Whiplash PCR Applied to solve NP search Problems

- Erik Winfree [1998]

Branching Programs

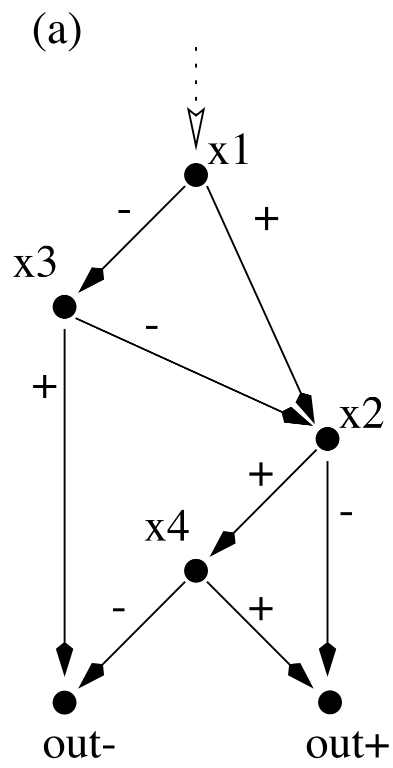

Figure 1: (a) A branching program for computing the μ-formula $(x_1 \vee \overline{x_3}) \wedge (\overline{x_2} \vee x_4)$. A possible input would be $x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 1$, which leads to output $^+$. The computation follows a path through the diagram, and thus can only access variables in the order prescribed. (b) A branching program which does not correspond to a μ-formula.

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Winfree's Whiplash PCR:

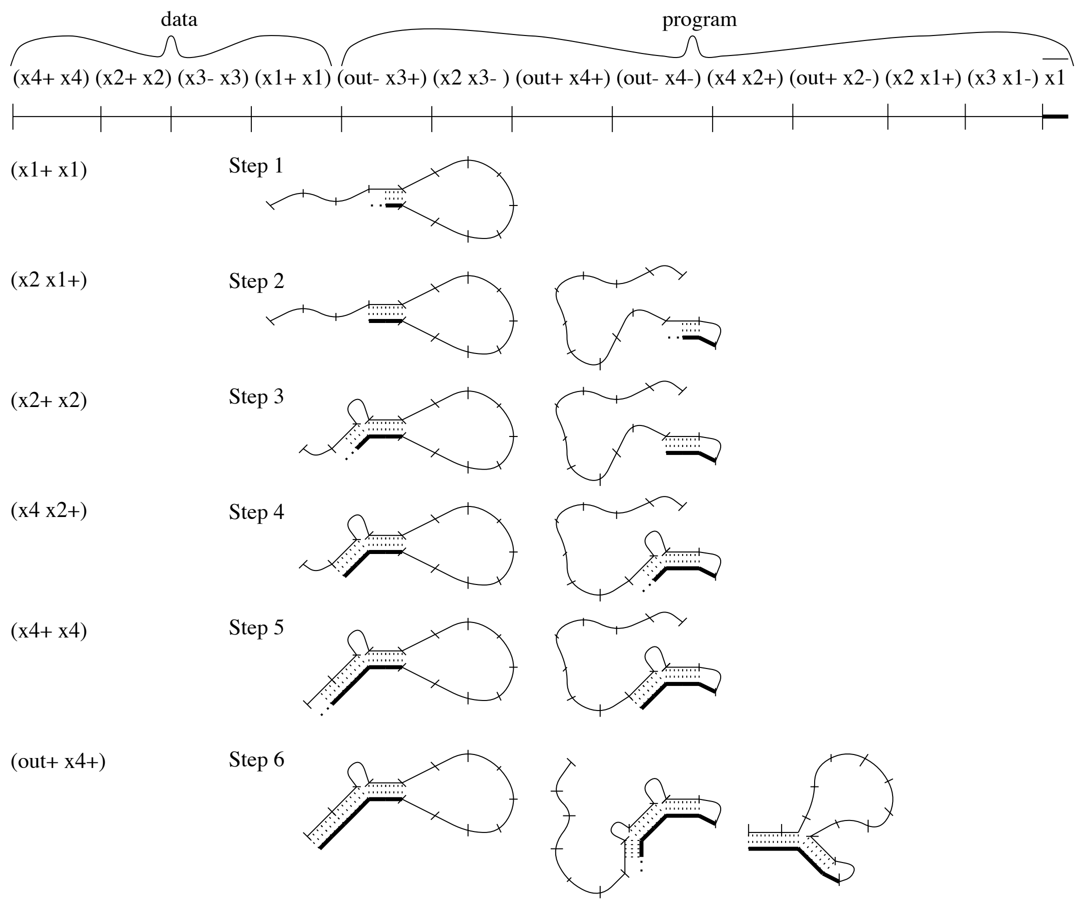

Figure 2: Probable secondary structures during the computation of the μ-formula $(x_1 \vee \overline{x_3}) \wedge (\overline{x_2} \vee x_4)$ on the input 1101. "Probable" is in the mind of the artist. Note that the tick marks denote the **stop** sequence; because the $3'$ head sequence will never contain the complement to the **stop** sequence, this will be the site of a small bulge in regions that are shown as double-stranded.

