat bath due to the relaxation of the particle into a potential minima. Only a tiny portion will be used for doing work. Thus a Brownian motor does not violate any law of thermodynamics it only turns one type of work into another one. Nevertheless the fluctuating force due to the heat bath is essential for a Brownian motor.

For more details and possible applications in biology and chemistry read the following review article:
 R.D. Astumian: *Thermodynamics and Kinetics of a Brownian Motor*, Science **276**, p. 917-922 (1997).

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I am a theoretical physicist working in nonlinear dynamics and physics of friction.

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Welcome to the Pendulum Lab! This is a [virtual laboratory](#) where you can do hands-on experiments at rigid pendula. Together with the material presented in the [lecture room](#), you can learn basic issues like harmonic oscillator and resonance but also advanced topics like parametric resonance, nonlinear dynamics, and chaos. Playing with the pendula in the lab, you only need curiosity and a browser which can run Java applets. However, to understand the mini-lecture, it is helpful to have some basic knowledge of calculus.

If you want to play with the Pendulum Lab offline, you are welcome to download the whole lab:

- [zipped \(348 Kb\)](#)
- [tarred and gzipped \(302 Kb\)](#)

[The Lab](#)

[The Lecture Room](#)

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