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Assembly Graph derived from Branching Programs for Evaluating Boolean Formula

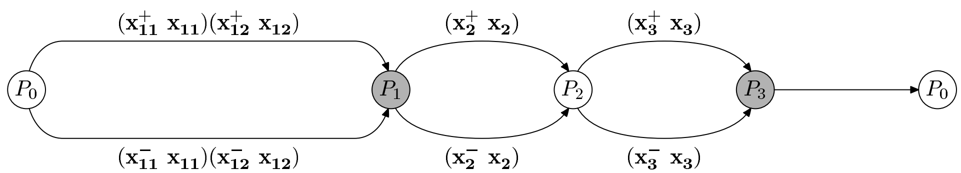

Figure 3: An assembly graph for generating input to the formula $(x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_3)$. Up to $2n+1$ oligos are required, and additional symbols P_i are used. For convenience, the node P_0 is written twice. Since there will be a restriction site in $\mathbf{P}_0$, this results effectively in paths from the leftmost node to the rightmost.

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Deriving GOTO program to Evaluate Boolean Formula

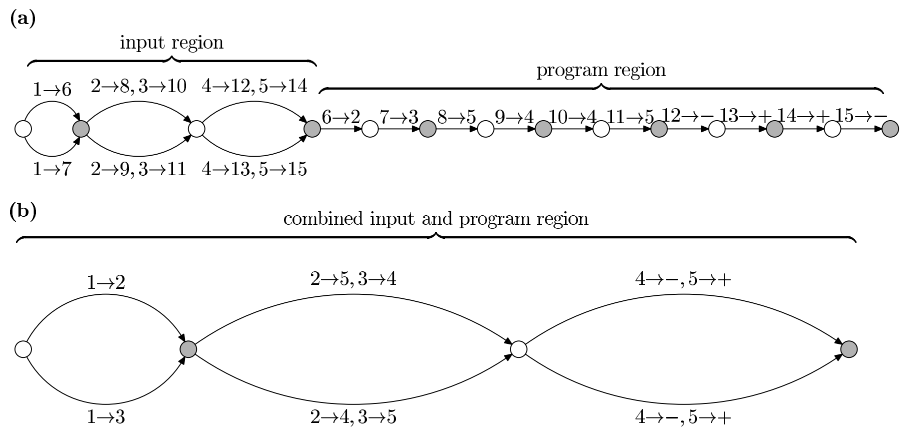

Figure 4: Reducing BP-SAT to GG-SAT: the $n = 3, \hat{n} = 5$ example. (a) The direct construction, combining the assembly graph from Figure 3 and the μ-formula program for $(x_{11} \vee \overline{x_2}) \wedge (\overline{x_{12}} \vee x_3)$. (b) The optimized construction obtained by following GOTO statements in the fixed region of (a). All GOTO programs are of length 5.

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Whiplash PCR

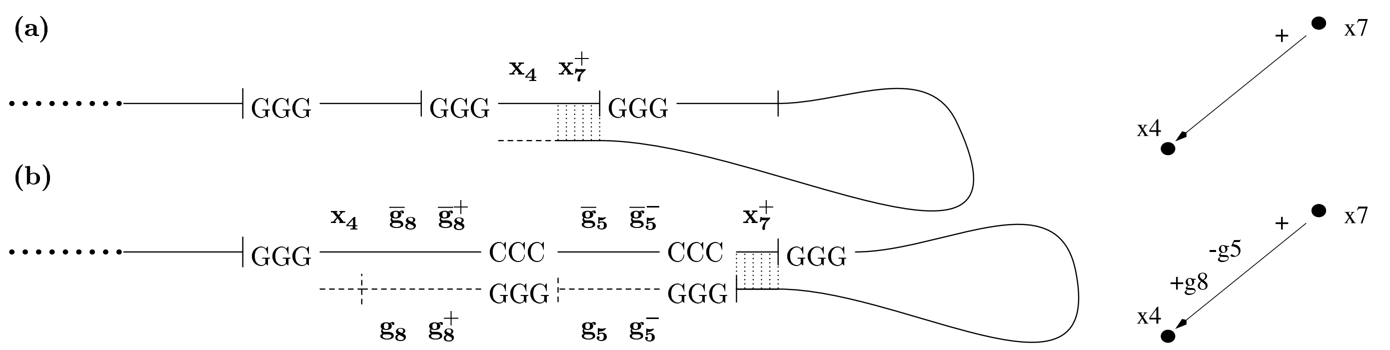

Figure 7: (a) The polymerization stop step on a standard frame, where a single symbol is copied, and its representation as an edge in a BP. (b) The polymerization stop step on an enhanced frame, where two hidden frames are made active, and its representation as an edge in a WOBP.

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GOTO program for Independent Set Problem

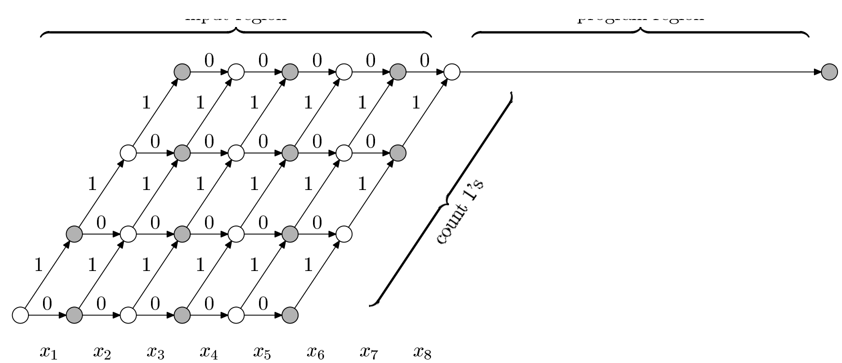

Figure 5: A GOTO graph for solving the Independent Set Problem. Inputs are generated in which exactly $k = 3$ out of $n = 8$ variables have value 1. The edge labels "0" and "1" in column i are shorthand for GOTO statements setting the value of variable x_i; as in FSAT, variables which are referenced more than once in the formula must be duplicated, and the corresponding edges in the graph will be labelled with more than one GOTO statement. Note that concentration ratios of the oligos could be adjusted to make all paths equally likely (for ligation-based assembly, at least; it is not so clear for assembly PCR).

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GOTO program for Hamiltonian Path Problem

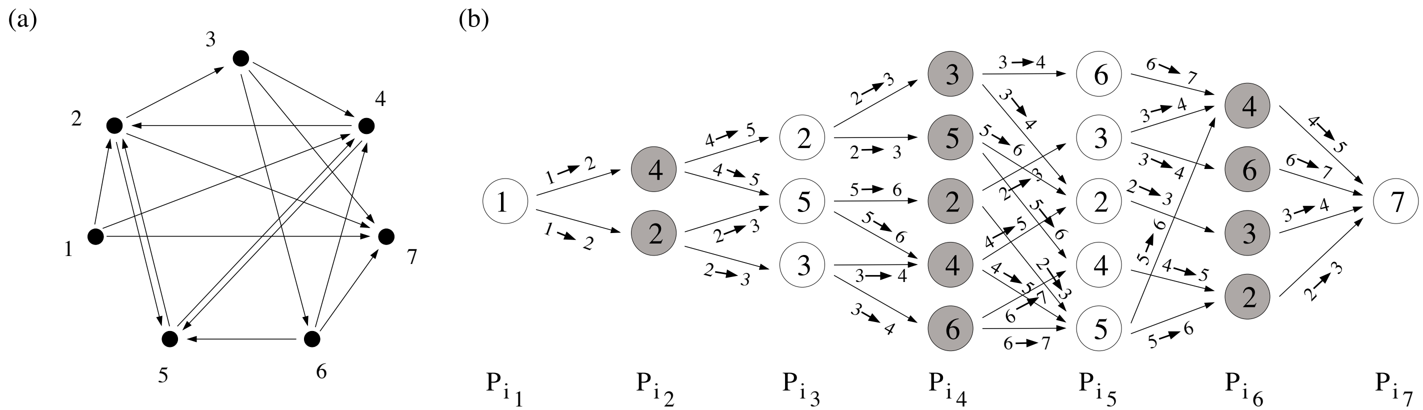

Figure 6: Solving the Hamiltonian Path Problem: A graph G (a) and its corresponding GOTO graph GG (b). This is Adleman's example with 2 additional edges added to prevent pruning from simplifying the GOTO graph to triviality. For convenience the nodes show only the vertex index i, and not the full symbol P_{i_k}.

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